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ON AN IMPROVED SOLUTION STRATEGY FOR A HYDRODYNAMICS PROBLEM

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Abstract
In this paper we purpose an improved solution strategy for the potential, steady state gravity channel flows.

Keywords:
free surface flows, inverse problems

1. INTRODUCTION

Let's study the inviscid, potential, steady state liniarized gravity channel flows. We assume that the fluid is freely moving over an arbitrary smooth topography and, in far field, has a uniform velocity. A Cartesian coordinates system with x axis following the unperturbed (at far distances) free surface and y axis directed vertically upwards is used.

We will develop a framework in order to improve the free surface flows solution, namely the free surface shape.

2. MODELING AND ANALYSIS

Let's denote by $\phi(x, y)$ the perturbation of the velocity potential $\Phi(x, y)$

$$\Phi(x, y) = Ux + \phi(x, y).$$

Here U denote the flow velocity in the far field.

Let's denote the free boundary by α. Then the inviscid fluid flows has a domain D defined by

$$D = \{(x, y) \in R^2 : -\infty < x < +\infty, 0 < y < \alpha(x)\}.$$

In the far field the flow has a uniform behavior and the velocity potential is constant $\frac{\partial \phi}{\partial x} = 0$ when $|x| \to \infty$.

So, the flow domain D is bounded by a rigid boundary $\Gamma = \{(x, y) : y = 0\}$ and by a free surface $S = \{(x, y) : y = \alpha(x)\}$.

The fluid flow is governed by Laplace's equation,

$$\Delta\phi = 0, \qquad x \in D,$$

subject to the slipping boundary conditions on the rigid boundary Γ

$$\frac{\partial\phi}{\partial n} = 0, \qquad x \in \Gamma,$$

and, respectively on the free surface S

$$\frac{\partial\phi}{\partial n} = 0, \qquad x \in S.$$

As well on the free boundary S due to the Bernoulli's integral we get the second condition which has to be imposed on the free surface. In the liniarized form it can be written as

$$\frac{\partial\phi}{\partial x} + g\alpha(x) = 0, \qquad x \in S.$$

Let's assume that, using some numerical method, an estimation of the velocity potential is established.
Let's denote this estimation by $\overline{\phi}$, which can be written as

$$\overline{\phi} = (\phi_{1,}, \ldots, \phi_m).$$

On the other hand we are looking for some functions α in some appropriate continuous space. However, because our intention is to give an estimation for the unknown function α using an finite representation, we will represent the function α by an number of n components

$$\overline{\alpha} = (\alpha_{1,}, \ldots, \alpha_n).$$

So the new unknown function belongs to R^n .

Obviously, for an appropriate approximation, the dimension of this space has to be as large as is possible. That is why we have $n > m$.

Since the velocity potential $\overline{\phi}$ has been established by some numerical method, let's denote with $\overline{\varepsilon}$ the errors introduced by the method employed.
Then, let's denote by $\tilde{\phi} = \overline{\phi} - \overline{\varepsilon}$ the data for our problem and let's make the assumption that there is a small $\delta > 0$ for which we have $\left\| \tilde{\phi} - \overline{\phi} \right\| < \delta$.

If we can definite an operator such that

$$A : R^n \to R^m$$

and

$$A(\overline{\alpha}) = \overline{\phi}$$

this problem can be reduced to the minimization one

$$\min(A(\overline{\alpha}) - \overline{\phi}) \tag{1}$$

The problem (1) has infinitely many solutions and among them we should select one which has physical relevance. In order to reach that solution we rewrite (1) in a new form by adding some additional information. So, a family of parameter dependent functional will be constructed and for an adequate parameter's selection some convergence result holds. We call a convergent regularization scheme ($[2], [4], [8]$), a pair $(T_a, a(\delta))$ defined by

- $T_a : R^m \to R^n$, $a(\delta) \in (0, \infty)$, a family of continuous functions such that

$$\lim_{\delta \to 0} \sup\left\{\left\|T_a\overline{\phi} - A^t\tilde{\phi}\right\|, \quad \left\|\overline{\phi} - \tilde{\phi}\right\| \le \delta\right\} = 0$$

with $a(\delta) > 0$ satisfying also

- $a(\delta) \to 0, \qquad \delta \to 0.$

Here $A^t : R^m \to R^n, A^t = (A'A)^{-1}A'$ is the pseudo inverse matrix of A while A' is its transpose.

Based on the Bernoulli's integral, we can define the operator A as

$$A = -\frac{1}{g} M^{-1} I_{m,n}$$

where $I_{m,n}$ is the unitary matrix of dimension $m \times n$ of the space and M is a $m \times m$ matrix, respectively the discretized expression of the $\frac{\partial}{\partial x}$ operator.

Further on the operator T_a can be constructed as

$$T_a = (A'A + aL)^{-1} A',$$

where L is a matrix incorporating prior information on the unknown function), $[8]$, for any parameter choice rule satisfying both $a(\delta) \to 0\,(\delta \to 0)$ and $\frac{\delta^2}{a(\delta)} \to 0\,(\delta \to 0)$.

In this framework there is an unique $\alpha^{a,\delta} \in R^n$ which solve the system $([4])$.

$$\alpha^{a,\delta} = T_a\overline{\phi}, \tag{2}$$

Then the shape or the free surface is determined by (2).

3. CONCLUSIONS

For an inviscid, potential, steady state liniarized gravity channel flow we have developed a mathematical framework to describe the free surface shape. It appears a promising tool for some other hydrodynamics problems.

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