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ON THE WAVE DISPERSION IN HEAT CONDUCTING LAYERED PLATES

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ABSTRACT:
In the present article, wave propagation in an isotropic layered thermoelastic plate is studied in the context of the generalized theory of thermoelasticity with thermal relaxation time. Commencing with analysis of waves projected along the x-axis, at an angle θ in a heat-conducting n-layered plate, transfer matrix relating the displacements, temperature, thermal stresses and temperature gradient within a layer to its wave amplitudes is obtained. The calculation is then carried forward for more specialized case of semi-infinite is absent or present. Some special cases have also been deduced and discussed. Applying appropriate boundary conditions on the plate outer boundaries a large variety of important physical problems can be solved, of these mentioned includes dispersion equation for a uniform free plate. Numerical calculations have been presented for a aluminum epoxy composite.
KEY WORDS: Generalized thermoelasticity, thermal relaxation times, free plate, layered, thermal waves.

1. INTRODUCTION

Thermal effects represent a primary origin of failure for a broad class of multilayered devices. Due to different mechanical and thermal properties between constituents, temperature changes can introduce residual stresses which may lead to interface debonding in ductile materials, or enhance crack formation in brittle. Propagation of elastic waves in layered media [4, 13, 19, 23], have long been of interest of researchers [1, 6] in the fields of geophysics, acoustics, and electromagnetic. Applications of these studies include such technologically important areas as earthquake prediction, underground fault mapping, and oil and gas exploration, architectural noise reduction. In contrast to elastic waves, only a few problems have been attempted for thermoelastic materials in dynamic coupled theory of thermoelasticity. Hence the study of thermo-mechanical interactions and wave phenomenon in anisotropic materials is justified and is of great importance and practical use in engineering applications especially in the context of generalized theory of thermoelasticity.

According to the theory of thermoelasticity, the temperature field is coupled with the elastic strain field. The classical theory of thermoelasticity predicts infinite speed of transportation, which contradicts the physical facts. Lord and Shulman [20], and Green and Lindsay [14] extended the coupled theory of thermoelasticity by introducing the thermal relaxation times in the constitutive equations to eliminate the paradox of infinite velocity of heat propagation, thus are called generalized theories of thermoelasticity. Banerjee and Pao [3] extended this theory to anisotropic heat conducting elastic materials. Dhaliwal and Sherief [12] treated the problem in more systematic manner. The works of Ignaczak [18], Hawwa and Nayfeh [16, 17], Green and Naghdi [15], Verma [26, 27] contain more detailed discussions on this phenomenon. The literature dedicated to such theories is quite large and its detailed review can be found in Nowacki [24], Chadwick [7-9] and Chandrasekharaiah [10, 11].

Dynamic problems in layered thermoelastic media are complicated than its counterpart in elasticity, because in thermoelasticity, solutions to both the heat conduction and thermoelasticity problems for all the layers are required. These solutions are also to satisfy the thermal and mechanical boundary and interface conditions. In the analysis of a layered thermoelastic medium, system of two simultaneous equations are to be solved for a large number of unknown constants. Bufler [5], Bahar[2] and Verma [28] proposed the transfer matrix method to study the isothermal elasticity problems in multilayered medium. Thangjitham, and Choi, [24] solved thermal stresses problem in multilayered anisotropic medium. Verma et al. [8] studied theoretical development for the multilayered plate with either all rigidly bonded interfaces or with one smooth interface and the remaining interfaces rigidly bonded is presented in the context of theories of generalized thermoelasticity for a plate consisting of an arbitrary number of isotropic layers.

In the present paper, wave propagation in arbitrary isotropic layered thermoelastic plate is studied in theory of the generalized thermoelasticity with thermal relaxation time. Commencing with analysis of waves in a heat-conducting n-layered plate of heat conducting media, the dispersion relations of thermoelastic waves are obtained by invoking continuity at the interface and boundary of conditions on the surfaces of layered plate. The calculation is then carried forward for more specialized case of semi-infinite is absent or present. Some special cases have also been deduced and discussed. Applying appropriate boundary conditions on the plate outer boundaries a large variety of important physical problems can be solved, of these mentioned includes dispersion equation for a uniform free plate. Numerical results reveal that at zero wave number limits, each figure displays four values of wave speeds corresponding to one quasi-longitudinal, two quasi-transverse, and one thermal mode. At relatively low values of the wave number, a rapid change is seen to take place in the slope of the quasi-longitudinal curve. As wave number increases, little change is seen. Curves also demonstrate, how the phase velocity varies with angle of propagation θ. Also when the thermal relaxation time $\tau_0 \to 0$, then the results obtained in the analysis reduces to conventional coupled thermoelasticity.

2. FORMULATION

The generalized coupled field equations governing dynamic thermoelastic processes for homogeneous heat conducting isotropic materials and in the absence of body forces and heat source can be written as

$$\mu\nabla\mathbf{u} + (\lambda+\mu)\nabla\nabla\mathbf{u} - \gamma\nabla T = \rho\ddot{\mathbf{u}} \tag{2.1}$$

$$K\nabla^2\theta + \rho C_e[\dot{T} + \tau_0\ddot{T}] = \gamma T_0(\nabla\dot{\mathbf{u}} + \tau_0\nabla\ddot{\mathbf{u}}), \tag{2.2}$$

where $\gamma = (3\lambda + 2\mu)\alpha_t$; λ and μ are the Lame' parameters and α_t the coefficient of thermal expansion; $\mathbf{u}$ is the displacement vector; T is the temperature change above the uniform reference temperature θ_0; ρ is the mass density; C_e is the specific heat at constant deformation; τ_0 is the thermal relaxation time. Consider a plate consisting of an arbitrary number, n, of homogeneous thermoelastic isotropic layers rigidly bonded at their interfaces. We shall assume that a plane wave propagates in the x-z plane at an arbitrary angle θ measured from the normal to the interface and the use the two sets of two-dimensional coordinate systems (x, z), is global coordinate system, which has its origin at the bottom layer of the plate such that x denotes the propagation direction and z is the normal to the interfaces. Hence layered plane will then occupy the space $0 \le z \le d$ where d denotes the total thickness of the plate. The second system is local for each sub-layer of the plate. Since the plate is made of n layers, the mth layer will then have its local coordinate $x^{(m)}$ and $z^{(m)}$ with local origin at bottom surface. Hence each layer occupy the space $0 \le z^{(m)} \le d^{(m)}$ where $d^{(m)}$ is its thickness. With this choice of co-ordinate system specialize the equations (2.1) and (2.2) for each layer reduce to

$$\mu[u_{,xx} + u_{,zz}) + (\lambda + \mu)(u_{,xx} + w_{,xz}) = \rho\ddot{u} + \gamma T_{,x}$$
$$\mu(w_{,xx} + w_{,zz}) + (\lambda + \mu)(u_{,xx} + w_{,zz}) = \rho\ddot{w} + \gamma T_{,z} \tag{2.3}$$
$$K[T_{,xx} + T_{,zz}] + \rho C_e(\dot{T} + \tau_0\ddot{T}) = \gamma T_0[(\dot{u}_{,x} + \dot{w}_{,z} + \tau_0(\ddot{u}_{,x} + \ddot{w}_{,z})$$

where τ_0 is the thermal relaxation time, λ and μ are Lame's constants, $\gamma = (3\lambda + 2\mu)\alpha_t$ is the thermoelastic coupling constant and α_t the coefficient of thermal expansion and all other symbols have their usual meanings as in Lord and Shulman [20]. The comma notation is used for spatial derivatives and the superposed dot denotes time differentiation.

3. ANALYSIS

Consider the formal solutions for Eqs. (2.1) and (2.2),

$$(u, w, T) = (U_1, U_2, U_3)e^{i\xi(x\sin\theta + \alpha z - ct)}, \tag{3.1}$$

for waves whose projected wave vector is along the x-axis, and for an angle of incidence θ, where ξ is the wave number , U_1 , U_2 and U_3 are the constant amplitudes related to displacements and temperature T, c is the phase velocity (= ω/ξ), ω is the circular frequency, α is the ratio of the z and x-directions wave numbers. This choice of solutions leads to the coupled equations

$$M_{mn}(\alpha)U_n = 0 \quad (m, n = 1,2,3), \tag{3.2}$$

where the summation convention is implied, and

$$M_{11} = c_2\alpha^2 + \sin^2\theta - \zeta^2,\ M_{12} = c_3\sin\theta\alpha,\ M_{13} = \sin\theta,\ M_{21} = M_{12},$$
$$M_{22} = c_2\sin^2\theta + \alpha^2 - \zeta^2, M_{23} = \alpha,\ M_{31} = \varepsilon_1\tau\omega^*\zeta^2\sin\theta, M_{32} = \varepsilon_1\tau\omega^*\zeta^2\alpha, \tag{3.3}$$
$$M_{33} = (\sin^2\theta + \alpha^2) - \omega^*\zeta^2\tau, \qquad c_3 = 1 - c_2,\ \ \varepsilon_1 = \frac{T_0\gamma^2}{\rho C_e(\lambda + 2\mu)}$$
$$\omega^* = C_e(\lambda + 2\mu)/K, \tau = (\tau_0 + i/\xi c), c_2 = \mu/(\lambda + 2\mu),\ \zeta^2 = \rho c^2/(\lambda + 2\mu). \tag{3.4}$$

The existence of nontrivial solution for U_1, U_2 and U_3 demands the vanishing of the determinant in Eqs. (3.2), and yields the polynomial equation

$$\alpha^6 + A_1\alpha^4 + A_2\alpha^2 + A_3 = 0, \tag{3.5}$$

where

$$P_1 = [3c_2\sin^2\theta - \{(1+\varepsilon_1)\omega^*\tau - 1\}c_2\zeta^2 - \zeta^2]/c_2$$
$$P_2 = [3c_2\sin^4\theta - \{1 + c_2 + c_2(1+\varepsilon_1)\omega^*\tau\}2\sin^2\theta\zeta^2 + \{1 + (c_2 + 1 + \varepsilon_1)\omega^*\tau\}\zeta^4]/c_2$$
$$P_3 = [\sin^4\theta + \omega^*\tau\zeta^4 - (1+\varepsilon_1)\omega^*\tau\sin^2\theta\zeta^2 - \sin^2\theta\zeta^2\}][c_2\sin^2\theta - \zeta^2]/c_2$$

Solving (3.5) for the six roots of α and using superposition results in the following formal solution relating the displacements, temperature, thermal stresses and temperature gradient within a layer to its wave amplitudes.

$$\begin{bmatrix} u \\ w \\ T \\ \bar{\sigma}_{zz} \\ \bar{\sigma}_{xz} \\ \bar{T}' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \dfrac{\alpha_1}{\sin\theta} & -\dfrac{\alpha_1}{\sin\theta} & \dfrac{\alpha_2}{\sin\theta} & -\dfrac{\alpha_2}{\sin\theta} & -\dfrac{\sin\theta}{\alpha_3} & \dfrac{\sin\theta}{\alpha_3} \\ S_1 & S_1 & S_2 & S_2 & 0 & 0 \\ D_1 & D_1 & D_1 & D_1 & D_3 & D_3 \\ D_4 & -D_4 & D_5 & -D_5 & D_6 & -D_6 \\ D_7 & -D_7 & D_8 & -D_8 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1E_1 \\ A_2E_2 \\ A_3E_3 \\ A_4E_4 \\ A_5E_5 \\ A_6E_6 \end{bmatrix} E\,, \qquad (3.6)$$

here α_1^2, α_2^2 and α_3^2 are roots of equation (3.5)

$$E_q = e^{i\xi\alpha_q z},\ E = e^{i\xi(x\sin\theta - ct)},\quad q = 1,2 \ldots 6$$

α_1^2, α_2^2 correspond to coupled longitudinal and thermal waves whereas α_3^2 corresponds to transverse wave which is not affected by the temperature variations

$$D_1 = c_2\left(\frac{c^2}{c_T^2} - 2\sin^2\theta\right),\ D_3 = -2c_2\sin\theta,\ D_4 = 2c_2\alpha_1, D_5 = 2c_2\alpha_2, D_6 = \frac{c_2}{\alpha_3}\left[\frac{c^2}{c_T^2} - (1+\sin^2\theta)\right]$$

$$D_7 = \alpha_1 S_1,\quad D_8 = \alpha_2 S_2,\ c_T^2 = \frac{\mu}{\rho},\quad \bar{\sigma}_{zz} = \frac{\sigma_{zz}}{i\xi}, \bar{\sigma}_{xz} = \frac{\sigma_{xz}}{i\xi}, \bar{T}' = \frac{T'}{i\xi}. \qquad (3.7)$$

The various parameters $\alpha_1, \alpha_2, \alpha_3$, S_1, S_2, D_1, etc. are specialized to the material of layer (m) under consideration. Putting (3.6) in the block matrix form

$$\mathbf{F_m} = \mathbf{R_m}\mathbf{B_m} \qquad (3.8)$$

in which $\mathbf{B_m} = (A_1, A_2, A_3, A_4, A_5, A_6)$, $\mathbf{F_m} = (\bar{\sigma}_{zz}, \bar{\sigma}_{xz}, \bar{T}', w, u, T)$ and $\mathbf{R_m}$ is the matrix, the elements of which are given explicitly in the (3.6).

Considering the origin of co-ordinates at the $(m-1)$th interface, the relation between the stress, thermal gradient, displacements and temperature vector $\mathbf{F_{m,m-1}}$ within the mth layer at the $(m-1)$th interface ,and the vector $\mathbf{B_m}$

$$\mathbf{F_{m,m-1}} = \mathbf{S_m}\mathbf{B_m} \qquad (3.9)$$

where $\mathbf{S_m}$ is derived from $\mathbf{R_m}$ by putting $z = 0$

Similarly stress, thermal gradient, displacements and temperature vector $\mathbf{F_{m,m}}$ within the mth layer at the mth interface is

$$\mathbf{F_{m,m}} = \mathbf{U_m}\mathbf{B_m} \qquad (3.10)$$

where $\mathbf{U_m}$ is derived from $\mathbf{R_m}$ by putting $z = H_m$

Using (3.8) to (3.10), $\mathbf{F_{m,m-1}}$ and $\mathbf{F_{m,m}}$ at the upper and bottom surface of mth layer we can related,

$$\mathbf{F_{m,m}} = \mathbf{G_m}\mathbf{F_{m,m-1}},\ \text{where } \mathbf{G_m} = \mathbf{U_m}\mathbf{S_m^{-1}} \qquad (3.11)$$

Using the boundary and continuity conditions $(\mathbf{F_{m,m-1}} = \mathbf{F_{m-1,m-1}})$ at each interface and considering that no slips occurs at the interface.
On applying (3.11) to each interface in turns gives

$$\mathbf{F_{n,n}} = \mathbf{G_n}\mathbf{G_{n-1}}\ldots\mathbf{G_1}\mathbf{F_{1,0}} = \mathbf{G}\mathbf{F_{1,0}} \qquad (3.12)$$

Free layered plate: If the semi-infinite medium is absent, consider a free layered plate. The characteristic equation for such a situation is obtained by invoking $(\bar{\sigma}_{zz} = \bar{\sigma}_{xz} = \bar{T}' = 0)$ stress-free as well as thermally insulated upper and bottom surfaces. Partitioning the 6×6 matrix $\mathbf{G}$ into four 3×3 sub-matrices $\mathbf{G^{(1)}}, \mathbf{G^{(2)}}$ etc., and partitioning the vector conformably

We obtain the characteristic equation as (3.12)

$$\left\{\{0,0,0\},\{w,u,T\}\right\}_{n,n}=\begin{bmatrix}\mathbf{G^{(1)}} & \mathbf{G^{(2)}}\\ \mathbf{G^{(3)}} & \mathbf{G^{(4)}}\end{bmatrix}\left\{\{0,0,0\},\{w,u,T\}\right\}_{1,0} \tag{3.13}$$

that is

$$\mathbf{G^{(2)}}\{w,u,T\}_{1,0}=0 \tag{3.14}$$

and

$$\{w,u,T\}_{n,n}=\mathbf{G^{(4)}}\{w,u,T\}_{1,0} \tag{3.15}$$

If $\{w,u,T\}_{1,0}$ is not to be null, therefore, $\mathbf{G^{(2)}}$ must be singular, that is

$$\det(\mathbf{G^{(2)}})=0 \tag{3.16}$$

The elements of $\mathbf{G^{(2)}}$ are functions of the elastic thermal constants and thickness of the layers. Equation (3.16) is the desired dispersion equation in the generalized theory of thermoelasticity. The numerical values of det($\mathbf{G^{(2)}}$) can be computed by the successive multiplication of the matrices $\mathbf{G_m}$ for each layer, and equation (3.16) can be solved by applying a suitable iterative method.

Semi-infinite medium:

If the Semi-infinite medium is present, then the problem reduces to that of a layered half space, pre-multiplying (3.12) by $\mathbf{S^{-1}_{n+1}}$ gives

$$\mathbf{B_{n+1}}=\mathbf{S^{-1}_{n+1}GF_{1,0}}=\mathbf{JF_{1,0}} \tag{3.17}$$

where $\mathbf{J}=\mathbf{S^{-1}_{n+1}G}$.

The conditions on the semi-infinite medium are that there be no sources at infinity and this implies that ϕ,ψ and T are expressible in terms of $\exp(-\alpha_i z)$ and $\exp(-\alpha_3 z)$, respectively , i.e. $A_1=A_2, A_3=A_4$, and $A_5=A_6$. Again partitioning J into four 3×3 sub-matrices

$$\left\{\{A_1,A_2,A_3\},\{A_1,A_2,A_3\}\right\}=\begin{bmatrix}\mathbf{J^{(1)}} & \mathbf{J^{(2)}}\\ \mathbf{J^{(3)}} & \mathbf{J^{(4)}}\end{bmatrix}\left\{\{0,0,0\},\{w,u,T\}\right\}_{1,0} \tag{3.18}$$

so that if is not to be null, $J^{(2)}-J^{(4)}$ must be singular. The dispersion equation for layered half space is therefore

$$\det(\mathbf{J^{(2)}}-\mathbf{J^{(4)}})=0 \tag{3.19}$$

Eq. (3.19) is to be solved with some iteration process, when this has been done; the displacement ratio on the top surface is given by

$$\left[\mathbf{J^{(2)}}-\mathbf{J^{(4)}}\right]\{w,u,T\}=0 \tag{3.20}$$

and the displacements, temperature ,stresses, and thermal gradient, at other depths can be obtained as for the free plate.

In both the half-space and free plate problems, the dispersion equation involves only the last two columns of $\mathbf{J}$ or $\mathbf{G}$, and there is no need to compute more than this. Further, the dispersion equation for a half space involves the matrices $\mathbf{J^{(2)}}-\mathbf{J^{(4)}}$. This can be obtained directly by pre-multiplying J by the matrix

$$\mathbf{I}_2=\begin{bmatrix}\mathbf{I} & -\mathbf{I}\\ \mathbf{I} & -\mathbf{I}\end{bmatrix} \tag{3.21}$$

in which $\mathbf{I}$ represents a 3×3 unit matrix. Thus

$$\mathbf{I_2J}=\begin{bmatrix}\mathbf{J}^{(1)}-\mathbf{J}^{(3)} & \mathbf{J^{(2)}}-\mathbf{J^{(4)}}\\ \mathbf{J}^{(1)}-\mathbf{J}^{(3)} & \mathbf{J^{(2)}}-\mathbf{J^{(4)}}\end{bmatrix}=\mathbf{I_2S^{-1}_{n+1}G_nG_{n-1}...G_1}. \tag{3.22}$$

Finally,

$$\left[\mathbf{J}^{(2)} \text{ - } \mathbf{J}^{(4)}\right] = \mathbf{M}_{n+1}\mathbf{G}_n \ldots \mathbf{E}_1$$

in which $\mathbf{M}_{n+1}$ is 3×6 matrix formed by the top three rows of $\mathbf{I}_2\mathbf{S}^{-1}_{n+1}$ and $\mathbf{M}_1$ is 6×3 matrix formed by the last three columns of $\mathbf{E}_1$. With this modification, the dispersion equations both for free plate and a half space involve only a 3×3 matrix and only this part of this matrix need to be computed. To make sure on these equations, it may be noted that they yield the dispersion equation for a uniform plate in the form

$$\left[\mathbf{G}^{(2)}\right] = 0 \tag{3.23}$$

which is in agreement with the exiting solutions by [30] for thermoelasticity and for elasticity Abubakar [1] when coupling constant is zero. Similarly, the dispersion equation for a medium half-space reduces to the usual Rayleigh wave equation Verma and Hasebe [30].

4. DISCUSSION AND NUMERICAL ILLUSTRATION

In this section, the numerical calculations are carried out to present phase velocity dispersion curves plotted as a function of the wave number assuming the thickness 'd' of the plate is fixed. These curves have been calculated from expression based on the dispersion relation in equation (3.23). The material chosen for this purpose of numerical evaluation is Aluminum epoxy composite. The physical data for such materials is given as

$\lambda = 7.59\times10^{11} dynes/cm^2$ $\qquad$ $\mu = 1.89\times10^{11} dynes/cm^2$

$K = 0.0149\times10^{11} dynes/cm^2$ $\qquad$ $\rho = 2.19\, gm/cm^3$

$C_e = 0.23 cal/C^0$ $\qquad$ $\omega_1 = 4.36\times10^{13} s^{-1}$

$\varepsilon_1 = 0.073$ $\qquad$ $\tau_0 = 6.131\times10^{-3} s$.

The analytical results (3.23) may further be simplified, once the number of layers, their properties, and geometric stacking are specified, and then one can present numerical results in two categories. In the first, variations of phase velocity with function of the product of frequency and unit cell thickness, for specified angle of incidence θ, and secondly the phase velocity with the angle of incidence θ for specified wave number.

In FIGURE (1), close examination of these figures reveals several interesting features. At zero wave number limit, each figure displays four values of wave speeds corresponding to one quasi-longitudinal, two quasi-transverse, and one thermal mode. At relatively low values of the wave number, a rapid change is seen to take place in the slope of the quasi-longitudinal curve. As wave number increases, slight change is observed.

In FIGURE (2), when the thermal relaxation time $\tau_0 \to 0$, then the results obtained in the analysis reduces to conventional coupled theory of thermoelasticity.

In FIGURES (3)-(5), explicit dependence of such curves on the propagation direction θ (measured from the normal to the surface of the plate) are given in for $\xi = 1.10^{-5}$ mm^{-1} , 2.5 mm^{-1} and 4 mm^{-1} respectively is demonstrated when thermal relaxation time is equal to.001.

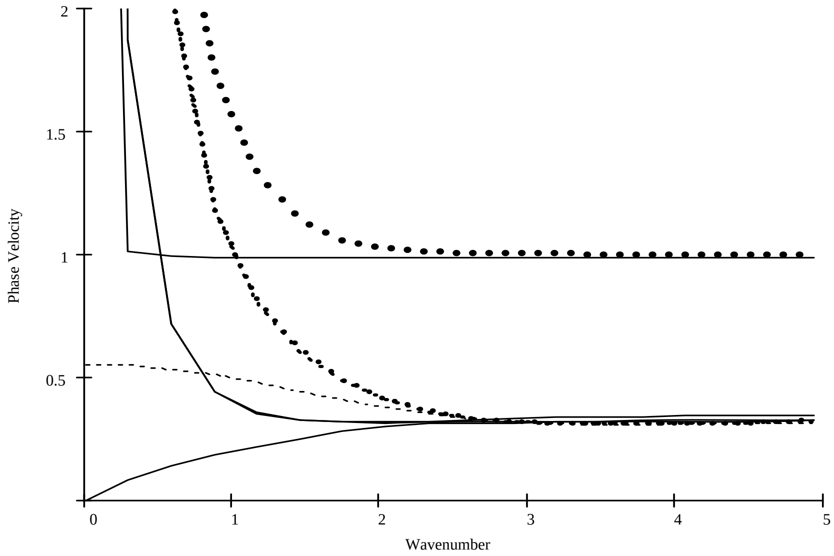

Figure 1. Dispersion curves for corresponding to antisymmetric modes (lines) and symmetric modes (dots)

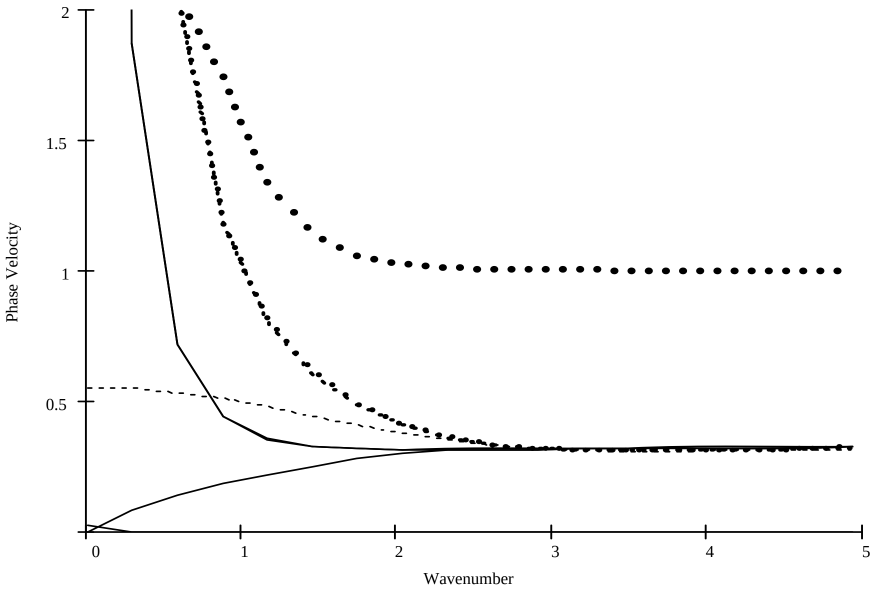

Figure 2. Dispersion curves for corresponding to antisymmetric modes (lines) and symmetric modes (dots) when τ_0 trends to 0

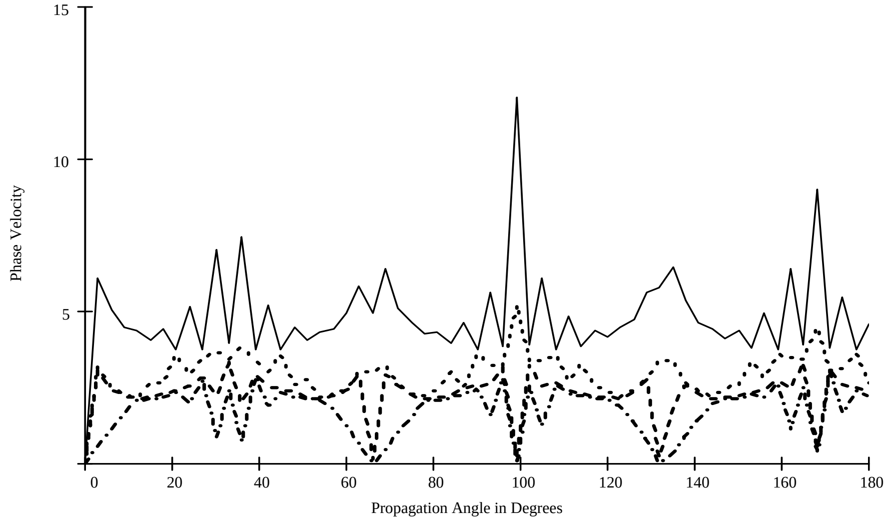

Figure 3. Variation of phase velocity of different modes with propagation angle when wave number is equal to 0.00001 mm-1 and thermal relaxation time = 0.001

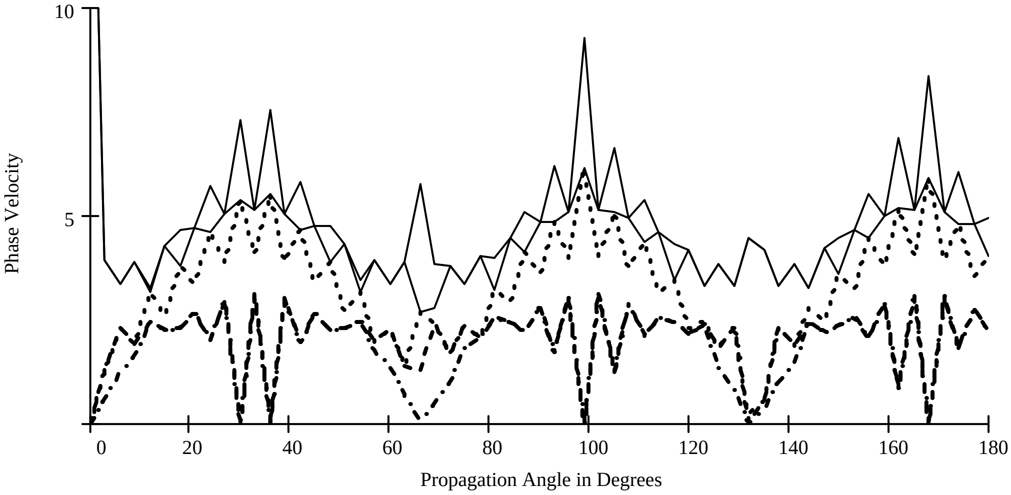

Figure 4. Variation of phase velocity of different modes with propagation angle when wave number is equal to 2.5 mm-1 and thermal relaxation time = 0.001

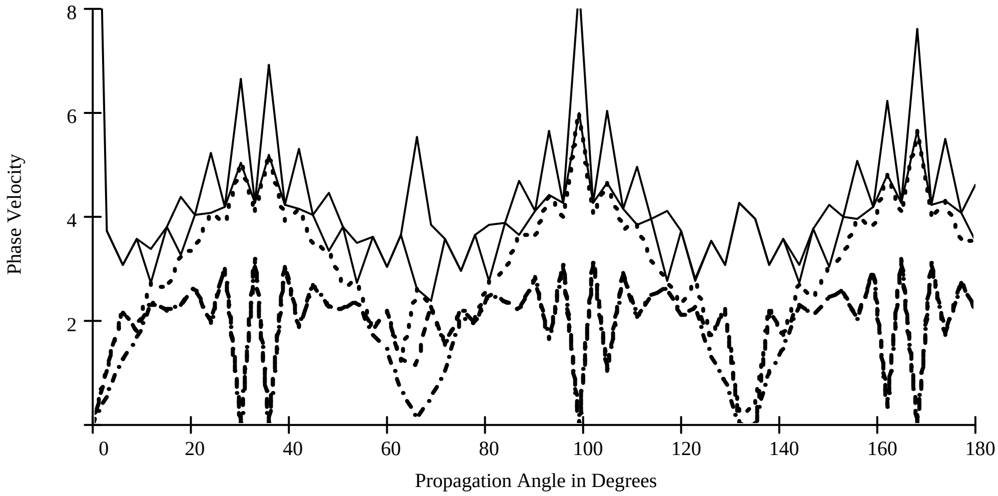

Figure 5. Variation of phase velocity of different modes with propagation angle when wave number is equal to 4 mm-1 and thermal relaxation time = 0.001

These curves demonstrate how the phase velocity varies with angle of propagation θ and when angle varies from $\theta = 0^0$ to $\theta = 180^0$ degrees. It is observed that phase velocity changing pattern divided into the three blocks from $\theta = 0^0$ to $\theta = 65.71^0$, second from $\theta = 65.71^0$ to $\theta = 131.817^0$ and the third from $\theta = 131.817^0$ to $\theta = 180^0$. Thus, curves as a function of wave number and hence display and demonstrate wave front dispersion behavior in generalized thermoelasticity.

5. CONCLUSIONS

Problem of thermoelastic wave propagation in multilayered media in the context of generalized thermoelasticity is solved. Analytical expressions have been derived that are easily adaptable to numerical illustrations for a thermoelastic plate consisting of an arbitrary number of layers, each layer possessing isotropic symmetry is chosen as a representative cell of medium. It is observed that at zero wave number limits, each figure displays four values of wave speeds corresponding to one quasi-longitudinal, two quasi-transverse, and one thermal mode. At relatively low values of the wave number, a rapid change is seen to take place in the slope of the quasi-longitudinal curve. As wave number increases, little change is seen. Curves also demonstrate, how the phase velocity varies with angle of propagation θ. Also when the thermal relaxation time $\tau_0 \to 0$, then the results obtained in the analysis reduces to conventional coupled theory of thermoelasticity.

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