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BURR DISTRIBUTION MODELING TRUNCATED

MAKSAY Ştefan, STOICA Diana

"POLITEHNICA" UNIVERSITY OF TIMISOARA,
ENGINEERING FACULTY OF HUNEDOARA

ABSTRACT:
Through the modeling herein we suggest the adaptation of Burr classic distribution densities to possible situations in real cases [3]. The classic distribution densities are truncated, obviously keeping the properties of a density. The new expressions of the density, having one extra parameter, can better approximate the experimental data. Further on, we introduced the Burr truncated repartition, accompanied by numerical examples.

KEYWORDS:
Burr distribution, truncated distribution.

1. THEORETICAL CONSIDERATION

The classical Burr law, having the distribution density

$$B(x,\alpha,\beta,\gamma) = \gamma\alpha\beta^{\alpha}x^{\gamma-1}(\beta + x^{\gamma})^{-(\alpha+1)}, \quad x \geq 0, \ \ \alpha,\beta,\gamma > 0 \tag{1}$$

will be approximated by a function of the form

$$BT(x,\alpha,\beta,\gamma,T) = \begin{cases} K(T)\cdot\gamma\alpha\beta^{\alpha}x^{\gamma-1}(\beta + x^{\gamma})^{-(\alpha+1)}, & x \in [0,T] \\ 0, & x \notin [0,T] \end{cases} \tag{2}$$

For the function *BT* be a distribution density we have these conditions

K(T) > 0 , T>0

and

$$\int_{-\infty}^{\infty} BT(x,\alpha,\beta,\gamma,T)\,dt = 1, \tag{3}$$

which determine the expression of the truncated density having the form

$$BT(x,\alpha,\beta,\gamma,T) = \begin{cases} \dfrac{1}{\int_0^T B(x,\alpha,\beta,\gamma)}\cdot\gamma\alpha\beta^{\alpha}x^{\gamma-1}(\beta + x^{\gamma})^{-(\alpha+1)}, & x \in [0,T] \\ 0, & x \notin [0,T] \end{cases}. \tag{4}$$

Figure 1 and 2 shows graphic representation of the functions *B* and *BT*, comparatively.

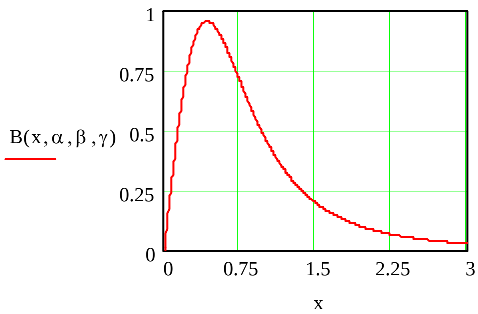

Figure 1. Graphic of function *B.*

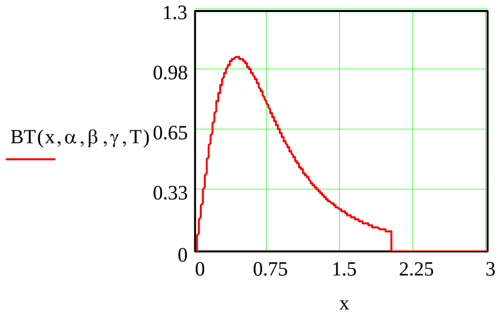

Figure 2. Graphic of function *BT.*

2. PRACTICAL CASE

A practical application results the following table of values

Table 1

$val^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.12	0.569	0.634	0.824	0.969	0.993	1.216	1.33	1.689	1.771	1.866	1.904	1.941	1.99	2.834
2	0.499	0.965	1.168	0.682	0.535	0.359	0.346	0.34	0.157	0.18	0.084	0.114	0.108	0.101	$1\cdot10^{-3}$

where the first line represents the values of independent variable x, and second line represents the values of dependent variable y.

Considering the values of dependent variable y, we used a program for the determination of the parameters α, β, γ of the classical function and we use least-squares method. The program is realized in MathCad [1].

ORIGIN ≡ 1

$x^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.12	0.569	0.634	0.824	0.969	0.993	1.216	1.33	1.689	1.771	1.866	1.904	1.941	1.99	2.834

$y^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.499	0.965	1.168	0.682	0.535	0.359	0.346	0.34	0.157	0.18	0.084	0.114	0.108	0.101	$1\cdot10^{-3}$

$i := 1..\,length(x)$ $n := length(x)$ $nrdi := 5$

$fmin := 10^{10}$ $T := 2$

$\alpha a := 1.2$ $\alpha b := 1.5$

$\beta a := 0$ $\beta b := 2$

$\gamma a := 1.5$ $\gamma b := 2.3$

```
prgt := | nt ← nrdi
        | fmin ← 10^10
        | for it ∈ 1.. nt + 1
        |   | βλ_it ← βa + (it − 1)·(βb − βa)/nt
        |   | for jt ∈ 1.. nt + 1
        |   |   | αλ_jt ← αa + (jt − 1)·(αb − αa)/nt
        |   |   | for kt ∈ 1.. nt
        |   |   |   | γλ_kt ← γa + (kt − 1)·(γb − γa)/nt
        |   |   |   | f ← Σ_{i=1}^{n} [γ·αλ_jt·β^(αλ_jt)·(x_i)^(γλ_kt−1)·[β + (x_i)^(γλ_kt)]^−(αλ_jt+1) − y_i]^2
        |   |   |   | if (f ≤ fmin)
        |   |   |   |   | βt ← βλ_it
        |   |   |   |   | αt ← αλ_jt
        |   |   |   |   | γt ← γλ_kt
        |   |   |   |   | fmin ← f
        |   |   |   | lt ← 1.. nt + 1
        | (αt, βt, γt, fmin)
```

$\alpha t := prgt_1$ $\beta t := prgt_2$ $\gamma t := prgt_3$ $prgt = \begin{pmatrix} 1.38 \\ 2 \\ 1.98 \\ 0.136 \end{pmatrix}$.

Using this program, we obtained the values of parameters α, β, γ

$\alpha t = 1.38$ $\beta t = 2$ $\gamma t = 1.98$.

For this value, substituted in expression (1), results the values of the coefficient of correlation and the deviation from the curve

$$rcl := \sqrt{1 - \frac{\sum_i (y_i - B(x_i, \alpha t, \beta t, \gamma t))^2}{\sum_i (y_i - \mathrm{mean}(y))^2}} \qquad rcl = 0.72 \quad . \qquad (5)$$

$$sd1 := \sqrt{\frac{1}{n} \cdot \left[\sum_i (y_i - B(x_i, \alpha t, \beta t, \gamma t))^2 \right]} \qquad sd1 = 0.229 \quad . \qquad (6)$$

Further on, we consider the problem with four parameters α, β, γ and T. In order to determine these parameters we use the least-squares method.

In this case using the following program in MathCAD [1]

ORIGIN ≡ 1

$x^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.12	0.569	0.634	0.824	0.969	0.993	1.216	1.33	1.689	1.771	1.866	1.904	1.941	1.99	2.834

$y^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.499	0.965	1.168	0.682	0.535	0.359	0.346	0.34	0.157	0.18	0.084	0.114	0.108	0.101	$1\cdot10^{-3}$

$i := 1..\,\mathrm{length}(x)$ $\quad n := \mathrm{length}(x)$ $\quad nrdi := 5$

$\alpha a := 1.2$ $\quad \alpha b := 1.5$

$\beta a := 0.5$ $\quad \beta b := .7$

$\gamma a := 1.8$ $\quad \gamma b := 2.3$

$Ta := 1.5$ $\quad Tb := 2.5$

```
prg := | nt ← nrdi
       | fmin ← 10^10
       | for it ∈ 1.. nt + 1
       |   βλ_it ← βa + (it − 1)·(βb − βa)/nt
       |   for jt ∈ 1.. nt + 1
       |     αλ_jt ← αa + (jt − 1)·(αb − αa)/nt
       |     for kt ∈ 1.. nt
       |       γλ_kt ← γa + (kt − 1)·(γb − γa)/nt
       |       for lt ∈ 1.. nt + 1
       |         Tλ_lt ← Ta + (lt − 1)·(Tb − Ta)/nt
       |         inte ← ∫_0^{Tλ_lt} B(x, αλ_jt, βλ_it, γλ_kt) dx
       |         f ← Σ_{i=1}^{n} [ if[ x_i < Tλ_lt, (1/inte)·γλ_kt·αλ_jt·(βλ_it)^{αλ_jt}·(x_i)^{γλ_kt−1}·[βλ_it + (x_i)^{γλ_kt}]^{−(αλ_jt+1)}, 0 ] − y_i ]^2
       |         if (f ≤ fmin)
       |           βt ← βλ_it
       |           αt ← αλ_jt
       |           δt ← γλ_kt
       |           Tt ← Tλ_lt
       |           fmin ← f
       | (αt, βt, δt, Tt, fmin)
```

$$f \leftarrow \sum_{i=1}^{n}\left[\mathrm{if}\left[x_i < T\lambda_{lt}, \frac{1}{inte}\cdot\gamma\lambda_{kt}\cdot\alpha\lambda_{jt}\cdot(\beta\lambda_{it})^{\alpha\lambda_{jt}}\cdot(x_i)^{\gamma\lambda_{kt}-1}\cdot\left[\beta\lambda_{it}+(x_i)^{\gamma\lambda_{kt}}\right]^{-(\alpha\lambda_{jt}+1)}, 0\right]-y_i\right]^2$$

$\alpha t := prg_1$ $\quad \beta t := prg_2$ $\quad \gamma t := prg_3$ $\quad Tt := prg_4$ $\quad prg = \begin{pmatrix}1.32\\0.58\\2.1\\2.1\\0.09\end{pmatrix}$.

we obtained the values for parameters

$$\alpha t = 1.32 \qquad \beta t = 0.58 \qquad \gamma t = 2.1 \qquad Tt = 2.1 .$$

With this values result the expression of distribution density

$$BT(x,\alpha t,\beta t,\gamma t,Tt) := \text{if}\left[x < Tt, \frac{1}{\int_0^{Tt} B(x,\alpha t,\beta t,\gamma t)\,dx}\cdot\gamma t\cdot\alpha t\cdot\beta t^{\alpha t}\cdot x^{\gamma t-1}\cdot\left(\beta t + x^{\gamma t}\right)^{-(\alpha t+1)}, 0\right] . \tag{7}$$

For these value substituted in above function we obtained the values of the coefficient of correlation and the deviation from the curve

$$rtr := \sqrt{1 - \frac{\sum_i \left(y_i - BT(x_i,\alpha t,\beta t,\gamma t,Tt)\right)^2}{\sum_i \left(y_i - \text{mean}(y)\right)^2}} \qquad rtr = 0.972 \tag{8}$$

$$str := \sqrt{\frac{1}{n}\cdot\left[\sum_i \left(y_i - BT(x_i,\alpha t,\beta t,\gamma t,Tt)\right)^2\right]} \qquad str = 0.077 . \tag{9}$$

Is to be noticed that the coefficient of correlation is higher.

With the values $\alpha t = 1.32$, $\beta t = 0.58$, $\gamma t = 2.1$, $Tt = 2.1$ substituted in the characteristic function

$$ex(t) := \int_{-2}^{6} BT(x,\alpha t,\beta t,\gamma t,Tt)\cdot\exp(i\cdot t\cdot x)\,dx \tag{10}$$

we obtained the graph given in Figure 3.

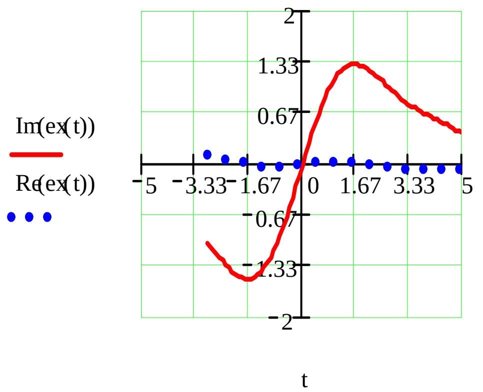

Figure 3. Graphic of characteristic function

3. CONCLUSIONS

For very high values of T substituted in relation (4), we obtained both for the distribution density and for the distribution function, respectively for the characteristic function, results known from the classical Burr distribution given in relation (1).

These studies have led to the conclusion that the best Burr modeling is the truncated one, introducing a new parameter, T. This is why it models better the points cloud obtained experimentally, which results in a superior coefficient of correlation.

As a conclusion, this type of modeling is recommendable for practical cases.

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