ANNALS OF THE FACULTY OF ENGINEERING HUNEDOARA
2004, Tome II, Fascicole 1

THE ANALYTICAL STUDY ABOUT THE STATE OF STRESS FOR CONTACT BETWEEN THE STEEL ROPE'S WIRES

VÎLCEANU Lucia*, GHITA Eugen**

* Engineering Faculty of HUNEDOARA, ROMANIA
** Mechanical Engineering Faculty of TIMIŞOARA, ROMANIA

Abstract

Examining the wires of an out-of work steel rope, they noticed that the local compression stress in between the wires had such high values that it produced an impression on the wire in the contact area. The contact stress appears between the wires of the same rope strand, of two adjacent rope strands and between the wires and the rope sheave.

The level of stress is function of the load, the geometry of the common contact surface and the Young's modulus. The practical observations are confirmed by the analytic calculations on the basis of Hertz's theory.

Keywords: local compression, contact stress, Hertz's theory.

1. Introduction

In order to perform a calculus at the contact between two rope's wires there is necessary to know the curvature radius and the contact forces both between wires and between a wire and the roll in contact. The distribution of forces on a bended wire around a roll is presented in Fig.1, where r is the curvature radius of the rope strand.

For the case of a wire rope composed from n_t rope strands, the average contact force on a wire has to be calculated with the formula

$$P_0 = \frac{4L\delta}{n_t D\ d} F \tag{1}$$

If a wire rope is wrapped up around a roll and is subjected to traction force F, then the interaction forces V and K actuate between the rope strands. The analytic relations in order to estimate the interaction forces are:

$$V = \frac{2\pi^2 (d - d_1)}{n_t L \sqrt{L^2 + \pi^2 (d - d_1)^2}} F \tag{2}$$

$$K = \frac{V}{2\cos\gamma} \tag{3}$$

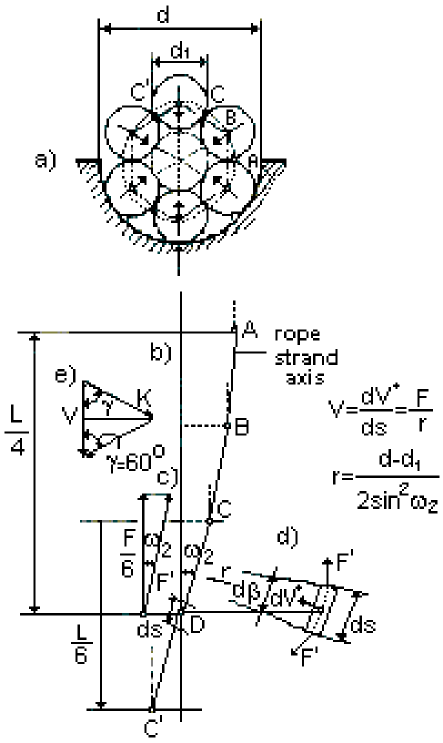

__Fig.1.__ Forces distribution on a bended wire around a roll

The pressing force on the wires belonging to a rope strand when the wire rope is subjected to traction is:

$$P_{0s} = \frac{\sin\omega_1 \cdot \mathrm{tg}\omega_1}{2n_t \sin\omega_2 \cos\gamma} \cdot \frac{\delta}{d - d_1} F \tag{4}$$

If the helix angles in a rope strand or a wire rope have the same value ($\omega_1 = \omega_2$), there will be obtained:

$$P_{0s} = \frac{\pi}{2n_t \cos\gamma} \cdot \frac{\delta}{L} F \tag{5}$$

For the characteristic case when $n_t = 6$ rope strands, $L = 7{,}5d$; $L' = 10d_1$; $\omega_1 = 15^0 40'$; $\omega_2 = 17^0 28'$; $\gamma = 60^0$; $d - d_1 = 2d/3$, according to (2), (3) and (5) the results will be:

$$V = K = \frac{1}{26{,}5d} F \quad \text{and} \quad P_{0s} = \frac{1}{15{,}9} \cdot \frac{\delta}{d} F \tag{5,a}$$

The interaction force between wires in the contact point or on the contact generatrix, for the case when the wire rope is wrapped on around a roll with the diameter D, will have the analytic expression:

$$P_0 = \frac{F \cdot \delta}{2n_t \sin\omega_1 \cdot \cos\gamma} \left[\frac{n_t - 1}{D} + \frac{\sin\omega_2 \cdot \mathrm{tg}\omega_2}{d - d_1} \right] \tag{6}$$

For the usual values $n_t = 6$; $L = 7{,}5d$; $L' = 10d_1$; $\omega_1 = 15^0 40'$; $\omega_2 = 17^0 28'$; $d - d_1 = 2d/3$; $\gamma = 60^0$, there will be obtained

$$P_0 = F \cdot \delta \left[\frac{2{,}8}{D} + \frac{1}{15{,}9d} \right] \tag{6,a}$$

2. The calculus of the contact pressure

The Hertz's theory is used to calculate the maximum p_0, respectively the average contact pressure p_m, in the framework of the elastic range:

$$p_0 = -\frac{3\cdot\sqrt[3]{P_0(E\cdot\Sigma\rho)^2}}{4\pi(\eta\chi)} = -\frac{3P_0}{2\pi ab} \tag{7}$$

$$p_m = -\frac{\sqrt[3]{P_0(E\cdot\Sigma\rho)^2}}{2\pi(\eta\chi)} \tag{8}$$

where: P_0 - the pressing force between the two bodies in contact, [N];

a, b – the half-axis of the elliptic contact patch, [mm];

E^* - the equivalent Young's modulus, $\frac{1}{E^*} = \frac{1}{2}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$ [MPa];

E_1, E_2 - the Young's modulus of the two bodies in contact, [MPa];

$\Sigma\rho$ - the amount of main curvatures of the surfaces in contact in the near vicinity of the contact point, [mm^{-1}]

$$\Sigma\rho = \rho_{11} + \rho_{12} + \rho_{22} + \rho_{21} = \Sigma\frac{1}{R_i} \tag{9}$$

The above-mentioned relationships may be adjusted (Fig.2) for the case of a wire belonging to a wire rope:

$$\rho_{11} = -\frac{1}{R_{11}};\quad \rho_{12} = \frac{1}{R_{12}};\quad \rho_{21} = \frac{1}{R_{21}};\quad \rho_{22} = -\frac{1}{R_{22}},$$

where: R_{11} – the curvature radius of the deformed shape of the wire rope, [mm];

R_{12} – the nominal radius of the wire rope, [mm];

R_{21} – the radius of the take-up roller, [mm];

R_{22} – the guidance radius of the take-up roller, [mm].

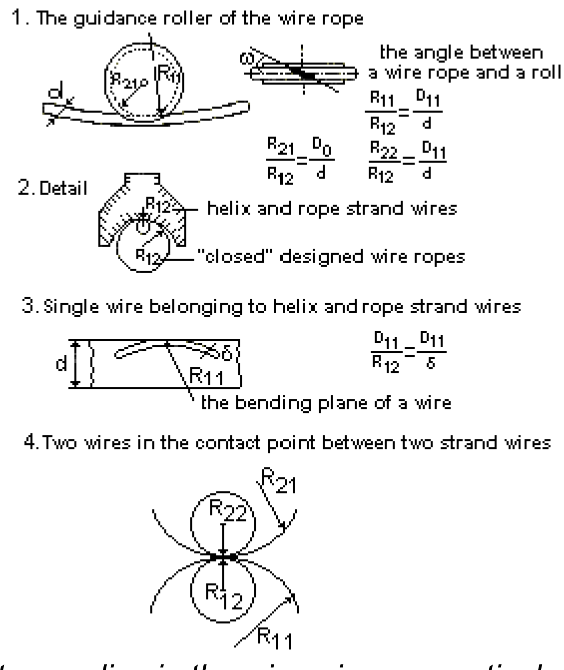

Fig.2. *The main curvature radius in the wire-wire, respectively wire-roll contact areas*

If the following notations are used: d – the diameter of the wire rope, [mm]; δ - the diameter of the wire, [mm]; D_{11} - the diameter of the deformed shape of the wire rope, [mm]; D_i – the diameter of the wire rope guidance, [mm]; D – the diameter of the wire rope take-up roller, [mm], the following mathematical relation will be obtained:

$$\Sigma\rho = 2\left[\frac{1}{D_{11}} + \frac{1}{\delta} + \frac{1}{D} + \frac{1}{D_i}\right] = \frac{2}{\delta}\left[1 + \frac{\delta}{D_{11}} + \frac{\delta}{D} + \frac{\delta}{D_i}\right] \tag{10}$$

The value ($\eta\chi$) is graphically obtained from the Fig.3, in function of the angle θ, where θ is defined by the relation

$$\cos\theta = \frac{\sqrt{(\rho_{11}-\rho_{12})^2 + 2(\rho_{11}-\rho_{12})(\rho_{21}-\rho_{22})\cos 2\omega + (\rho_{21}-\rho_{22})^2}}{\Sigma\rho} \tag{11}$$

In the relation (11) it was noticed ω, the angle between the axis of the wire rope or of the component wire and the symmetry plane of the roller (Fig.2).

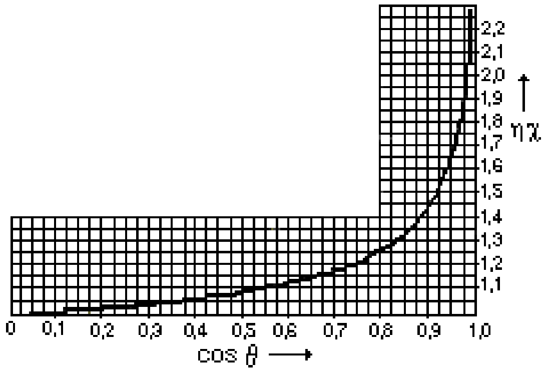

Fig.3. *The value ($\eta\chi$) in function of the angle θ*

For a general case, it will be obtained:

$$\cos\theta = \frac{\sqrt{\left(\frac{1}{D_{11}} - \frac{1}{\delta}\right)^2 + 2\left(\frac{1}{D_{11}} - \frac{1}{\delta}\right)\left(\frac{1}{D} - \frac{1}{D_i}\right)\cos 2\omega + \left(\frac{1}{D} - \frac{1}{D_i}\right)^2}}{\frac{1}{D_{11}} + \frac{1}{\delta} + \frac{1}{D} + \frac{1}{D_i}} \tag{12}$$

By using the notation

$$M = 1 + \frac{\delta}{D_{11}} + \frac{\delta}{D} + \frac{\delta}{D_i}, \tag{13}$$

for the case $\omega = 0$, the maximum pressure will became

$$p_0 = -\frac{0{,}38}{\eta\chi} \cdot \sqrt[3]{E^2} \cdot \sqrt[3]{M^2} \cdot \sqrt[3]{\frac{P_0}{\delta^2}} \tag{14}$$

Finally, the following relation will be obtained

$$p_0 = -k_3 \cdot \frac{\sqrt[3]{M^2}}{\eta\chi} \cdot \sqrt[3]{P_0} \tag{15}$$

For the external layer wires in contact with the guidance in the roll, the value of the coefficient k_3 is dependent by the character of the material of the guidance roll, in accordance with the Table 1, [3].

The value of the coefficient $\boldsymbol{k_3}$ **Table 1**

Material of the roll	Steel	Cast iron	Hard rubber	Lining	
				Leather	Wood
k_3	280	230	13	2,8	7

3. The analytic relations to calculate the contact pressure for the traction wire ropes composed by rope strands

By using the relations (7), (9), (11), adjusted for the same type of wire rope, the following relations to calculate the contact pressure will be obtained for different contact geometries:

a) The contact between the wires and the roll

a.1) – Cross knitted wire ropes ($\omega = 0$)

$$p_0 = -\frac{280}{\eta\chi}\cdot\sqrt[3]{\left(1+\frac{\delta\sin^2\omega_1}{d_1}+\frac{\delta}{D}-\frac{\delta}{D_i}\right)^2}\cdot\sqrt[3]{\frac{P_0}{\delta^2}} \qquad (16)$$

and

$$\cos\theta = \frac{1}{M}\left|1-\frac{\delta\sin^2\omega_1}{d_1}-\frac{\delta}{D}-\frac{\delta}{D_i}\right| \qquad (17)$$

The usual values for the quantities between brackets are presented in Table 2.

The usual values for wire ropes **Table 2**

Wire ropes with six rope strands	d_1/δ	$d_1/\sin^2\omega_1\delta$		D_i/δ	D/δ	
		$L'=10d_1$	$L'=8d_1$		Roll	Puttee
7 wires in a rope	3	33	22	10...12	80…200 (100)*	500...1000
19 wires in a rope	5	55	37	18...22		500...1000
37 wires in a rope	7	78	52	23...25		500...1000

* - for parallel knitted wire ropes

a.2) - Parallel knitted wire ropes ($\omega = 27^0 ... 30^0$)

The analytic expression of the pressure p_0 remain as in the mentioned above case, but the value of $\cos\theta$ has a different one:

$$\cos\theta = \frac{1}{M}\sqrt{\left(1-\frac{\delta\sin^2\omega_1}{d_1}\right)^2 - 1{,}2\left(1-\frac{\delta\sin^2\omega_1}{d_1}\right)\left(\frac{\delta}{D}+\frac{\delta}{D_i}\right)+\left(\frac{\delta}{D}+\frac{\delta}{D_i}\right)^2} \qquad (18)$$

The usual values for the quantities between brackets are found in the Table 2, and the values of the average contact force $\mathbf{P_0}$ has to be calculated according to the formula (1).

b) The contact between two component wires of a wire rope

For the both knitting methods (parallel, respectively in cross), the following mathematical relations for p_0 and $\cos\theta$ will be obtained:

$$p_0 = -\frac{444}{\eta\chi}\cdot\sqrt[3]{\left(1+\frac{\delta\sin^2\omega_1}{d_1}\right)^2}\cdot\sqrt[3]{\frac{P_0}{\delta^2}} \qquad (19)$$

and
$$\cos\theta = \left[\frac{1-\dfrac{\delta\sin^2\omega_1}{d_1}}{1+\dfrac{\delta\sin^2\omega_1}{d_1}}\right]\cos\omega \tag{20}$$

The interaction force P_0 in the formula (19) will be calculated according to the formula (6).

4. Conclusions

After the level of the normal stresses in the wire is calculated, the equivalent normal stress will be obtained according to one of the strength theories. The mathematical expressions of the main stresses in the contact patch between two bodies will be, [2]:

$\sigma_1 = \dfrac{2+\nu\dfrac{b}{a}}{\nu\left(1+\dfrac{b}{a}\right)}p_0$ - the longitudinal stress along the main axis of the contact ellipse,

$\sigma_2 = \dfrac{2\dfrac{b}{a}+\nu}{\nu\left(1+\dfrac{b}{a}\right)}p_0$ - the transversal stress along the secondary axis of the contact ellipse,

$\sigma_3 = -\dfrac{3P_0}{2\pi ab} = -p_0$ - the vertical stress along the normal direction on the contact patch.

In accordance with the shape modifying strength theory, the equivalent stress will be, [1]
$$\sigma_{echV} = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1} \tag{21}$$

For the particular cases of wire ropes or even wires in contact, the maximum contact pressure p_0 is accompanied by the traction stress σ_t and the bending stress σ_i. The bending stress σ_I has a negative value because on the contact area between the wire rope and the roll, the bending became compression. So, the main stress along the wire is
$$\sigma_1 = \frac{2+\dfrac{b}{a}\nu}{\nu\left(1+\dfrac{b}{a}\right)}p_0 + (\sigma_t - \sigma_i) \tag{22}$$

The researches of A. Dumas [3] conduce to the value of the average accepted pressure $p_{0\max} = \dfrac{HBS}{2}$, where HBS is the hardness of the surface which has been estimated according to the Brinell's procedure, or $p_{o\max} \cong 2\sigma_r$, where $\boldsymbol{\sigma_r}$ is the ultimate strength of the wire material.

REFERENCES:

[1] Babeu, T. – The Basic Theory of Strength of Materials, Mirton Publishing House, Timisoara, 1998.

[2] Ghita, E. – Strength and working life-time of solid bodies in contact, Mirton Publishing House, Timisoara, 2000.

[3] Vîlceanu, L. – Contributions in the analysis of the influence of the local compresion effect about the working life-time of the component wires in a wire-rope, Ph.D. Thesis, Defended at POLITEHNICA University of Timişoara, 2001.