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CONSIDERATIONS UPON THE MOTION OF THE POROUS MATERIALS WITH ELASTIC SKELETON

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Abstract

The porous materials are used in the attenuation of the sound waves.

In the paper are established the differential equations of motion for porous materials with metallic skeleton.

We obtain relations that allow the determination of porous material impedances and the absorption coefficients of sound for different porous materials applied on the acoustic screens.

Keywords: *motion, porous materials, elastic skeleton.*

1. THE EQUATIONS OF MOTION

Let us consider a porous material with cylindrical open pores having radius equal to R, placed parallel on the length of the material. The air friction force with viscosity ν at laminar flowing through an element of the material with transversal section 1 cm^2 and thickness dx, according to Poiseuille law, is

$$P_f = \sigma_P \cdot v_P \cdot h \cdot dx \tag{1}$$

where

$$\sigma_P = \frac{8 \cdot \nu}{R^2} \tag{2}$$

is the resistance of the air in the pores of the material, $v_P = \frac{Q}{h \cdot t}$ - the linear speed of the air on the length of the pore's axle, Q – the flow of the air through the pores, t – the time necessary for passing air and h – the porosity.

The expression *(1)* of friction force of the air from the walls of the pores is also available for non uniform and non cylindrical pores and non laminar or oscillating flowing of the air, but the air resistance is not expressed anymore through the simple formula *(2).*

Let us consider a material with parallel pores of different diameters situated under the angle θ as to the normal at the surface of the material from which we isolate an element 1 cm^2 in section and dx in thickness.

In order to ease the following deductions, suppose that all the pores came together; the projection of their surface on the plan perpendicular to the normal equals h (fig. 1).

By replacing a few parallel pores with the equivalent pore we are influencing the value of the friction force. This force is the one given by the formulas *(1)* and *(2)*. The inertness forces against (on) the element and the air contained in it did not change.

Suppose that on the surface of the isolated element of the skeleton and of the air, there are operating the sonorous pressure p_s and p_a. On the unit of the surface of the material will operate the forces $p_1 = p_s(1-h)$ and $p_2 = p_a \cdot h$

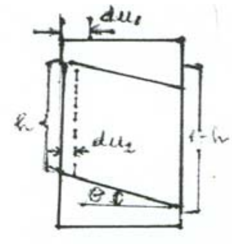

Fig 1.

We shall use the following notations: ρ_1 for the density of the skeleton of the material and v_1 for its speed in direction of the normal at the surface of the material; in that case, the inertness force corresponding to the element of the skeleton is $\rho_1 \cdot dX \cdot \frac{\partial v_1}{\partial t}$.

The air also takes part in two motions: one portable with an acceleration equal with the acceleration of the skeleton $\frac{\partial v_1}{\partial t}$ and a relative motion with relative acceleration $\frac{\partial v_p}{\partial t} = \frac{1}{\cos\theta} \cdot \frac{\partial (v_2 - v_1)}{\partial t}$ where v_p is the linear speed of the air on the length of the axis of the pore in its relative motion, and v_2 represents the linear speed of the air on the direction of the normal at the surface of the element. The inertness forces corresponding to the air are equal to $\rho_2 \cdot dx \cdot \frac{\partial v_1}{\partial t}$ and $\rho_2 \cdot \frac{dx}{\cos\theta} \cdot \frac{(v_2 - v_1)}{\partial t}$ where $\rho_2 = p \cdot h$ is the mass of the air contained in the unit of the volume and ρ represent the density of the air in the atmosphere.

During the air motion in the pore also appears an interaction force between the air and the skeleton with the normal component P_n and the tangential one P_f.

Also on the volume cut from the skeleton operate the force T perpendicular on the ox axis in perpendicular direction on the normal, due to the deformation of the material (fig. 2).

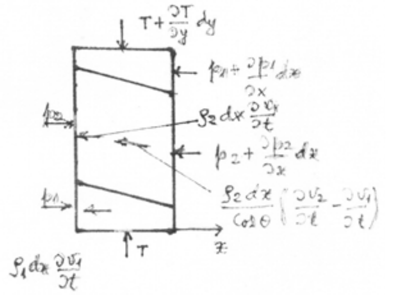

Fig. 2

In the same way we may consider separately the section of the skeleton of the volume element cut and the air contained in this volume which operate on it (fig. 3 and 4).

Projecting the forces which operate on the volume element cut from the porous material on ox axis (fig. 2) after making the reduction and simplifications necessary, we obtain

$$-\frac{\partial p_1}{\partial x}-\frac{\partial p_2}{\partial x}=\rho_1\cdot\frac{\partial v_1}{\partial t}+\rho_2\cdot\frac{\partial v_2}{\partial t} \tag{3}$$

Furthermore, by projecting on the axle of the posre the forces which operate on the air, situated in the volume element cut from the element (fig. 4) and using the expression *(1)* where $v_p=\frac{v_2-v_1}{\cos\theta}$ results

$$-\frac{\partial p_2}{\partial x}=\rho_2\cdot\frac{\partial v_2}{\partial t}+\rho_2\cdot(\varepsilon-1)\cdot\left(\frac{\partial v_2}{\partial t}-\frac{\partial v_1}{\partial t}\right)+\sigma\cdot h^2\cdot(v_2-v_1) \tag{4}$$

where $\varepsilon=\frac{1}{\cos^2\theta}$ is the structure factor and $\sigma=\sigma_p\cdot\frac{\varepsilon}{h}$ represents the resistance of the material faced up by the air which is different from the one established in the expression *(2)*. If the pores are not parallel and have a chaotic orientation, the structure factor has a more complex formula.

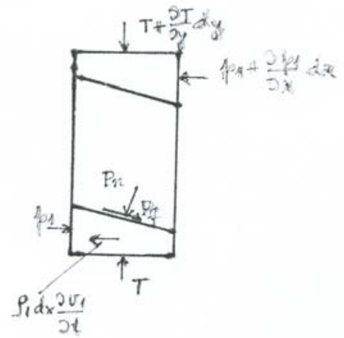

Fig. 3

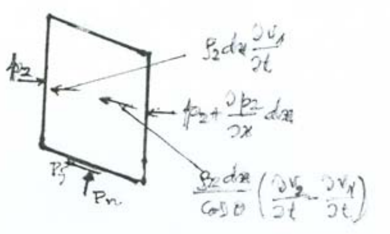

Fig. 4

The equation obtained can receive a symmetrical formula if we introduce the expression of $\frac{\partial p_2}{\partial x}$ from the equation *(4)* in the equation *(3)*

$$-\frac{\partial p_1}{\partial x}=\rho_1\cdot\frac{\partial v_1}{\partial t}+\rho_2\cdot(\varepsilon-1)\cdot\left(\frac{\partial v_1}{\partial t}-\frac{\partial v_2}{\partial t}\right)+\sigma\cdot h^2\cdot(v_1-v_2) \tag{5}$$

Two of the expressions *(3)-(5)* are the motion equations for porous materials with elastic skeleton.

These equations are universal because they are valid for stationary process and also for non stationary ones. For instance, if we consider $\frac{\partial}{\partial t}=i\cdot\varpi$ (for harmonic oscillation) the expressions *(4)-(5)* appear in the motion equation obtained by Zwikker, Kosten [4] and L. Beranek [1].

Beranek introduces in the equation *(1)* an additional term that estimates the damages at the friction in skeleton; the damages at the friction in the air are not take considered.

We can estimate easier the damages of the friction in the skeleton and in air through the complex modules of elasticity.

2. THE CONTINUITY EQUATIONS

Next, we will establish the continuity equations. For the porous material skeleton, based on Hooke's law, we may write $-p_1 = E_1 \cdot (\varepsilon_1 + \varepsilon_1^{'})$. Here $E_1 = E_s \cdot (1 + i \cdot \eta_s)$ is the complex module of elasticity E_s and η_s is the dynamic module of elasticity and the coefficient of the damage of the skeleton; ε_1 - the deformation of the skeleton in the direction of the ox axis; $\varepsilon_1^{'}$ - the supplementary deformation on the same axis determined by the air pressure.

For the various porous materials (glass wool, mineral wool, polymer etc.) due to the fact that the elasticity module of the air is a lot less significant than the elasticity module of the material used to produce the skeleton of the porous materials we may consider $\varepsilon_1 >> \varepsilon_1^{'}$. Subsequently, $-p_1 = \varepsilon_1$ and by differencing this equality in function of time we obtain

$$-\frac{\partial p_1}{\partial t} = E_1 \cdot \frac{\partial v_1}{\partial x} \qquad (6)$$

We can obtain the continuity equation for the air from the equation of status for the polytrophic process PV^n = const., where P and V represent the pressure and the volume, and n – the polytrophic index. From the equation of status we obtain the increase of pressure $-\Delta P = P \cdot n \cdot \frac{\Delta V}{V}$ where ΔV is the increase of the volume.

The increase of ΔP pressure equals the increase of sonorous pressure ΔP_a $\Delta P = \Delta P_a = \frac{\Delta P_2}{h}$.

Multiplying the air pressure by the polytrophic index leads to the complex module of air elasticity $P \cdot n = E_2 = E_a \cdot (1 + i \cdot \eta_a)$, where E_a and η_a are the dynamic module of elasticity and the damage coefficient for air. These relations lead to $-\Delta P_2 = E_2 \cdot h \cdot \frac{\Delta V}{V}$.

In the isolated element of the volume, the initial volume of air (fig. 1) is V = hdx.

If we consider that at the modification of the hydrostatic pressure the skeleton volume does not change in the initial conditions because the elasticity module of the air is much less significant than the elasticity module of the material, used to make the skeleton, then at the compression of the skeleton to the value Δu_1 and the air to the value Δu_2, the air volume in the element adjusts to the value $\Delta V = h \cdot \Delta u_2 + (1 - h) \cdot \Delta u_1$.

Replacing the values of volumes V and ΔV in expression of Δp_2, after dividing with Δt and effecting the limit for $\Delta t \to 0$ we obtain

$$-\frac{\partial p_2}{\partial t} = h \cdot E_2 \cdot \frac{\partial v_2}{\partial x} + (1-h) \cdot E_2 \cdot \frac{\partial v_1}{\partial x} \tag{7}$$

where $v_{1,2} = \frac{\partial u_{1,2}}{\partial t}$.

The expressions *(6)* and *(7)* represent the continuity equations for some porous materials (glass wool, mineral wool, polymer etc.). For other materials these equations are not so accurate,

3. THE INTEGRATION OF MOTION EQUATION

We will solve the equations *(4)-(7)* for the harmonic vibrations $\left(\frac{\partial}{\partial t} = i \cdot \varpi\right)$.

Introducing the expression *(6)* in formula *(5)* and the expression *(7)* in formula *(4)* we obtain a system of two equations

$$\frac{\partial^2 v_2}{\partial x^2} = \left[\frac{E_1}{h \cdot E_2} \cdot \frac{\rho_2 \cdot \varpi^2 - i \cdot \varpi \cdot s}{i \cdot \varpi \cdot s} - \frac{1-h}{h}\right] \cdot \frac{\partial^2 v_1}{\partial x^2} + \left[\frac{(\rho_1 \varpi^2 - i \cdot \varpi \cdot s)(\rho_2 \varpi^2 - i \cdot \varpi \cdot s) + \varpi^2 \cdot s^2}{i \cdot \varpi \cdot s \cdot h \cdot E_2}\right] \cdot v_1 \tag{8}$$

$$v_2 = -\frac{E_1}{i \cdot \varpi \cdot s} \cdot \frac{\partial^2 v_1}{\partial x^2} - \frac{\rho_1 \cdot \varpi^2 - i \cdot \varpi \cdot s}{i \cdot \varpi \cdot s} \cdot v_1 \tag{9}$$

where $s = i \cdot \varpi \cdot \rho_2 \cdot (\varepsilon - 1) + \sigma \cdot h^2$ is the coefficient which characterizes the link between the vibrations of the skeleton and of the air.

If we eliminate v_2 from the expression *(8)* and *(9)*, we obtain the equation

$$\frac{\partial^4 v_1}{\partial x} + A \cdot \frac{\partial^2 v_1}{\partial x^2} + B \cdot v_1 = 0 \tag{10}$$

where

$$A = \frac{\rho_1 \varpi^2}{E_1} - \frac{i \cdot \varpi \cdot s}{h \cdot E_1} + \frac{\rho_2 \cdot \varpi^2}{h \cdot E_2} - \frac{i \cdot \varpi \cdot s}{h \cdot E_2} \tag{11}$$

$$B = \frac{\rho_1 \varpi^2}{E_1} \cdot \frac{\rho_2 \cdot \varpi^2}{h \cdot E_2} - \frac{\rho_1 \cdot \varpi^2}{E_1} \cdot \frac{i \cdot \varpi \cdot s}{h \cdot E_2} - \frac{\rho_2 \cdot \varpi^2}{h \cdot E_2} \cdot \frac{i \cdot \varpi \cdot s}{E_1} \tag{12}$$

The solution of the equation *(10)* is

$$\varpi_1 = i \cdot \varpi \cdot \left(C_1 e^{\gamma_1 \cdot x} + C_2 e^{-\gamma_1 \cdot x} + C_3 e^{\gamma_2 \cdot x} + C_4 e^{-\gamma_2 \cdot x}\right) \cdot e^{i \cdot \varpi \cdot t} \tag{13}$$

where

$$\gamma_{1,2} = \sqrt{-\frac{A}{2} \mp \sqrt{\frac{A^2}{4} - B}} \tag{14}$$

From the continuity equation *(6)* as well as from the solution *(13)* we obtain

$$p_1 = E_1\left(-\gamma_1 \cdot C_1 e^{\gamma_1 \cdot x} + \gamma_1 \cdot C_2 e^{-\gamma_1 \cdot x} - \gamma_2 \cdot C_3 e^{\gamma_2 \cdot x} + \gamma_2 \cdot C_4 e^{-\gamma_2 \cdot x}\right) \cdot e^{i \cdot \varpi \cdot t} \tag{15}$$

From the expression *(9)* and *(13)* results

$$v_2 = \left(C_1 a_1 e^{\gamma_1 \cdot x} + C_2 a_1 e^{-\gamma_1 \cdot x} + C_3 a_2 e^{\gamma_2 \cdot x} + C_4 a_2 e^{-\gamma_2 \cdot x}\right) \cdot i \cdot \varpi \cdot e^{i \cdot \varpi \cdot t} \tag{16}$$

Finally from the continuity equation *(7)* and the expressions *(13)* and *(16)* we obtain

$$p_2 = E_2\left(-\gamma_1 \cdot b_1 \cdot C_1 e^{\gamma_1 \cdot x} + \gamma_1 \cdot b_1 \cdot C_2 e^{-\gamma_1 \cdot x} - \gamma_2 \cdot b_2 \cdot C_3 e^{\gamma_2 \cdot x} + \gamma_2 \cdot b_2 \cdot C_4 e^{-\gamma_2 \cdot x}\right) \cdot e^{i \cdot \varpi \cdot t} \tag{17}$$

where

$$\begin{aligned} b_1 &= a_1 \cdot h + (1-h) & \qquad b_2 &= a_2 \cdot h + (1-h) \\ a_1 &= 1 - \frac{E_1 \cdot \gamma_1^2 + \rho_1 \cdot \varpi^2}{i \cdot \varpi \cdot s} & \qquad a_2 &= 1 - \frac{E_1 \cdot \gamma_2^2 + \rho_1 \cdot \varpi^2}{i \cdot \varpi \cdot s} \end{aligned} \tag{18}$$

From the expressions *(13)-(17)* we observe that in the porous material with elastic skeleton, two waves propagate simultaneously with constants γ_1 and γ_2. These waves are coupled and they propagate in the game time in the skeleton as well as in the air.

The relations *(13), (15), (16)* and *(17)* allow the determination of porous materials impedances and absorption coefficients of sound for different porous materials applied on acoustic screens.

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