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CONSIDERATIONS IN PHYSICAL SURFACE ABOUT A FLUID MOTION IN BOUNDARY LAYER WITH SLIDING PHENOMENA

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ABSTRACT:
Considering a sliding boundary layer in the neighborhood of an unlimited plane plaque we study the velocity distributions in physical surface.

KEYWORDS: Sliding boundary layer, Polynomial velocity profile
AMS: 76D10;

1. INTRODUCTION

Let us envisage a fluid stream past a semi-infinite plane plaque with the "attack" angle zero. Suppose that the fluid is viscous incompressible while the flow is plane (in Oxy). The plane plaque is considered located on the real axis Ox, its "attack" edge being at O.

According to the well known boundary layer approximation (Prandtl), the Navier -Stokes equations

$$\rho\vec{v}.\nabla u = -\frac{\partial p}{\partial x} + \mu\Delta u,\ \ \rho\vec{v}.\nabla v = -\frac{\partial p}{\partial y} + \mu\Delta v,\ \ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

lead to

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

where ρ, p and $\vec{v}(u, v)$, are the mass density, the pressure and the plane velocity respectively while μ is the viscosity coefficient.

To these equations one attaches the boundary conditions

$$u(x,0) = L_1 \frac{\partial u}{\partial y}(x,0), \quad v(x,0) = 0, \quad u(x,\infty) = u_\infty, \tag{4}$$

the first one signifying the fact that the fluid slides on the plaque surface (in stead of the adherence on the plaque, i.e. of the classical non-slip condition $u(x,0) = 0$).

2. RESULTS

By using the expression of v which comes from (3), the above relation (2) leads to

$$\rho\left[u\frac{\partial u}{\partial x} - \left(\int_0^y \frac{\partial u}{\partial x}dy\right)\frac{\partial u}{\partial y}\right] = \mu\frac{\partial^2 u}{\partial y^2}. \tag{5}$$

Integrating this integral-differential equation across the boundary layer, namely from $y = 0$ to $y = \delta(x)$ - the upper border of the boundary layer, we get the integral relation

$$\rho u_\infty^2 \frac{d}{dx}\int_0^\delta \frac{u}{u_\infty}\left(\frac{u}{u_\infty} - 1\right)dy = -\tau_w, \tag{6}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w. \tag{7}$$

In a previous paper we have suggested the approach of this equation by using a velocity profile (within the boundary layer) of a polynomial form.

In this paper we solve this equation by considering again a velocity profile of a polynomial type (within the boundary layer) but this time of higher (5^{th}) degree which seems to be a more accurate approach.

Precisely we suppose that

$$\frac{u}{u_\infty} \equiv \bar{u} = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + a_5\eta^5, \quad 0 \le \eta \le 1, \tag{8}$$

and

$$\frac{u}{u_\infty} \equiv \bar{u} = 1, \quad \eta \ge 1 \text{, where } \eta \equiv \frac{y}{\delta(x)}. \tag{9}$$

The coefficients a_i can be determined by using the appropriate conditions

$$\bar{u} = L\frac{\partial \bar{u}}{\partial \eta}, \quad \frac{\partial^2 \bar{u}}{\partial \eta^2} = 0, \quad \text{for} \quad \eta = 0, \tag{10}$$

$$\bar{u}=1, \qquad \frac{\partial \bar{u}}{\partial \eta}=0, \qquad \frac{\partial^2 \bar{u}}{\partial \eta^2}=0, \qquad \frac{\partial^3 \bar{u}}{\partial \eta^3}=0, \quad \text{for } \eta=1, \tag{11}$$

where $L=\frac{L_1}{\delta(x)}$.

Following the calculations, by using the 5th degree polynomial approximation, it results for the non dimensional profile of the horizontal component of the velocity the expression

$$\bar{u}=\frac{1}{2+5L}\left(5L+5\eta-10\eta^3+10\eta^4-3\eta^5\right). \tag{12}$$

The thickness of the impulse losses

$$\theta=\delta\int_0^1 \frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\eta, \tag{13}$$

has in this case the following form

$$\theta=\frac{\delta}{(2+5L)^2}\left(\frac{495L+80}{198}\right). \tag{14}$$

The local tension between two neighboring layers

$$\tau=\mu\left(\frac{\partial u}{\partial y}\right), \tag{15}$$

has the expression

$$\tau=\frac{5\mu\ u_\infty}{\delta\,(2+5L)}\left(1-6\eta^2+8\eta^3-3\eta^4\right). \tag{16}$$

The local stress on the plaque has the structure

$$\tau_w=\frac{5\mu\ u_\infty}{\delta(2+5L)}. \tag{17}$$

Replacing then the velocity (12) and the local stress on the plaque (17), in the integral relationship (6), it comes out that

$$\rho\ u_\infty^2\ \frac{d}{dx}\left[\frac{\delta}{(2+5L)^2}\left(\frac{495L+80}{198}\right)\right]=\frac{5\ \mu\ u_\infty}{\delta\,(2+5L)}, \tag{18}$$

from where, by integrating, we get

$$\delta(x)=2\ \sqrt{(2+5L)\frac{495}{80+459L}\ \frac{\mu\ x}{\rho\ u_\infty}}, \tag{19}$$

due to $\delta(0)=0$.

The mechanical considerations about dynamical phenomena's in physical surface its obtains from analytical equations in plan (x, η), considering the relation

$$y = \eta \cdot \delta(x) \tag{20}$$

thus

$$y = \eta \cdot 2\sqrt{(2 + 5L)\frac{495}{80 + 459L}\frac{\mu\ x}{\rho\ u_\infty}} \tag{21}$$

In Figure 1 the profile of the non dimensional velocity in physical surface is sketched while in Figure 2 the influence of the L parameter on the velocity's profile, as in physical surface, is presented.

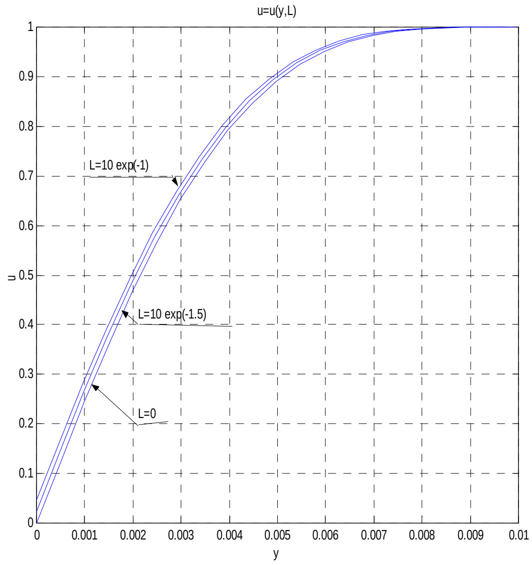

Figure 1. Profile of the non dimensional velocity $\bar{u}$.

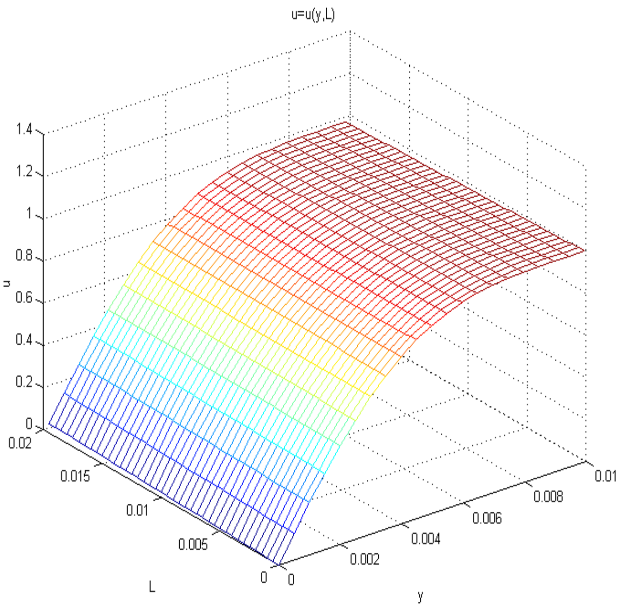

Figure 2. The influence of the L parameter on the velocity's profile

In Figure 3, in the section x=ct, the profile of the local stress between two neighboring layers, in physical surface functions of y and L, is represented.

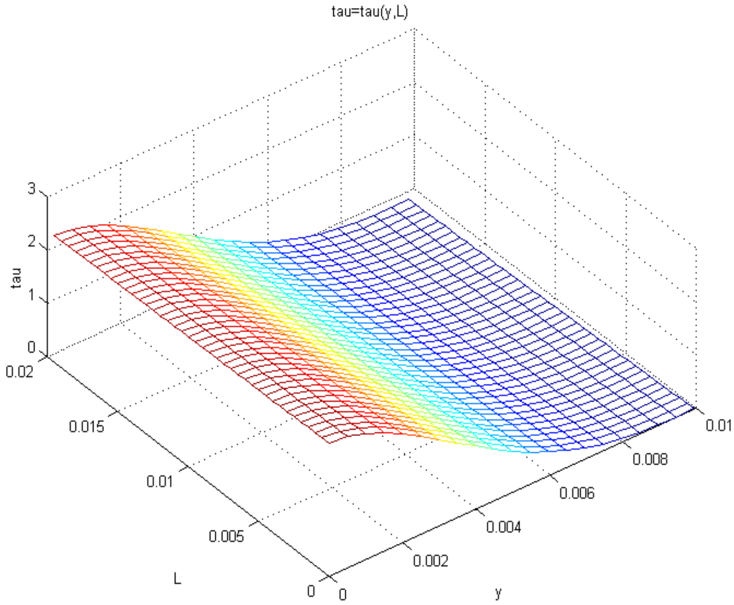

Figure 3. Profile of the local stress between two neighboring layers as function of η and L (in the section $x = ct$.)

Now if we retake different developments of the non dimensional velocity distribution considered within the sliding phenomena, namely

$$u3 = 1/(2 + 3L) \cdot (3L + 3\eta - \eta^3) \,, \tag{22}$$

(in the case when a 3rd degree polynomial form is used) or

$$u4 = 1/(1 + 2L) \cdot (2L + 2\eta - 2\eta^3 + \eta^4) \,, \tag{23}$$

(when a 4th degree polynomial form is used) or

$$uP = 2\eta - 2\eta^3 + \eta^4 \,, \tag{24}$$

(if the classical Polhausen method is taken into consideration, i.e. the sliding effects are neglected) then denoting by u5 the representation for the velocity profile inside the boundary layer by a 5th degree polynomial form as we envisaged in this paper

$$u5 = \frac{1}{2 + 5L}\left(5L + 5\eta - 10\eta^3 + 10\eta^4 - 3\eta^5\right). \tag{25}$$

The corresponding graph, in physical surface, is given in the Figure 4.

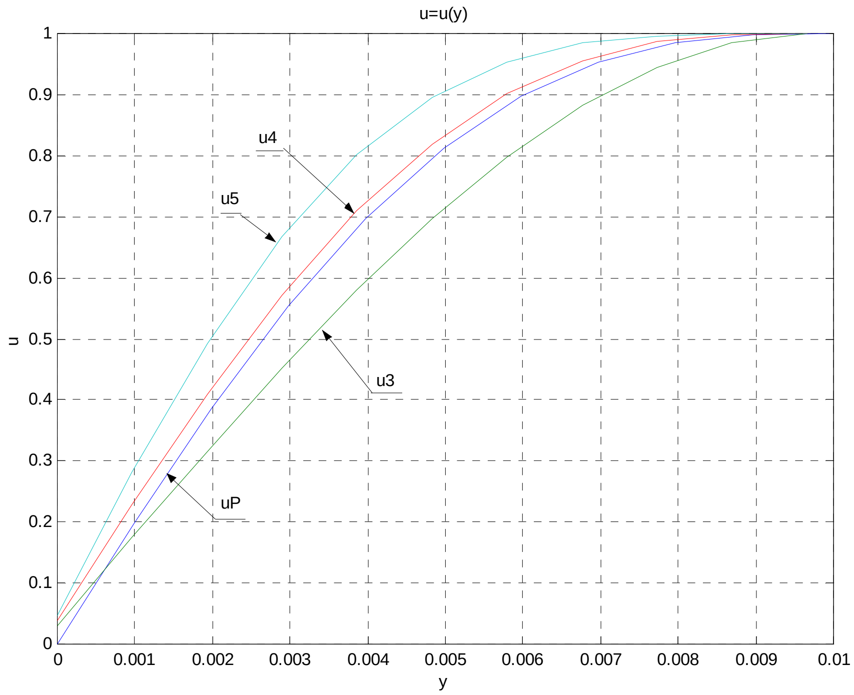

Figure 4. The velocity profiles in physical surface

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