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THE NORMAL REPARTITION MODELING TRUNCATED

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RESUME

Cette note, elaborée en but didactique et de recherche, propose une réduction de la répartition normale pour l'obtention d'un modelage amélioré pour un monage de points. Dans la suite, on présente one application qui accompogne les considérations théoriques.

In this note, we propose a truncated of normal allotment law to obtain an optimal modeling for a cloud' points.

The normal law, having the distribution density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad x \in R, \qquad (1)$$

will be approximated with a function of the form

$$g(x, K(T)) = \begin{cases} K(T)\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}, & x \in [-T, T] \\ 0, & x \notin [-T, T] \end{cases} \qquad (2)$$

For the function $g(x, K(T))$ be a distribution density we have these conditions

$$K(T) > 0, \quad T > 0$$

and

$$\int_{-\infty}^{\infty} g(t, K(T))\,dt = 1, \qquad (3)$$

which determine the expression of the truncated density having the form

$$g(x, K(T)) = \begin{cases} \left(\int_{\frac{-T-m}{\sigma\sqrt{2}}}^{\frac{T-m}{\sigma\sqrt{2}}} e^{-t^2} dt\right)^{-1} \frac{1}{\sigma\sqrt{2}} e^{-\frac{(x-m)^2}{2\sigma^2}}, & x \in [-T, T]. \\ 0, & x \notin [-T, T] \end{cases} \qquad (4)$$

In figure 1 is graphic represented the functions f and g in comparison.

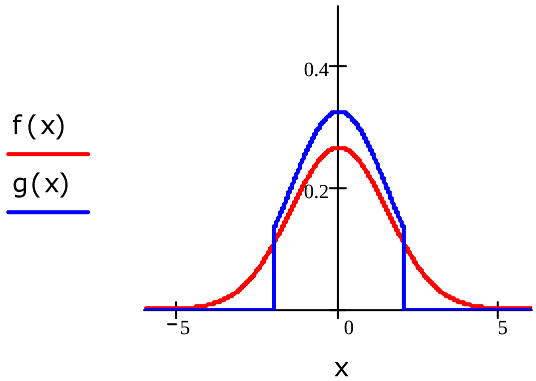


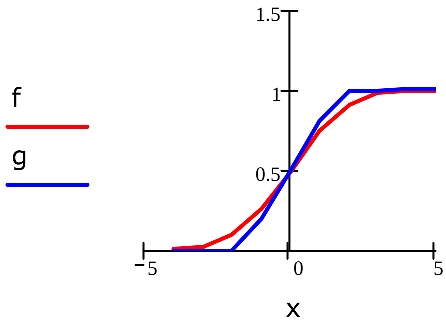
Figure 1

The corresponding distribution functions is represented in figure 2

$i := 1 .. 10$ $\qquad x_i := -4 + (i - 1)$ $\qquad T := 10$

$$F(x) := \int_{-20}^{x} f(x)\, dx \qquad f_i := F(x_i)$$

$$G(x) := \int_{-T}^{x} g(x)\, dx \qquad g_i := G(x_i)$$



Application I

In a practical application results the following table of values

$val^T =$

	1	2	3	4	5	6	7	8	9	10
1	0.208	0.269	0.286	0.319	0.328	0.353	0.359	0.369	0.378	0.393
2	0.757	0.752	0.751	0.747	0.646	0.744	0.643	0.742	0.541	0.739

where first line represented the values independent variable x1, and second line represented the values dependent variable yCT, to corresponding the practical problem.

To determine the values of T we impose the minimization of the function

$$fCt(T) := \sum_{i=1}^{n}\left[\left(\int_{\frac{-T-m}{\sigma\sqrt{2}}}^{\frac{T-m}{\sigma\sqrt{2}}} e^{-t^2}\,dt\right)^{-1}\cdot\frac{1}{\sigma\cdot\sqrt{2}}\cdot e^{-\frac{(x1_i-m)^2}{2\cdot\sigma^2}} - yCT_i\right]^2 \qquad (5)$$

and using the following program

ORIGIN ≡ 1

$n := 10 \qquad i := 1..\,n$

$x := (0.208\ \ 0.269\ \ 0.286\ \ 0.319\ \ 0.328\ \ 0.353\ \ 0.359\ \ 0.369\ \ 0.378\ \ 0.393)$

$x1 := x^T \qquad \sigma := 1.5 \qquad m := 0$

$yCT^T =$

	1	2	3	4	5	6	7	8	9	10
1	0.757	0.752	0.751	0.747	0.646	0.744	0.643	0.742	0.541	0.739

$$fCt(T) := \sum_{i=1}^{n}\left[\left(\int_{\frac{-T-m}{\sigma\sqrt{2}}}^{\frac{T-m}{\sigma\sqrt{2}}} e^{-t^2}\,dt\right)^{-1}\cdot\frac{1}{\sigma\cdot\sqrt{2}}\cdot e^{-\frac{(x1_i-m)^2}{2\cdot\sigma^2}} - yCT_i\right]^2$$

$Tinitt := 0.1 \qquad Tfint := 5 \qquad p := 5000$

$jt := 1..\,p+1$

$$T_{jt} := Tinitt + (jt-1)\cdot\frac{(Tfint - Tinitt)}{p}$$

$fCT_{jt} := fCt\left(T_{jt}\right)$

$fmin := \min(fCT) \qquad fmin = 0.044$

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indice := | for  jt ∈ 1.. p
          |     | ind ← jt
          |     | break  if  (fCT_jt − fmin) = 0
          | ind
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indice = 344

Using this program we obtain the values of T which minimization the function above

$T_{indice} = 0.7174 \qquad Tf := T_{indice}$

For this value of T results that deviation from the curve has this values

$$abatt := \sqrt{\frac{1}{n}\cdot\sum_{i=1}^{n}\left[\left(\int_{\frac{-Tf-m}{\sigma\sqrt{2}}}^{\frac{Tf-m}{\sigma\sqrt{2}}} e^{-t^2}\,dt\right)^{-1}\cdot\frac{1}{\sigma\cdot\sqrt{2}}\cdot e^{-\frac{(x1_i-m)^2}{2\cdot\sigma^2}} - yCT_i\right]^2} \qquad (6)$$

$abatt = 0.066$

We note that the function

$$g(x,K(T)) = \begin{cases} \left(\displaystyle\int_{\frac{-T-m}{\sigma\sqrt{2}}}^{\frac{T-m}{\sigma\sqrt{2}}} e^{-t^2} dt \right)^{-1} \dfrac{1}{\sigma\sqrt{2}} e^{-\frac{(x-m)^2}{2\sigma^2}} & , x \in [-T, T] \\ 0 \quad , & x \notin [-T, T] \end{cases}, \qquad (7)$$

represent a truncated of the normal allotment law, and the classic case is obtained for $T \to \infty$.

In the practical case these results show us, that is more advantage to use the truncated modeling, because the classical modeling is the practical case of these.

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