STUDY OF THE LOW-PASS AND HIGH-PASS, FIRST- AND SECOND ORDER ACTIVES FILTERS

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Abstract
The author has realized studies of the low-pass, high-pass, band-pass and band-rejection active filter with operational amplifiers. The studies has included building and simulation on PC this circuits realized with operational amplifiers. This paper presented the work above low-pass and high-pass first- and second order filters. Those are presented electrical schemes of the filters and the graphical results of the simulation and the conclusions after the comparison between the simulation and the physical circuits.

Keywords
low-pass high-pass first- and second order filters, operational amplifier

1. Above active filters theory

1.1 Low-pass filter
The general transfer function of a low-pass filter is:

\[ A(s) = \frac{A_0}{\prod_i (1 + a_is + b_is^2)}; \]  \hspace{1cm} (1)

with \( A_0 \) being the pass band gain.

The filter coefficients distinguish between types and order filters [1].

For a first-order filter, \( b_i = 0 \):

\[ A(s) = \frac{A_0}{1 + a_is}; \] \hspace{1cm} (2)

and for a second-order filter:

\[ A_2(s) = \frac{A_0}{(1 + a_is + b_is^2)}; \] \hspace{1cm} (3)
The higher-order filter is built with cascading connection of first-order and/or second-order filters.

The transfer function of the First-Order Noninverting Low-Pass Filter (fig. 1) is:

\[
A(s) = \frac{1 + \frac{R_2}{R_3}}{1 + \omega_0 R_1 C_1 s} ; \tag{4}
\]

with the relations between the parameters:

\[
A_0 = 1 + \frac{R_2}{R_3} ; \tag{5}
\]

\[
a_1 = \omega_0 R_1 C_1 ; \tag{6}
\]

The coefficient \(a_1\) is taken from the tables [1].

To dimension the circuit, specify the corner frequency \(f_c\), the DC gain \(A_0\) and capacitor \(C_1\) and then solve for resistor \(R_1\) and \(R_2\):

\[
R_1 = \frac{a_1}{2 \pi f_c C_1} ; \tag{7}
\]

\[
R_2 = R_3(A_0 - 1) ; \tag{8}
\]

The transfer function of Second Order Unity-Gain Sallen-Kay Low-Pass Filter (fig. 3) is:

\[
A(s) = \frac{1}{1 + \omega_c C_1 (R_1 + R_2) s + \omega_c^2 R_1 R_2 C_1 C_2 s^2} ; \tag{9}
\]

with the relations between the parameters:

\[
A_0 = 1
\]

\[
a_1 = \omega_c C_1 (R_1 + R_2) ; \tag{10}
\]

\[
b_1 = \omega_c^2 R_1 R_2 C_1 C_2 ; \tag{11}
\]

Given \(C_1, C_2\) with the condition:

\[
C_2 \geq C_1 \frac{4 b_1}{a_1^2} ; \tag{12}
\]

The resistor values for \(R_1\) and \(R_2\) are calculated through:

\[
R_{1,2} = \frac{a_1 C_2 \pm \sqrt{a_1^2 C_2^2 - 4 b_1 C_1 C_2}}{4 \pi f_c C_1 C_2} ; \tag{13}
\]
1.2 High-pass Filter

By replacing the resistors of the low-pass filter with capacitors and the capacitors with resistors a high-pass filters created.

The general transfer function of a high-pass filter is then:

\[
A(s) = \frac{A_0}{\prod_i \left(1 + \frac{a_i s + b_i}{s^2}\right)}; \quad (14)
\]

The transfer function of a single stage is:

\[
A(s) = \frac{A_0}{1 + \frac{a_i s}{s}}; \quad (15)
\]

for a First-Order Noninverting High-Pass Filter, and:

\[
A(s) = \frac{A_0}{\left(1 + \frac{a_i s + b_i}{s^2}\right)}; \quad (16)
\]

for a Second Order Sallen-Kay Unity-Gain High-Pass Filter.

The transfer function of the First-Order Noninverting High-Pass Filter (fig.5) is:

\[
A(s) = \frac{sC R_{R} R_{R}}{sC_1 1 + \omega_c + \omega_c \omega_c}; \quad (17)
\]

with the relations between the parameters:

\[
A_\infty = 1 + \frac{R_2}{R_3}; \quad (18)
\]

\[
a_1 = \frac{1}{\omega_c R_1 C_1}; \quad (19)
\]

To dimension the circuit, specify the corner frequency \(f_c\), the DC gain \(A_0\) and capacitor \(C_1\) and then solve for resistor \(R_1\) and \(R_2\):

\[
R_1 = \frac{1}{2\pi f_c a_i C_1}; \quad (20)
\]

\[
R_2 = R_3(A_\infty - 1); \quad (21)
\]

The transfer function of Second Order Unity-Gain Sallen-Kay High-Pass Filter (fig.7) is:
\[ A(s) = \frac{1}{1 + \frac{2}{\omega_c R_1 C} \frac{1}{s} + \frac{1}{\omega_c^2 R_1 R_2 C^2} \frac{1}{s^2}}; \quad (22) \]

with the relations between the parameters

\[ a_1 = \frac{2}{\omega_c R_1 C}; \quad (23) \]
\[ b_1 = \frac{1}{\omega_c^2 R_1 R_2 C^2}; \quad (24) \]

Given \( C \), the resistor values for \( R_1 \) and \( R_2 \) are calculated through:

\[ R_1 = \frac{1}{\pi f_c C a_1}; \quad (25) \]
\[ R_2 = \frac{a_1}{4 \pi f_c C b_1}; \quad (26) \]

2. CONCLUSIONS

Electrical schemas are presented in fig.1, fig.3, fig.5, fig.7. For the simulated schemas the solutions can be visualized in fig.3, fig.5, fig.7, respectively fig.8. For real schemas the graphical characteristics were obtained from point by point method.

The conclusions presented have been obtained in conditions when the values of the real components differed with maximal 2% of the values from simulated circuits.

Four principal conclusions are:
- characteristics gain-frequency are very appropriate for the two cases
- real filter can present some suprise or decrease and oscillates amortized on the front an high-pass filter and an low-pass filter (which not appear through the simulated program). This excluding to be difficulty.
- for any situation gain value has sensible differed for the two cases.
- Electronics Workbench program can be utilized simple with successful in the study of active filters.
Fig. 1 First-Order Noninverting Low-Pass Filter

Fig. 2 Graphical solutions after the circuit’s from fig1 simulation on PC
Fig. 3 Second Order Unity-Gain Low-Pass Filter

Fig. 4 Graphical solutions after the circuit’s from fig.3 simulation on PC
Fig. 5 First-Order Noninverting High-Pass Filter

Fig. 6 Graphical solutions after the circuit’s from fig.5 simulation on PC
3. REFERENCES

1. Thomas Kugelstadt - Active Filter Design Techniques, 2001, SUA