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CONSIDERATION ON THE RELATION CAUSE – EFFECT AT THE MEASUREMENT DEVICE FOR MECHANICAL SIZES BASED ON THE ELASTIC ELEMENTS

CARDEI Petru, JERCALEANU Chiriac,
INMA Bucharest, E-mail icsit@sunu.rnc.ro

Abstract

This paper propose some reflection about the measuring of the mechanical sizes process (strength, moments,), using elastic elements (flexors) in the measuring devices. For a good measurement, generally, is theoretically required a certain type of dependence between the state physical sizes of the elastic element and the loads of the system, which we try to measure. There is possible as the dependence between the state physical sizes of the elastic element and the loads of the system to be a one - to - one function? If not, there is a restriction of the function between the loads and the state physical size, where this function is a one - valued function? How is implicate in this problem the principle of Saint - Venant and the equation of the elastic continuous media? These are only few questions at the paper try to find some partial answers.

Keywords: *cause, effect, measurement*

INTRODUCTION

In the direct problem of the elasticity there are given the body geometry and material properties, the limit conditions, the initial conditions (if the problem is dynamic) and the loads. The equation of the linear elasticity gives the solution: the displacements vector and the strain and stress tensors. In the case of this paper we have the inverse problem.

On the elastic system is measured the strain (and by calculus, the stress) in a certain number of system locations. The question is if there is some relation between the physical and geometrical characteristics of the elastic system and the stress calculated (on the basis of the measured strain in certain locations) which can gives the interaction components.

THE DEVICE FOR THE MEASUREMENT OF THE INTERACTION

The geometry of the device for the measurement of the interaction parameters (forces and moments) is shown in the figure 1.

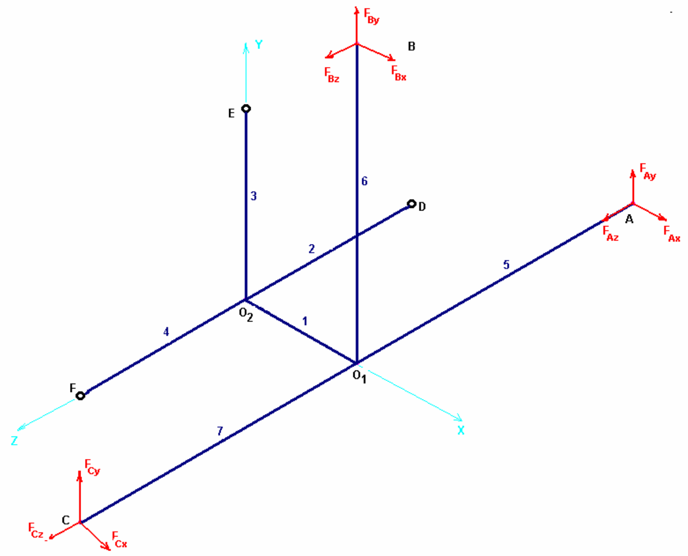

Fig. 1 *The scheme of the interaction measurement device.*

The points where the device is coupled at the tractor are D, E, F and the points where the device is coupled at the farm machinery are A, B, C. Be *L* the length of the beams AO_1, BO_1 and CO_1, *l* the length of the beams DO_2, EO_2 and FO_2, *d*, the length of the beam O_1O_2. I suppose that in the points A, B, C action the strength:

$$\vec{F}_A = F_{Ax}\vec{i} + F_{Ay}\vec{j} + F_{Az}\vec{k},\ \vec{F}_B = F_{Bx}\vec{i} + F_{By}\vec{j} + F_{Bz}\vec{k},\ \vec{F}_C = F_{Cx}\vec{i} + F_{Cy}\vec{j} + F_{Cz}\vec{k}\ , \tag{1}$$

where $\vec{i}, \vec{j}, \vec{k}$, are the unit vectors of the axes O_2x, O_2y, O_2z. The point O_2 is the origin of the axes system. In these conditions, the components of the resultant strength and resultant moment are (according to [3] or [4]) :

$$F_x = F_{Ax} + F_{Bx} + F_{Cx},\ \ F_y = F_{Ay} + F_{By} + F_{Cy},\ \ F_z = F_{Az} + F_{Bz} + F_{Cz},. \tag{2}$$

Then the longitudinal stress (see [1]) has the next expression:

$$\sigma(x,y,z) = \frac{F_x}{A} + \frac{y(d-x)F_y}{I_z} - \frac{z(d-x)F_z}{I_y} + \frac{zL(F_{Cx} - F_{Ax})}{I_z} - \frac{yLF_{Bx}}{I_y} + \frac{L(F_{Ay} + F_{Bz} - F_{Cy})}{I_p} r\ , \tag{3}$$

where *x* is the coordinate along the beam and *y* and *z* are the coordinates in the plane of the cross section of the beam (see figure 2) and $r=(y^2+z^2)^{1/2}$, I_y and I_z are the moment of inertia of the beam cross section and I_p is the polar moment of the beam cross section. Between the moments of inertia there is the relation $I_p=I_y+I_z$. For simplicity I suppose that $I_y = I_z = I$.

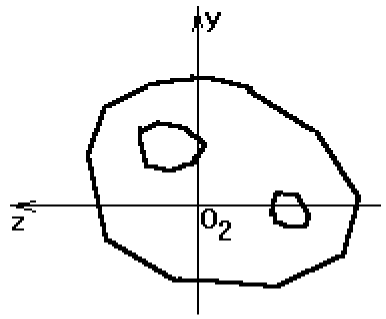

Fig. 2 *The coordinates in the beam cross section.*

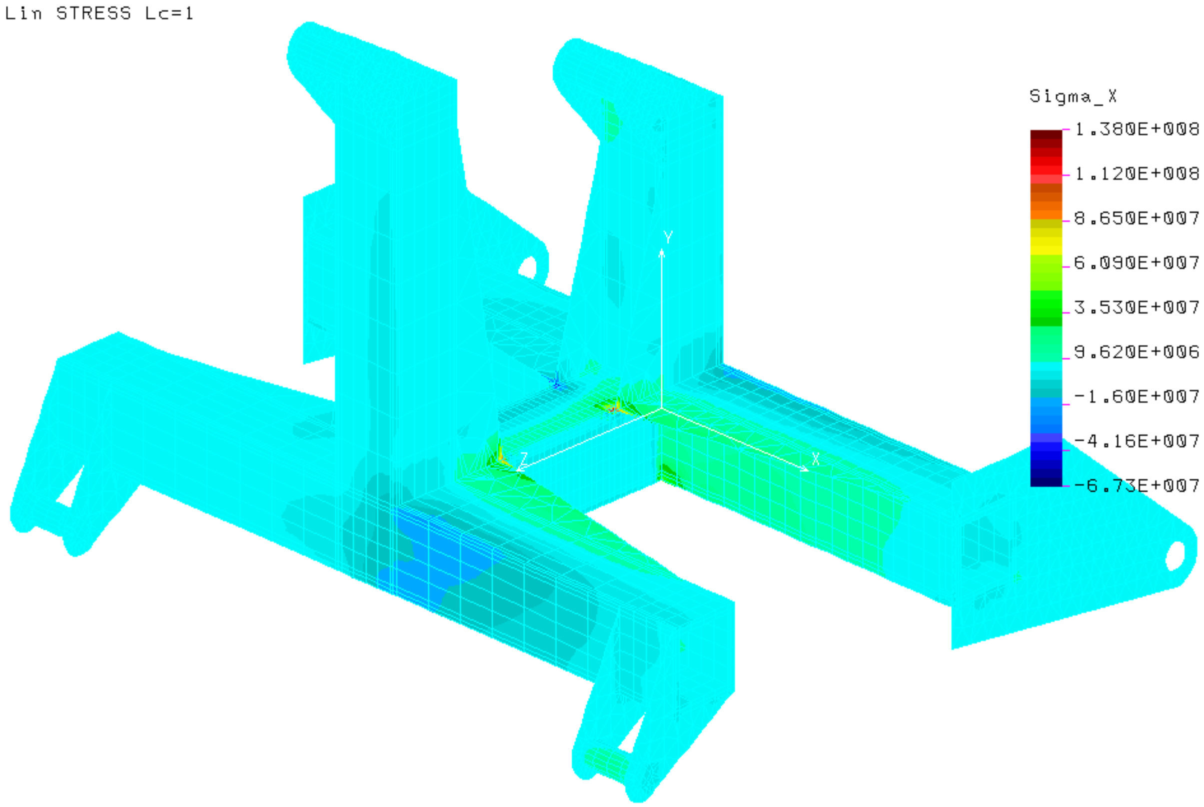

Fig. 3 *The measurement system and the location of the six strain gauge.*

The 3-dimensional real geometry of the measurement device is shown in the figure 3. We suppose placed six strain gauges in the six measurement points on the central beam . If we write down the variables:

$$u_1 = \frac{F_x}{A}, u_2 = \frac{F_y}{I}, u_3 = \frac{F_z}{I}, u_4 = \frac{L(F_{Ax} - F_{Cx})}{I}, u_5 = \frac{L}{I} F_{Bx}, u_6 = \frac{LF_{Ay} + LF_{Bz} - LF_{Cy}}{I_p}, \quad (4)$$

and we write the relation (3) for each measurement point, *i*= 1,… ,6, then is obtaining next system of linear equations:

$$u_1 + (d - x_i) y_i u_2 - (d - x_i) z_i u_3 + z_i u_4 - y_i u_5 + \sqrt{y_i^2 + z_i^2}\, u_6 = \sigma_{mi}, \quad i = 1, ..., 6, \quad (5)$$

where σ_{mi} is the stress calculated starting from the measured strain (with the strain gauge) in the point *i* (*i*= 1,… ,6). The matrix form of the system (5) is:

$$Tu = \sigma_m, \quad (6)$$

where:

$$T = \begin{pmatrix} 1 & (d-x_1)y_1 & -(d-x_1)z_1 & z_1 & -y_1 & \sqrt{y_1^2+z_1^2} \\ 1 & (d-x_2)y_2 & -(d-x_2)z_2 & z_2 & -y_2 & \sqrt{y_2^2+z_2^2} \\ 1 & (d-x_3)y_3 & -(d-x_3)z_3 & z_3 & -y_3 & \sqrt{y_3^2+z_3^2} \\ 1 & (d-x_4)y_4 & -(d-x_4)z_4 & z_4 & -y_4 & \sqrt{y_4^2+z_4^2} \\ 1 & (d-x_5)y_5 & -(d-x_5)z_5 & z_5 & -y_5 & \sqrt{y_5^2+z_5^2} \\ 1 & (d-x_6)y_6 & -(d-x_6)z_6 & z_6 & -y_6 & \sqrt{y_6^2+z_6^2} \end{pmatrix}, \ u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix}, \ \sigma_m = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}. \quad (7)$$

Then the solution there is and is unique if and only if det T $\neq$0. In this case the solution is:

$$u = T^{-1}\sigma_m . \qquad (8)$$

Using the relations (4) is obtained:

$$F_x = u_1 A, F_y = u_2 I, F_z = u_3 I, F_{Cx} - F_{Ax} = u_4 \frac{I}{L}, F_{Bx} = u_5 \frac{I}{L}, F_{Ay} + F_{Bz} - F_{Cy} = u_6 \frac{I_p}{L}. \qquad (9)$$

and finally, with (2):

$$F_{Ax} = \frac{Au_1 - \frac{I}{L}(u_5 + u_4)}{2}, F_{Cx} = \frac{Au_1 - \frac{I}{L}(u_5 - u_4)}{2}. \qquad (10)$$

The formula (3) is valid only for the central zone of the beam O_1O_2 (see figure 1 and 3), thus we try to locate the measurement points in this zone, where the boundary effects are slow .

The main disadvantage of this procedure for the calculus of the interaction between the tractor and the farm machinery is the high sensibility of the linear operator at the errors of the experimental strain (or stress).

The experimental error has a component which results from the difference between the simple solution (3) and the real solution (the real solution obtained solving the elasticity equations).

It is possible to consider that these errors are equal for any measurement location. In this case, the numerical study shows that the strength error is equal with the measured stress error. But, if, for example, at least one of measured stresses errors differs from the others and even only by sign, not necessarily so in absolute value, then the strength errors increase very much.

For example, if the errors of the measured strain give the calculated stress value error for all location, 0.1 % then the strength error is, also, 0.1 %, but if the errors of the measured strain are 0.1% for five of the strain gauges and -0.1% for the sixth strain gauge, then the strength error is 12.85%. These aspects are in study for the moment. We know that, for multiple connected domains and concentrated strengths there is theorems of oneness for the solution (see [2] and [5]).

In certain conditions results if that the strength fields are slowly modified, then the stress state is also slowly modified. Unfortunately we cannot obtain an analytical solution for our problem and then the numerical solution is the unique solution that we can obtain. This solution is the reference solution. With this solution we can simulate the process of measurement of the interaction between the tractor and farm machinery.

In this conditions we use for the calculus a method based on the experimental (simulation) results. Using the experimental results is possible to obtain functions which give the loads if we know the longitudinal stresses on the beam O_1O_2 .

SIMULATION OF THE MEASURING. RESULTS, PRECISION

For example we consider a measurement device like this plotted in the figure 1, with next characteristics: d= 0.3 m, L= 0.5 m, l= 0.3 m, S= 0.1 m, all the beams of the measurement device are square pipe with the 0.39 m side and 0.008 m thickness (see figure 3). The coordinates of the strain gauge location on the beam O_1O_2, are given in the table 1.

***Table 1** Measurement location coordinates and the theoretical longitudinal stresses.*

Element level	**x, m**	**y, m**	**z, m**
1540	0.175000	0.024000	-0.04000
987	0.182714	-0.008000	0.04000
986	0.182714	0.008000	0.04000
1539	0.175000	0.008000	-0.04000
988	0.182714	-0.024000	0.04000
1537	0.175000	-0.024000	-0.04000
985	0.182714	0.024000	0.04000
699	0.175000	0.040000	0.04000

Using the simulation data, is obtain next linear function for the loads:

$$\begin{aligned}
&F_x(\sigma_{z1540},\sigma_{z987},\sigma_{z986},\sigma_{z1539},\sigma_{z988},\sigma_{z1537},\sigma_{z985},\sigma_{z699}) = 0.0566\cdot\sigma_{z1540} - \\
&35.8466\cdot\sigma_{z987} - 55.5556\cdot\sigma_{z986} - 32.066\cdot\sigma_{z1539} - 0.8866\cdot\sigma_{z988} + 0.1981\cdot\sigma_{z1537} + \\
&63.8283\cdot\sigma_{z985} - 61.1098\cdot\sigma_{z699}; \\
&F_y(\sigma_{z1540},\sigma_{z987},\sigma_{z986},\sigma_{z1539},\sigma_{z988},\sigma_{z1537},\sigma_{z985},\sigma_{z699}) = 0.0257\cdot\sigma_{z1540} - \\
&16.3015\cdot\sigma_{z987} + 25.2643\cdot\sigma_{z986} + 14.5822\cdot\sigma_{z1539} + 0.4032\cdot\sigma_{z988} - 0.0901\cdot\sigma_{z1537} - \\
&29.0263\cdot\sigma_{z985} + 27.7901\cdot\sigma_{z699}; \\
&F_z(\sigma_{z1540},\sigma_{z987},\sigma_{z986},\sigma_{z1539},\sigma_{z988},\sigma_{z1537},\sigma_{z985},\sigma_{z699}) = 0.6187\cdot\sigma_{z1540} + \\
&894.0309\cdot\sigma_{z987} + 651.8893\cdot\sigma_{z986} + 1180.6579\cdot\sigma_{z1539} + 8.6551\cdot\sigma_{z988} + 0.8084\cdot\sigma_{z1537} - \\
&199.5535\cdot\sigma_{z985} - 139.5446\cdot\sigma_{z699}
\end{aligned} \tag{11}$$

The functions (11) are calculated using the least – squares method with experimental (numerical experiments) data. The performance of this method can be estimated using the table 2.

***Table 2**. Some results of the (11) formula.*

The real resultant loads, N			The calculated resultant loads, N		
F_x	F_y	F_z	F_x	F_y	F_z
431	-196	925	433	-197	943
431	-196	1735	431	-196	1781
431	-196	2235	429	-195	2234
431	-196	1475	433	-197	1452

CONCLUSIONS

The main conclusion of this stage of the research is that the dependence between the loads and the body (the measurement device) stresses is not a globally one – valued function.

Then we try to find a local one – valued dependence between the loads and the body stresses and the local three-dimensional interval will be theoretically estimated. The experimental data processing will made using the result of this estimation (function like (11)).

REFERENCES

[1] Hute, Engineer's Handbook, Technical Printing House, Bucharest, 1995;
[2] Lazăr Dragoş, Principles of Mechanics of the Continuous Media, Technical Printing House, Bucharest, 1983;
[3] Nicolae Posea, Material Strength, Didactic and Pedagogical Printing House, Bucharest, 1979;
[4] Victor Drobota, Material Strength, Didactic and Pedagogical Printing House, Bucharest, 1982;
[5] W. Kecs, P.P. Teodorescu, Introduction into the Theory of Distributions with Applications in Technique, 1975;