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MATHEMATICAL MODEL OF THE MEAN FLOW STRESS OF MAGNESIUM ALLOY AZ31 OBTAINED BY LABORATORY HOT ROLLING

[1.]Miroslav LEGERSKI, [1.]Stanislav RUSZ, [1.]Ivo SCHINDLER,
[2.]Eugeniusz HADASIK, [1.]Marcel JANOŠEC

[1.]VŠB – TECHNICAL UNIVERSITY OF OSTRAVA, FACULTY OF METALLURGY AND MATERIALS ENGINEERING, OSTRAVA, CZECH REPUBLIC
[2.]SILESIAN UNIVERSITY OF TECHNOLOGY, KATOWICE, POLAND

ABSTRACT:
An experiment leading to obtaining the model of the mean flow stress (MFS) of magnesium alloy AZ31 was realized in laboratory rolling mill. It was based on the measurement of forces during hot rolling of flat samples with varying thickness. It resulted from mathematical and statistical processing of MFS values that these could be described by a simple function of just two independent variables – temperature (200 to 450 °C) and equivalent height strain (ca 0.2 to 0.7). The increasing strain resulted in decreasing deformation resistance. The effect of equivalent strain rate (ca 10 to 80 s^{-1}) could be neglected.

KEYWORDS:
Magnesium Alloy, Hot Rolling, Forming Factor, Mean Flow Stress

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1. INTRODUCTION

Magnesium alloys, for which low specific weight (ca 1800 kg/m^3) and good strength properties after heat treatment are characteristic, are currently demanded mainly in aeronautical and automotive industry. Most magnesium alloys are processed in high temperatures due to unfavourable formability at the room temperature. It is caused above all by magnesium itself, which crystallises in the hexagonal system with tight arrangement. By this magnesium differs from most technical metals that crystallise in the cubic system. Formability of magnesium alloys depends significantly on temperature and strain rate and on grain size. Influence of softening processes on final mechanical properties is important [1-3].

2. METHODOLOGY

Samples with thickness graded in size (see Figure 1) were used for gaining data on MFS at various rolling modes [10]. The aim of the experiment was to reach MFS values of magnesium alloy AZ31, containing 2.82 % Al, 0.80 % Zn and 0.37 % Mn. Methodology of this experiment was described e.g. in [12,13] in details.

Each sample was carefully measured and then heated in the furnace to homogenization of structure in temperature 470 °C within 30 minutes. After pulling out of the furnace the sample was cooled down in the air to deformation temperature 200 to 450°C. Each sample was, after partial cooling, inserted for 5 minutes into the furnace heated to forming temperature. The heated sample was immediately after discharging the furnace rolled in the two-high stand A of the mill Tandem [4].

Adjustment of the roll gap and revolutions of rolls were changed (2.8 to 3.5 mm and 40 to 160 rpm, respectively) at various temperatures (200 to 450 °C). With particular samples the temperature was changed, as well as adjustment of the roll gap (and hence the total reduction corresponding to specific grades of each sample) and nominal revolutions of rolls N. By all here specified factors the achieved mean equivalent strain rate γ [s^{-1}] is given.

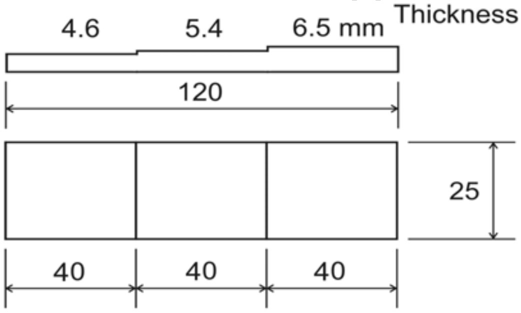

Figure 1. Initial shape of the sample with thickness graded in size

The registered total roll forces F [N] and actual revolutions of rolls, as well as dimensions of the rolled products, serve for automatic calculation of height strain $e_h = ln(H_0/H_1)$, mean strain rate γ and mean flow stress σ_m [MPa] for each element of the rolled sample. For the calculation the following formulae are used [5]:

$$\gamma = \frac{2}{\sqrt{3}} \cdot \frac{v_r}{\sqrt{R \cdot (H_0 - H_1)}} \cdot e_h \tag{1}$$

where H_0, or H_1 [mm] is entry, or exit thickness of the rolling stock in a given location; v_r [mm/s] is real circumferential speed of rolls with radius R [mm]. Mean flow stress is calculated as follows [14]:

$$\sigma_m = \frac{F}{Q_{Fv} \cdot \sqrt{R \cdot (H_0 - H_1)} \cdot B_m} \tag{2}$$

where Q_{Fv} is a forming factor, corresponding to a specific rolling mill stand, and B_m [mm] is mean width of the rolling stock in a given place (an average value of the width before and after rolling). The factor $\sqrt{R \cdot (H_0 - H_1)}$ represents contact length of the roll bite, i.e. l_d [mm]. Credibility of calculation of MFS is influenced most of all by an exact estimate of the forming factor, which – as matter of fact – transfers deformation resistance to values of equivalent flow stress (i.e. of that which corresponds to a defined uniaxial stress state). Values of Q_{Fv} for both stands of the rolling mill Tandem were obtained by previous research and they are described in relation to aspect ratio l_d / H_m by equations of type [6]:

$$Q_{Fv} = A - B \cdot exp\left(-C \cdot \frac{l_d}{H_m}\right) + exp\left(D \cdot \frac{H_m}{l_d}\right) \tag{3}$$

where A ... D are constants for a given facility, verified e.g. by comparison of power/force parameters, determined during laboratory rolling, torsion test, or industrial rolling; H_m [mm] is mean thickness in a given location.

Figure 2 shows the course of the measured roll force, together with values of the actual height reduction, strain rate, dimensions of the sample before and after rolling, and MFS values calculated from these values in particular parts of the sample.

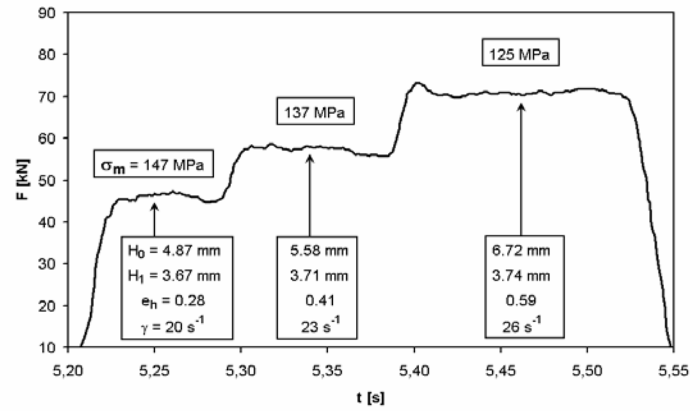

Figure 2. Example of measured roll force in rolling of the graded in thickness specimen at temperature 340 °C

3. DETERMINATION OF THE FORMING FACTOR

For calculation of MFS knowledge of the forming factor of the mill stand is necessary. The forming factor Q_{Fv} highlights influence of the mean stress exerting on the contact surface between the rolled metal and work rolls in the roll bite and in the rolling direction on the roll force size. The experiment was carried out with samples from low-carbon steel grades ČSN 11 523, 12 040 and Cr-Ni austenitic steel grade ČSN 17 251. For each of these materials a model of flow stress σ in the expression according to Eq. (4) was derived, based on continuous torsion tests. From these torsion tests values of mean flow stress σ_{m-t} may be calculated by integration and then compared with values of mean deformation resistance σ_{d-r}, obtained by rolling in analogous conditions.

$$\sigma = G \cdot e^{J} \cdot exp\left(-J \cdot \frac{e}{e_p}\right) \cdot \gamma^{\left(K-\frac{M}{T}\right)} \cdot exp(-N \cdot T) \tag{4}$$

where e is true strain [-], e_p is strain to peak [-], γ is strain rate [s^{-1}], T is temperature [K] and G ... N are material constants.

Pertinent material constants for three investigated steels were achieved in the statistic program Unistat by non-linear regression. Mean flow stresses σ_{m-t} corresponding to parameters of rolling of particular samples are reached by integration of the specific flow stress curve – see Eq. (4) - from initial to final deformation e_1 :

$$\sigma_{m-t} = \frac{1}{e_1} \cdot \int_0^{e_1} \sigma_e(e)de \tag{5}$$

Flat samples were rolled with a various reduction size at various forming temperatures. The initial height of individual samples varied in the range of 4 - 30 mm. For reaching of a wide range of the aspect ratio $l_d\ /H_m$, height reductions in the range of 10 – 50 % were realized, according to power possibilities of the laboratory mill TANDEM [4]. In this way for each sample values of mean deformation resistance σ_{d-r} were reached, in accordance with the equation:

$$\sigma_{d-r} = \frac{F_v}{l_d \cdot B_m} \tag{6}$$

The value of the forming factor Q_{Fv} for each sample was calculated by means of the mean deformation resistance achieved from roll forces σ_{d-r} and mean flow stress σ_{m-t}.

$$Q_{Fv} = \frac{\sigma_{d-r}}{\sigma_{m-t}} \tag{7}$$

A final equation for the forming factor, with evaluated constants according to Eq. (3) and corresponding to the mill stand A of the rolling mill Tandem, has the following expression [8, 9]:

$$Q_{Fv} = 4{,}0483 - 4{,}7198 \cdot exp\left(-0{,}0842 \cdot \frac{l_d}{H_m}\right) + exp\left(0{,}2475 \cdot \frac{H_m}{l_d}\right) \tag{8}$$

In Fig. 3 values of the forming factor related to aspect ratio for particular types of steel are seen, as well as dependence of Q_{Fv} based on the Eq. (8).

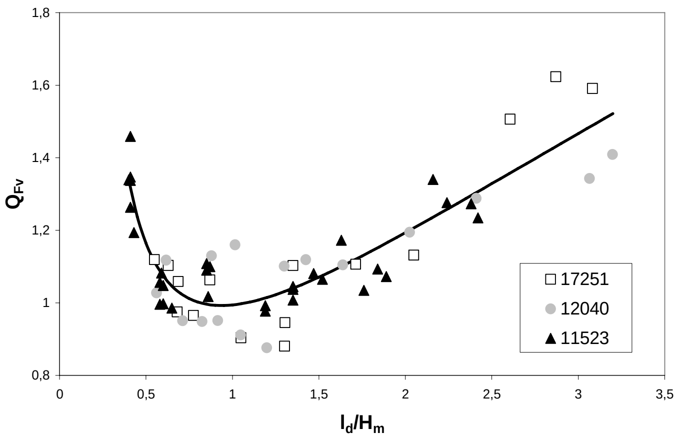

Figure 3. Graphic expression of the relationship between the forming factor of the stand A of laboratory mill Tandem and aspect ratio l_d/H_m (the curve corresponds to eq. (8))

4. DEFORMATION RESISTANCE MODELS

Based on previous experience [14] a simple model for description of MFS of the investigated material in relation to strain, temperature and strain rate was chosen [11]:

$$\sigma_m = a \cdot e_h^{\ b} \cdot exp(-c \cdot e_h) \cdot \gamma^d \cdot exp(-g \cdot T) \tag{9}$$

where σ_m is MFS which is predicted (calculated according to the developed model) and ***a*** ... ***g*** are calculated material constants, which were determined on the basis of methods of the multiple non-linear regression with use of the statistical program Unistat.

The basic form of Eq. (9) includes members for hardening, softening, speed and temperature. However, from viewpoint of practice it is desirable its further simplification, which would accelerate prediction of MFS. It was found by mathematical processing that the effect of strain rate (in range of ca 10 – 80 s^{-1}) is statistically insignificant in the given case, and that is why a simplified equation in the following form was derived:

$$\sigma_m = 357.5 \cdot e_h^{\ 0.006} \cdot exp(-0.30 \cdot e_h) \cdot exp(-0.00234 \cdot T) \tag{10}$$

By further investigation it was found that influence of deformation, expressed only by one member, is sufficient and the subsequent simplification of the mathematical model has no significant influence on the value of gained MFS, calculated according to this equation. A final, totally simplified form of the mathematical model with quantified constants looks like this:

$$\sigma_m = 352.2 \cdot exp(-0.28 \cdot e_h) \cdot exp(-0.00232 \cdot T) \tag{11}$$

A graphic confirmation of the possible simplification of the equation of type (9) one can see in specific graphs in Fig. 4, which show relative deviations Δ of values MFS calculated according to Eqs. (10) and (11) from values determined in experimental way. The deviation Δ was calculated as a quotient of the residuum and the MFS value which was found out in the experimental way.

5. CONCLUSIONS

Based on laboratory rolling of flat samples in the temperature range of 200 – 450 °C, actual height reductions of ca 0.2 – 0.7 and strain rates ca 10 – 80 s-1, the MFS values of the magnesium alloy of type AZ31 were obtained, after recalculation from roll forces. Model of the forming factor for mill stand A was developed from values measured in rolling of flat products in the laboratory rolling mill Tandem and from a model describing flow stress curves of these steels on the basis of hot torsion tests. The resulting equation describes with good accuracy the function $Q_{Fv} = f(l_d/H_m)$ in the whole range of applied temperatures and deformations, regardless of friction coefficient.

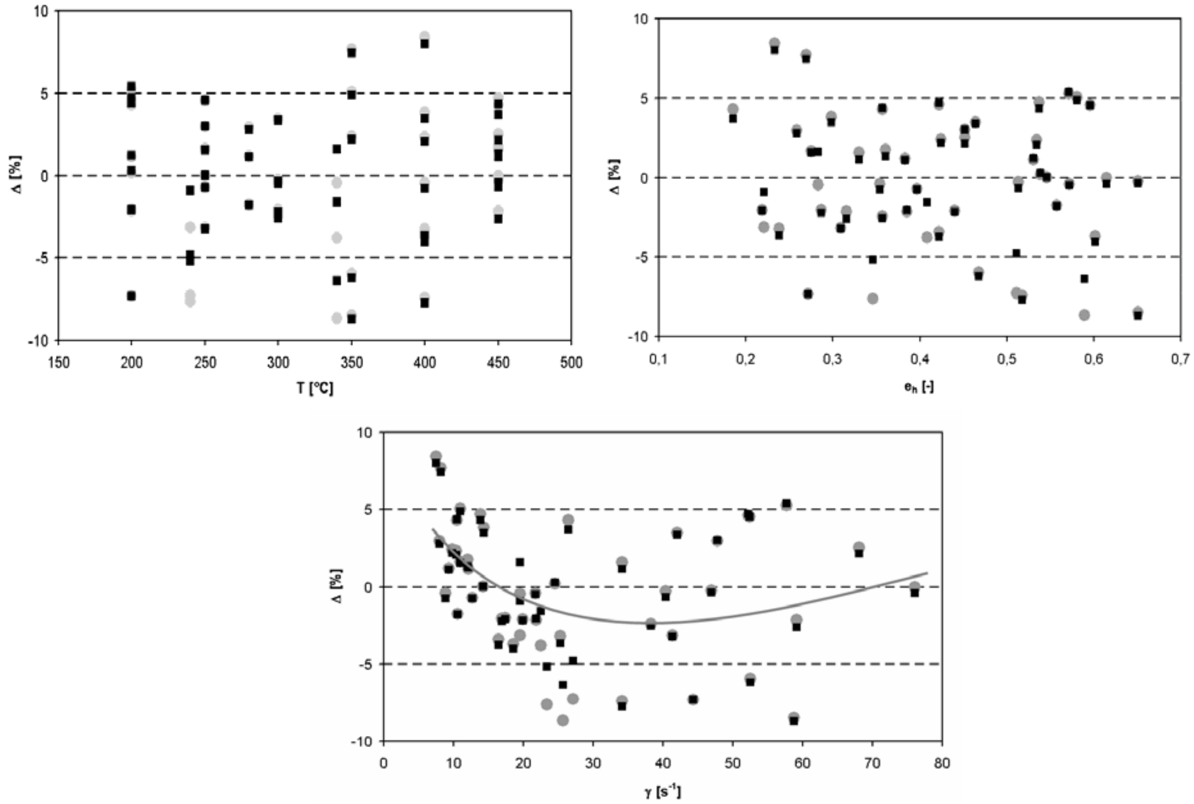

Figure 4. Relative errors of the MFS values calculated according to Eqs. (10) ● and (11) ■ in comparison with the values obtained experimentally

As far as accuracy of the developed models of deformation resistance of the AZ31 alloy is concerned, the root of mean square error 6.7 and value $R^2 = 0.958$ resulted from calculations for Eq. (10), for Eq. (11) the root of mean square error was 6.2 and value $R^2 = 0.964$. It means that the simplified Eq. (11) describes the given relationship of deformation resistance even better than Eq. (10). The satisfactory accuracy of calculation of MFS found according to both equations can also be seen from graphs in Fig. 4. Dispersion variance of experimental deviations and deviations of MFS values which were found by recalculation according to Eqs. (10) or (11) is satisfactory in the whole range and relative errors do not surpass ±10 %. So, the model (11) would be very suitable for implementation in the control system of the mill which rolls strips from the given alloy in operational conditions.

Compared with articles [7, 8, 12, 13], which were published sooner and used for description of MFS a similar mathematical model, a certain trend in the case of relative deviations Δ for strain and strain rate in apparent. In contrast with previous works where a trend of these deviations was apparent just in the case of temperature, the dispersion of these deviations in dependence on temperature is here very uniform.

Neglecting of the speed member in the resulting model of MFS was possible by a relatively narrow range of applied strain rates, and also by the fact that a relationship between variables e_h and γ exists – see Eq. (1).

The unconventional description of deformation influence only by the softening member in Eq. (11) can be justified only in the case of description of the mean flow stress, it would not be applicable for description of the stress-strain curve, mainly in initial phases of fierce increase in deformation resistance. The developed models for MFS have strongly utilitarian character and therefore the needed experimental data were obtained only for actual height reductions eh higher than ca 0.2.

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