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CONVECTION EFFECTS ON FLOW PAST AN INCLINED PLATE WITH VARIABLE SURFACE TEMPERATURES IN WATER AT 4^0C

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Abstract
This paper deals with the unsteady free convection flow of water at its maximum density past a semi-infinite inclined plate with variable surface temperature of the plate varying as x^n, a power function of the distance from the leading edge. The governing partial differential equations are transformed into a set of dimensionless governing equations, which are solved numerically using an implicit finite-difference method. The dimensionless governing equations are unsteady, two-dimensional, coupled and non-linear integro partial differential equations. The transient velocity and temperature profiles, local and average skin friction and the Nusselt number are shown graphically. The effects of exponent n on velocity and temperature profiles, skin friction and rate of heat transfer have been analysed. It is observed that the velocity decreases owing to lower temperature of the water.
Keywords:
finite-difference, water, skin friction, velocity, heat transfer

1. INTRODUCTION

Buoyancy driven flows are of common occurrence in technological, atmospheric, and oceanic phenomena, the buoyancy stratification being achieved often by the temperature field.

Due to numerous applications, the study of the natural convection heat transfer from a different geometry of the surfaces has received much attention in recent years both theoretically and experimentally. On the other hand, very little attention has been given for the problem of natural convection over an inclined plate or slightly inclined to the horizontal plate. Free convection heat transfer from an inclined surface is very frequently encountered in engineering devices and natural environment. The unsteady natural convection flow past a semi-infinite vertical plate was first solved by Hellums and Churchill [1] using explicit finite difference scheme. As explicit finite difference scheme has its own deficiencies, a more efficient implicit finite difference scheme has been used by Soundalgekar and Ganesan [2]. Natural convection over an inclined plate was first studied experimentally by Rich [3]. Chen *et al.* [4] have obtained a numerical solution for the problem of natural convection over an inclined plate with variable surface temperature. Ganesan and Ekambavannan [5] studied the problem of transient free convective flow of an incompressible viscous fluid past a semi-infinite isothermal inclined plate by an implicit finite difference method. Ekambavannan and Ganesan [6] solved the unsteady natural convection boundary layer flow over a semi-infinite inclined plate with the wall temperature varying as the axial coordinate using an implicit finite difference scheme, which is unconditionally stable. As the problem was governed by nonlinear system of coupled equations, it was solved by finite-difference method. The fluid considered was air, water at normal temperature and pressure. However, if the water is assumed to be of 4^0C, then its density is maximum and hence the usual Boussinesq's approximation is not valid. Under normal temperature and pressure, the difference between the density at any two specific points is a linear function of the difference between the temperature at the same two specific points and is defined as

$$\Delta\rho = -\rho\beta(\Delta T)$$

where β is the co-efficient of thermal expansion.

However, in case of water at 4^0C, this relation is modified to

$$\Delta\rho = -\rho\,\gamma\,(\Delta T)^2$$

where $\gamma = 8X10^{-6}(^0C)^{-2}$

Taking this fact into account, Soundalgekar [7] studied the free convection flow of water at 4^0C past a semi-infinite vertical plate by integral method. Jaiswal *et al*. [8] studied unsteady free convection effects on flow past an impulsively started infinite vertical plate in water at 4^0C. Takhar and Ram [9] studied MHD forced and free convection flow of water at 4^0C through a Porus Medium in the presence of a uniform transverse magnetic field. The coupled non-linear differential equations are solved by shooting numerical technique for the point boundary value problems. Kumaran and Pop [10] analyzed a theoretical investigation of the boundary layer flow over a vertical flat plate embedded in a porous medium filled with water at 4^0C. In a large number of applications, the surface heating conditions are non uniform and the induced buoyant flow is laminar. So we considered variable surface temperatures. However, unsteady free convection flow of water at 4^0C past semi-infinite inclined plate with variable surface temperatures has not been studied in the literature because of non-linear, unsteady, coupled and integro partial differential equations of the problem. Now it is proposed to study the problem of free convection flow of water at 4^0C past a semi-infinite inclined plate with variable surface temperatures.

2. FORMULATION OF THE PROBLEM

The angle of inclination of the plate with the horizontal is assumed to be ϕ. The x-axis is measured along the plate and y-axis is measured along normal to the plate. Initially, the plate and the fluid are assumed to be at the same temperature T'_∞. Subsequently when time t´>0, the temperature on the plate is supposed to be raised suddenly and maintained at a higher temperature in the form $T_w'(x) = T_\infty' + ax^n$. Then under the usual Boussinesq's approximation, the flow of water at 4^0C can be shown to be governed by the following equations [Ref:4,7] The schematic diagram of the of the physical system is shown in Figure1.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\gamma\cos\phi\frac{\partial}{\partial x}\int_y^\infty (T' - T'_\infty)^2 dy + g\gamma\sin\phi(T' - T'_\infty)^2 + \nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2} \quad (3)$$

Initial and Boundary conditions are as follows:

$$
\begin{aligned}
&t' \le 0 : u = 0,\quad v = 0,\quad T' = T'_\infty, \\
&t' > 0 : u = 0,\ v = 0,\ T_w'(x) = T_\infty' + ax^n, && \text{at } y = 0 \\
&\qquad u = 0,\quad T' = T'_\infty, && \text{at } x = 0 \\
&\qquad u \to 0,\quad T' \to T'_\infty, && \text{as } y \to \infty
\end{aligned} \tag{4}
$$

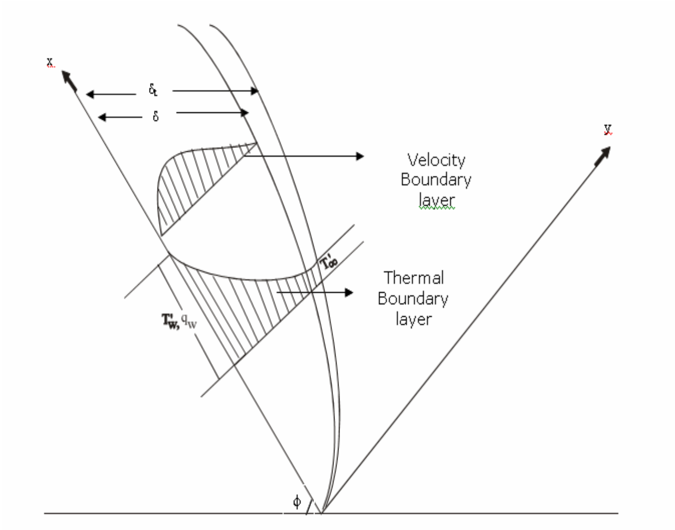

Fig.1 Schematic diagram of Physical System

Introducing the following non-dimensional quantities

$$
X = \frac{x}{L},\quad Y = \frac{y}{L} Gr^{1/4},\quad U = \frac{uL}{\upsilon} Gr^{-1/2},\quad V = \frac{vL}{\upsilon} Gr^{-1/4},
$$
$$
t = \frac{\upsilon t'}{L^2} Gr^{1/2},\qquad T = \frac{T' - T'_\infty}{T'_w(L) - T'_\infty}, \tag{5}
$$
$$
Gr = \frac{g\gamma L^3 (T'_w(L) - T'_\infty)}{\upsilon^2},\qquad Pr = \frac{\nu}{\alpha},
$$

Governing equation reduces to the following form:

$$
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}
$$

$$
\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = Gr^{-1/4}\cos\phi \frac{\partial}{\partial X}\int_Y^\infty T^2 dY + T^2 \sin\phi + \frac{\partial^2 U}{\partial Y^2} \tag{7}
$$

$$
\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \tag{8}
$$

The corresponding initial and boundary conditions in non-dimensional quantities are given by

$$
\begin{aligned}
&t \le 0 : U = 0,\quad V = 0,\quad T = 0, \\
&t > 0 : U = 0,\quad V = 0,\quad T = X^n, && \text{at } Y = 0 \\
&\qquad U = 0,\quad T = 0, && \text{at } X = 0 \\
&\qquad U \to 0,\quad T \to 0, && \text{as } Y \to \infty
\end{aligned} \tag{9}
$$

3. NUMERICAL TECHNIQUE

An implicit finite difference scheme of Crank-Nicolson type has been used to solve the governing non-dimensional equations (6) – (8) under the initial and boundary conditions (9). The method of solving the above finite difference equations using Crank-

Nicolson type has been discussed by Soundalgekar and Ganesan [2]. The region of integration is considered as a rectangle with sides X_{max} (= 1.0) and Y_{max} (= 15.0) where Y_{max} corresponds to Y = ∞ which lies very well outside the momentum and thermal boundary layers. Appropriate mesh sizes ΔX = 0.05, ΔY = 0.25 and time step Δt = 0.01 are considered for calculations. Computations are repeated until the steady state is reached. The steady-state solution is assumed to have been reached when the absolute difference between values of velocity U as well as temperature T at two consecutive time steps are less than 10^{-5} at all grid points. The Crank-Nicolson implicit finite difference scheme is always stable and convergent as explained in the Ref [5].

4. RESULTS AND DISCUSSION

The Prandtl number for water at 4⁰C is 11.4, whereas at normal temperature of 20⁰C, it is 7.0. So to understand the effect of cooling of the temperature of water, we have replaced T^2 by T and computed U and T for Pr = 7.0

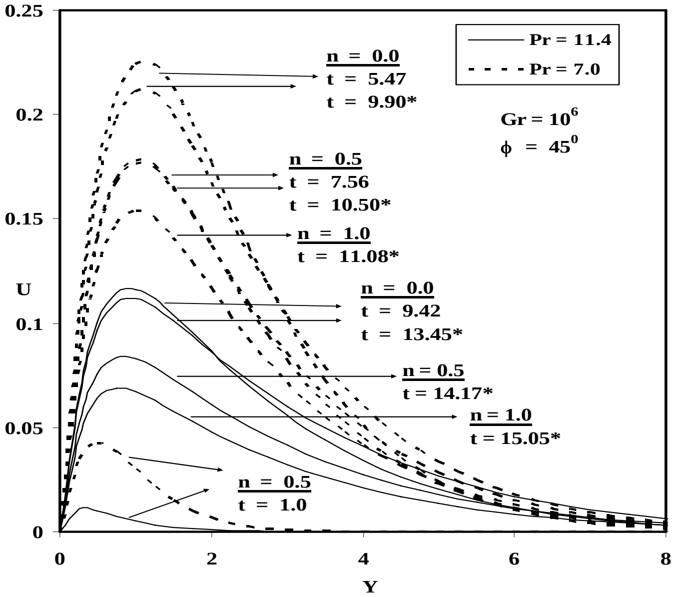

Fig.2 Transient velocity profiles at X=1.0 for different n (* - steady state)

Transient velocity and temperature profiles are shown in Figures 2 and 3 respectively for different values of exponent n. Temporal maximum in velocity and temperature profiles are observed. When n increases, the temperature gradient along the plate near the leading edge decreases. That is, the impulsive force along the plate decreases with increasing n. Due to these facts, the difference between the temporal maximum and steady state value decreases with increasing n. The figure 2 shows that the velocity decreases owing to lowering temperature of the water. Physically, this is due to the density becoming maximum which opposes the motion of water. Time taken to reach the steady state is more for lowering temperature of water. The result indicates that for small temperature differences the magnitudes of the convection currents are significantly reduced from those that what would obtain for water at 20⁰C. The increase in the value of n reduces temperature on the surface up to X=1.0. Therefore velocity decreases with increasing value of n.

To illustrate the effects of Grashof number and inclination angle ϕ on velocity and temperature, Figures 4 and Figure 5 present the distribution of U and T respectively at steady state level near the plate at X = 1.0. When ϕ increases, the normal component of the buoyancy force decreases near the leading edge, which causes an impulsive driving force to fluid motion along the plate. That is, the impulsive force along the plate decreases with increasing ϕ. Time taken to reach the steady state decreases as ϕ increases. Since the tangential component of the buoyancy force increases with ϕ and dominates in the down stream, the steady state velocity increases with ϕ but lower temperature is experienced for systems of a higher value of ϕ. It is observed that the time taken to reach the steady state is more for lower values of Grashof number compared to higher value of Grashof number. It is also observed that velocity decreases with the increasing value of Gr. From the numerical results it is observed that there is no appreciable change in the temperature distribution due to change in Gr.

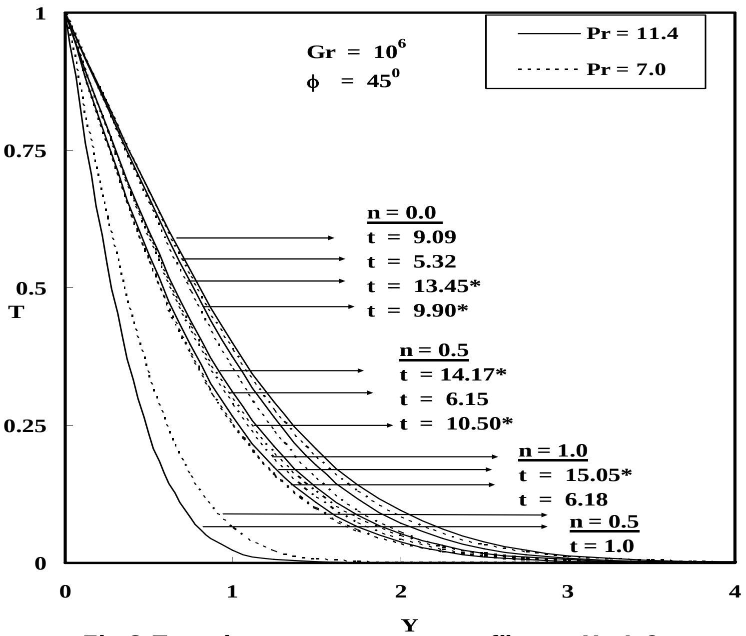

Fig.3 Transient temperature profiles at X=1.0 for different n (* - steady state)

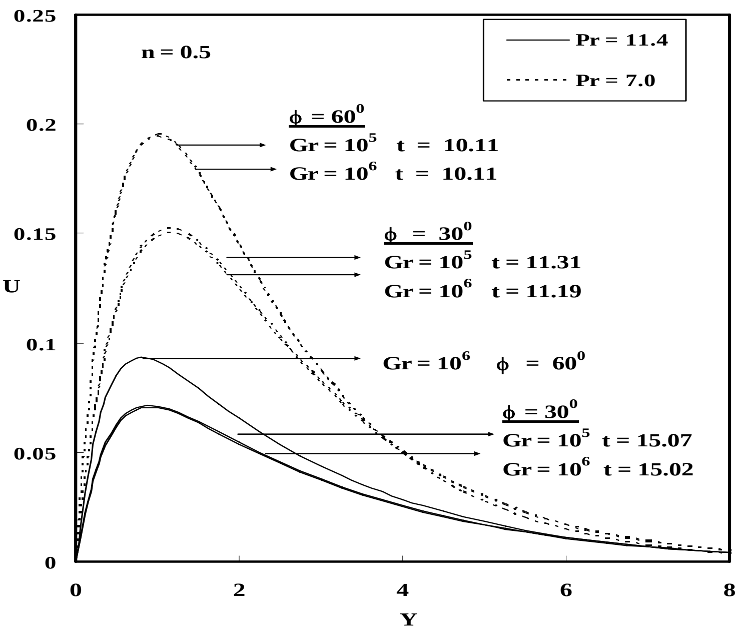

Fig.4 Steady state velocity profiles at X=1.0 for different Gr and ϕ

Local skin friction is shown in Figure 6. The local skin friction increases due to a rise in temperature from 4⁰C to 20⁰C. The local wall shear stress decreases as n increases or ϕ decreases. This is because of the fact that the velocity gradient decreases near the plate as n increases or ϕ decreases which are shown in Figure 2 and 4 respectively.

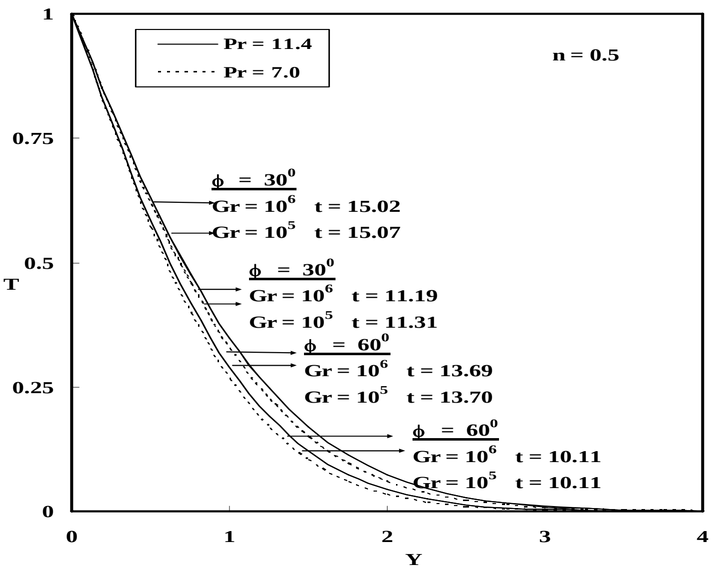

Fig.5 Steady state temperature profiles at X=1.0 for different Gr and ϕ

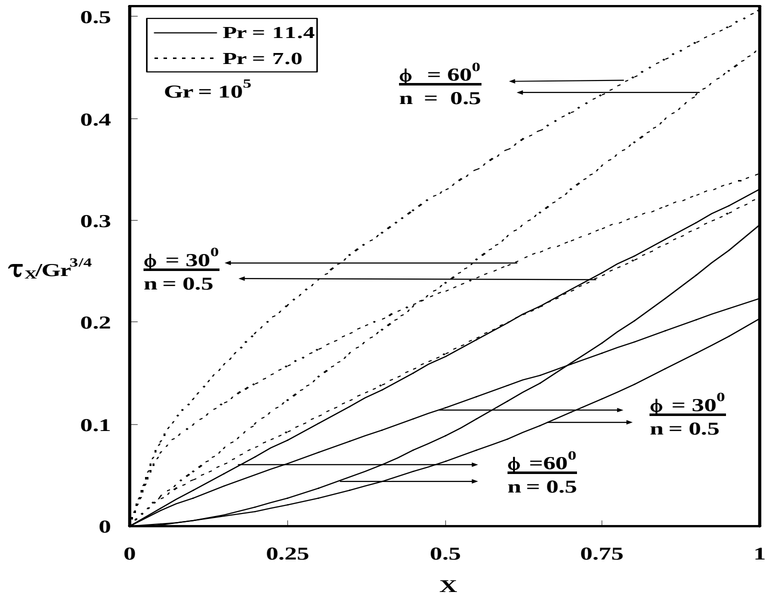

Fig6. Local skin friction

Local Nusselt number is shown in Figure 7 and it is observed that Nusselt number increases with n. However, it is observed that the above trend is reversed near the leading edge. Local Nusselt number increases with ϕ. The rate of heat transfer decreases due to a fall in temperature of water from 20⁰C to 4⁰C.

Average skin friction and Nusselt number are plotted in Figures 8 and 9 respectively. Average skin friction decreases as n increases or ϕ decreases. This is due to the fact that the tangential component of buoyancy force, which directly contributes to the motion in the streamwise direction, decreases with ϕ, and the velocity gradient near the plate decreases as n increases. Figure 9 show that there is no change in average Nusselt number in the initial period with respect to ϕ or n. This reveals that initially heat transfer is due to conduction only. An increase in the value of n decreases the average Nusselt number. The average skin friction increases due to rise in temperature of water from 4⁰C to 20⁰C. Average rate of heat transfer decreases due to rise in temperature from 4⁰C to 20⁰C up to time t = 4.0 and then reverse trend is observed.

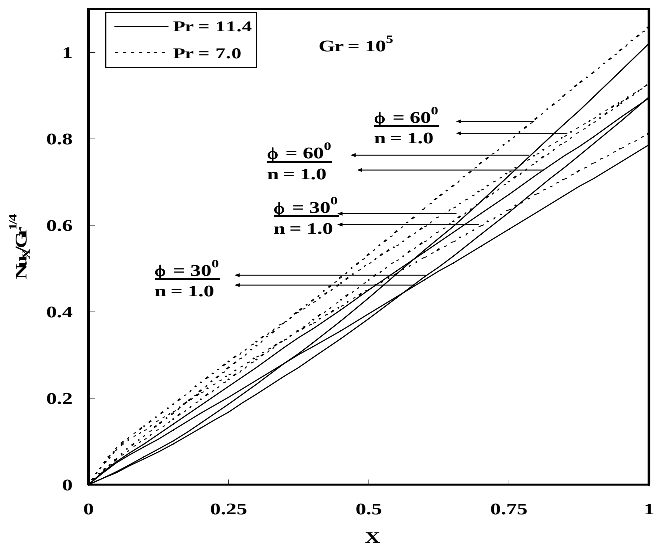

Fig.7 Local Nusselt Number

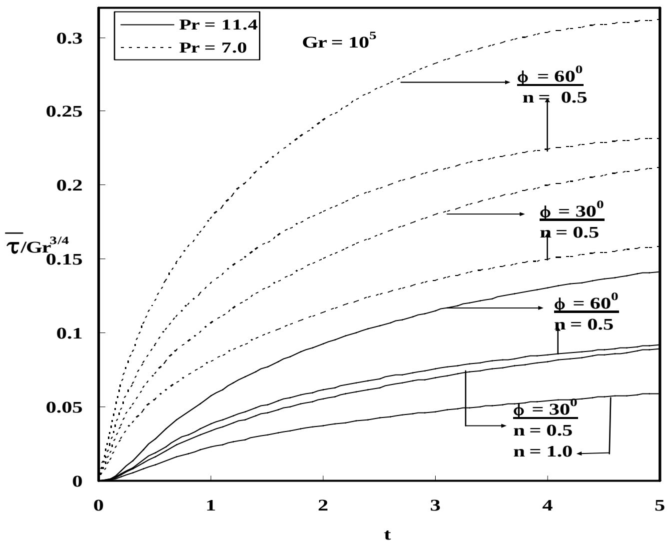

Fig.8 Average skin friction

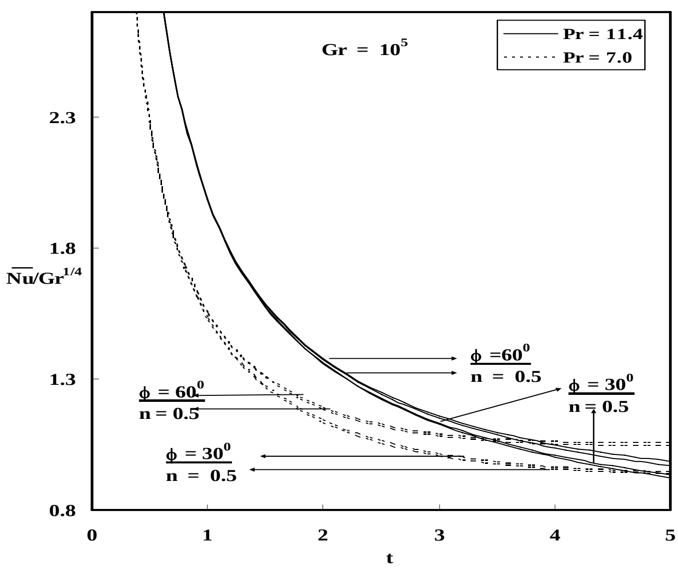

Fig.9 Average Nusselt Number

5. CONCLUSION

A detailed numerical study has been carried out for the convection flow past an inclined plate with variable surface temperature in water at 4⁰C. The governing boundary layer equations are transformed into non-dimensional form and then solved by an implicit finite difference scheme of Crank-Nicolson Method. Conclusions of the study are as follows:

- Velocity decreases owing to lowering temperature of the water.
- Velocity decreases with increasing value of exponent n.
- Time taken to reach the steady state is more for lower values of Grashof number compared to higher value of Grashof number.
- The local wall shear stress decreases as n increases or ϕ decreases.
- The rate of heat transfer decreases due to a fall in temperature of water from 20⁰C to 4⁰C.
- The average skin friction increases due to rise in temperature of water from 4⁰C to 20⁰C.

NOMENCLATURE

a constant
Gr Grashof number
g acceleration due to gravity
L reference length
n exponent in power law variation for surface temperature
$\overline{Nu}$ dimensionless average Nusselt number
Nu_X dimensionless local Nusselt number
Pr Prandtl number
T' temperature
T dimensionless temperature
t' time
t dimensionless time
u,v velocity components in x, y directions respectively
U, V dimensionless velocity components in X, Y directions respectively
x spatial coordinate along the plate
X dimensionless spatial coordinate
y spatial coordinate along upward normal to the plate
Y dimensionless spatial coordinate along upward normal to the plate

Greek symbols

α thermal diffusivity
β volumetric coefficient of thermal expansion
γ $8X10^{-6}(^0C)^{-2}$
ϕ angle of inclination of the plate with horizontal
ν kinematic viscosity
τ_X dimensionless local skin friction
$\bar{\tau}$ dimensionless average skin friction

Subscripts

w conditions on the wall
∞ free stream condition

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