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INFLUENCE OF THE ELLIPSOMETRICAL READINGS WITH RESPECT TO THE OPTICAL CONSTANTS OF SURFACES

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ABSTRACT
The graphical representation of the variation of optical constants $\bar{n}$ *and* $\bar{k}$ *with respect to the ellipsometrical magnitudes* Δ *and* Ψ *allows a better understanding of the way in which ellipsometrical measurements can be used in determining the optical constants of solid or liquid surfaces.*
The precision of measuring the ellipsometrical magnitudes has a greater influence on the precision of determining the optical constants of surfaces when these optical constants have high values.

KEYWORDS:
ellipsometry, optical constants, refraction index, specular reflection.

1. INTRODUCTION

Ellipsometry is a highly precise and minute optical method of determining the optical properties of surfaces and superficial films developed on solid or liquid surfaces. With this aim we measure the change in the polarization state of a radiation reflected on the surface separating two optically different media. The modification in the polarization state of the reflected radiation is expressed by ellipsometrical magnitudes, which are experimentally measurable: Δ and Ψ. The ellipsometrical magnitude Δ expresses the modification in the phase of the reflected radiation, while ellipsometrical magnitude Ψ expresses the amplitude attenuation of the incident radiation after specular reflection [1,2].

The fundamental equation of ellipsometry:

$$\tilde{\rho} = \tan\Psi e^{i\Delta} = -\frac{\cos(\varphi_0 + \tilde{\varphi})}{\cos(\varphi_0 - \tilde{\varphi})} \tag{1}$$

and Snell's equation:

$$n_0 \sin\varphi_0 = \tilde{n}_s \sin\tilde{\varphi} \tag{2}$$

allow an expression of ellipsometrical magnitudes Δ ,Ψ and of the complex ratio $\tilde{\rho}$ depending on the relative refraction index of the reflecting surface with respect to the incidence medium:

$$\tilde{n} = \frac{\tilde{n}_s}{n_0} = \overline{n} - i\overline{k} \qquad (3)$$

One pair of values Δ, Ψ of the ellipsometrical magnitudes measured for the incidence angle φ_0 is enough for determining the optical constants $\overline{n}$ and $\overline{k}$ of the reflecting surface.

At a first stage, we calculate the value of the complex ratio $\tilde{\rho}$ out of the values of ellipsometrical magnitudes Δ and Ψ:

$$\tilde{\rho} = \tan\Psi\cos\Delta + i\tan\Psi\sin\Delta = \mathrm{Re}\,\tilde{\rho} + i\,\mathrm{Im}\,\tilde{\rho} \qquad (4)$$

We then calculate the complex refraction index of the reflecting surface out of the value of complex ratio $\tilde{\rho}$

$$\tilde{n} = \overline{n} - i\overline{k} = n_0 \tan\varphi_0 \sqrt{1 - \frac{4\tilde{\rho}\sin^2\varphi_0}{(1+\tilde{\rho})^2}} \qquad (5)$$

The graphical representation of the variation of optical constants $\overline{n}$, $\overline{k}$ and complex ratio $\tilde{\rho}$ with respect to the ellipsometrical magnitudes Δ and ψ allows a better understanding of the way in which ellipsometrical measurements can be used in determining the optical constants of solid or liquid surfaces.

2. THE GRAPHICAL EXPRESSION OF OPTICAL CONSTANTS $\overline{n}$ AND $\overline{k}$ WITH RESPECT TO ELLIPSOMETRICAL ANGLES Δ AND Ψ

Figures 1 and 2 presents the dependence of optical constants $\overline{n}$ and $\overline{k}$ on ellipsometrical magnitudes Δ and Ψ [3].

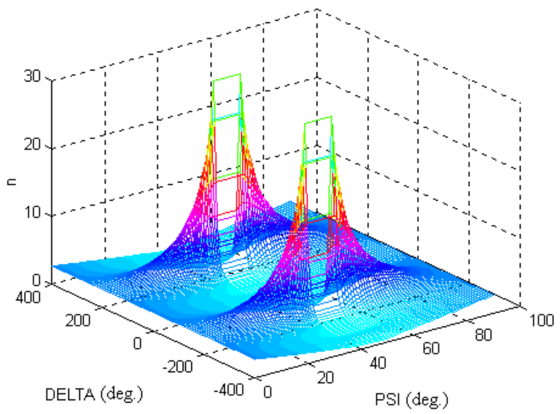

Fig. 1 Dependence of optical constants $\overline{n}$ on ellipsometrical magnitudes Δ and Ψ.

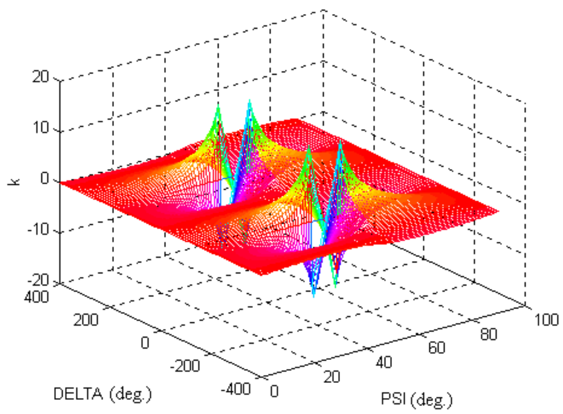

Fig. 2 Dependence of optical constants $\overline{k}$ on ellipsometrical magnitudes Δ and Ψ.

The analysis of relations (4) and (5), as well as that of figures 1 and 2 leads to the following interpretations:

- If $\Psi = 0$, both the real and imaginary part of complex ratio $\tilde{\rho}$ are equal to zero. As a result, the values of the optical constants are: $\overline{n} = n_0 \tan\varphi_0$ and $\overline{k} = 0$ (for any values of Δ).

- If $\Psi=\pi/2$, the value of the complex ratio $\tilde{\rho}$ tends to the infinite. In this case $\bar{n}=n_0\tan\varphi_0$ and $\bar{k}=0$ (for any values of Δ).
- If $\Psi=\pi/4$, the complex ratio has the expression:

$$\tilde{\rho}=\cos\Delta+i\sin\Delta \qquad (6)$$

and relation (5) expressing the refraction index will have the expression:

$$n=n_0\tan\varphi_0\sqrt{1-\frac{2\sin^2\varphi_0}{(1+\cos\Delta)}}=\bar{n} \qquad (7)$$

The refraction index is a real magnitude, optical constant $\bar{k}$ being equal to 0. In this case, the optical constant $\bar{n}$ is different from zero, when $\Delta\in\left(-(\pi-2\varphi_0);+(\pi-2\varphi_0)\right)$.

Figure 3 presents a detail of the central area of figure 1 in which $\varphi_0=70^0$, $\bar{n}=0$ when $\Delta=40^0$ and $\Psi=45^0$ (in accordance with the condition previously mentioned).

- For other values of Ψ, different from those previously mentioned, when $\Delta=0$ or $\Delta=\pi$, $\mathrm{Re}\,\tilde{\rho}=\rho$ (real value) and $\mathrm{Im}\,\tilde{\rho}=0$. The optical constants are $\bar{n}\neq 0$ and $\bar{k}=0$.

If $\Delta=0$, then $\mathrm{Im}\,\tilde{\rho}=0$ and $\mathrm{Re}\,\tilde{\rho}=\tan\Psi$. For any values of Ψ the optical constants are $\bar{k}=0$ and:

$$n=\tan\varphi_0\sqrt{1-\frac{4\tan\Psi\sin^2\varphi_0}{(1+\tan\Psi)^2}} \qquad (8)$$

Figure 4 show that for $\Psi=0$, $\pi/2$ or $\pi/4$ optical constant $\bar{k}=0$ (for any values of Δ).

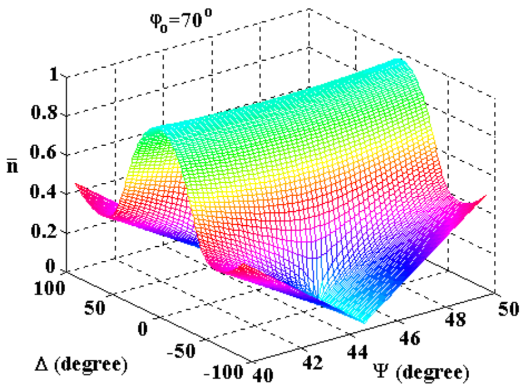

Fig. 3. A detail of the central area of figure 1

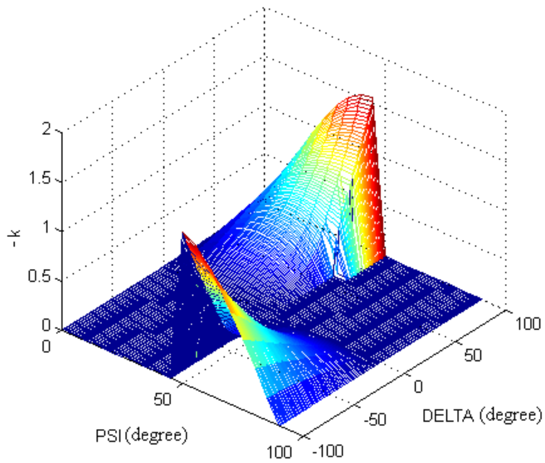

Fig. 4 Values of $\bar{k}$ are 0 for $\Psi=0$, $\pi/2$ or $\pi/4$

These results are summarized in table 1.

The analysis of Figure 1 shows that errors in the experimental determination of magnitudes Δ and Ψ influence to a large extent the error of determination for optical constants $\bar{n}$ and $\bar{k}$ when these optical constants have high values [4]. When determining the optical constants of surfaces, great errors appear when both ellipsometric magnitudes Δ and Ψ

are about the values $\Delta=\pi$ and $\Psi=\pi/4$. Great errors in determining $\bar{n}$ also appear when the ellipsometric angles have values next to $\Delta=\pm(\pi - 2\varphi_0)$ and $\Psi=\pi/4$, as it can be noticed in Figure 3.

Table 1

Δ	Ψ	$Re\,\tilde{\rho}$	$Im\,\tilde{\rho}$	$\bar{n}$	$\bar{k}$
any values	0	0	0	$n_0 \tan \varphi_0$	0
0	any values	$\tan \Psi$	0	relation (8)	0
0	$\pi/4$	1	0	$n_0 \sin \varphi_0$	0
$\pi/4$		$\sqrt{2}/2$	$\sqrt{2}/2$	relation (7)	
$\pi/2$		0	1	$n_0 \tan \varphi_0$	
0	$\pi/2$	∞	∞	$n_0 \tan \varphi_0$	0
$\pi/4$		∞	∞		
$\pi/2$		∞	∞		

3. Conclusions

The interpretation of the three-dimensional graphical representations of the optical constants dependence on ellipsometrical angles Δ and Ψ allows a better understanding of the way ellipsometrical measurements can be used in determining the optical constants of solid or liquid surfaces.

In the case of reflecting surfaces lacking superficial films, the measurable ellipsometrical angles Δ and Ψ used in determining the optical constants of the surfaces rank within the domain of values: $\Delta\in[0,\pi]$ and $\Psi\in[0,\pi/4]$. The errors of experimental measuring of the ellipsometrical angles Δ and Ψ have a stronger influence on the errors in determining the optical constants, especially when they have high values.

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