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THERMOELASTIC DISTURBANCES IN TRANSVERSELY ISOTROPIC HALF-SPACE

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ABSTRACT
In this article attempt is made to study the fracas due to a thermal line load in a homogeneous transversely isotropic half-space in linearized theory of generalized thermo elasticity. A combination of Fourier and Laplace transform technique is applied to obtain the solutions of governing equations. The transformed solutions are then inverted using Cagniard technique for small times. The results obtained theoretically, for temperature, thermal stresses are computed numerically for a crystal of zinc, and found that variations in stresses and temperature are more prominent at small times and decrease with passage of time. The results obtained theoretically are represented graphically.
KEY WORDS: Transversely isotropic, thermo elasticity, thermal stresses, Cagniard technique. AMS Subject Classification: 74A15, 74F05, 74H45, 74K20, 74L05

1. INTRODUCTION

Study of thermally induced disturbances in anisotropic bodies is essential for a comprehensive study of their response due to an exposure to temperature fields, which may in turn occur in service or during the manufacturing stages. For example, during the curing stages of filament bound bodies, thermal disturbances may be induced from the heat buildup and cooling processes. The level of these disturbances may well exceed the ultimate strength. The theory of thermoelasticity Nowacki [15,16] that includes such thermal disturbances has aroused considerable interest in the last century, but a systematic research started only after thermal waves—called second sound—were first measured in materials like solid helium, bismuth, and sodium fluoride.

Thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past thirty years. The literature dedicated to such theories is quite large and its detailed review can be found in Chandrasekharaiah [3, 4]. Lord and Shulman [13], Green and Lindsay [8], and Hetnarski and Ignaczak [12] are among the non-classical theories, which are common use in engineering applications.

Lord and Shulman theory introduces a single time constant to dictate the relaxation of thermal propagation, as well as the rate of change of strain rate, and the rate of change of heat generation. Green and Lindsay theory, on the other hand, the thermal and thermo-mechanical relaxations are governed by two different time constants. Dhaliwal and Sherief [6] extended theory of generalized thermoelasticity to anisotropic solids. Hawwa and Nayfeh [11] studied the general problem of thermoelastic waves in anisotropic periodically laminated composites. Verma and Hasebe [18, 19] studied the wave propagation in plates of general anisotropic media in generalized thermoelasticity. Verma [20, 21] studied thermoelastic problems by considering equation for anisotropic heat conducting solids with thermal relaxation time. Harinath [9, 10] considered the problems of surface point and line source over a homogeneous isotropic thermoelastic half-space in thermoelasticity. De Hoop [9] modified and used a method originally presented by Cagniard [2] to solve the disturbances that are generated by an impulsive, concentrated load applied along a line on the free surface of a homogeneous isotropic elastic half-space. Nayfeh and Nasser [14] developed the displacements and temperature fields in a homogeneous isotropic generalized thermoelastic half-space subjected on the free surface to an instantaneously applied heat source using the Cagniard-De Hoop [9] method Sharma [17] studied the Transient Generalized thermoelastic waves in a transversely isotropic Halfspace using the same method.

In this paper, Cagniard-De Hoop method is used to study the transient behavior of homogeneous transversely isotropic linearized thermoelastic material. The motions are caused in the half-space by a thermal line load on its free surface. The thermal relaxation time of the heat conduction is also included in the analysis to ensure that thermal wave speed remains finite. In using Cagniard-De Hoop method, the strong coupling between thermal and elastic motions, which, however, suggests that we seek solutions for small values of the thermoelastic coupling coefficient. Therefore we express solutions in terms of a small thermoelastic coupling coefficient. Only the approximated short time solutions are considered for thermoelastic response due to the existence of the thermal damping term, which makes the short time solution meaningful. A combination of Laplace and Fourier transforms is applied to obtain the solutions of governing equations of transversely isotropic thermoelastic solid half-space, which are subjected to thermal line load on its free surface. The resulting equations are then inverted using Cagniard-De Hoop small times. The results obtained theoretically have been verified numerically and illustrated graphically for single crystal of zinc.

2. BASIC GOVERNING EQUATIONS AND FORMULATION

The basic field equations of generalized thermoelasticity governing thermoelastic interaction in homogeneous anisotropic solids proposed by Dhaliwal and Sherief [6] are:
Equation of Motion

$$\sum_{j=1}^{3}\left(\frac{\partial \tau_{ij}(\mathbf{u})}{\partial x_j}\right)+f_i=\rho\frac{\partial^2 u_i}{\partial t^2} \qquad i=1,2,3 \tag{2.1}$$

Energy equation

$$K_{ij}T_{,ij}-\rho C_e\left(\dot{T}+\tau_0\ddot{T}\right)=T_0\beta_{ij}\left(\dot{u}_{i,j}+\tau_0\ddot{u}_{i,j}\right) \tag{2.2}$$

Stress- Strain-Temperature relations

$$\tau_{ij}=C_{ijkl}e_{kl}-\beta_{ij}T,\quad \beta_{ij}=C_{ijkl}\alpha_{kl} \tag{2.3}$$

the summation convention is implied; ρ is the density, t is the time, u_i is the displacement in the x_i direction, K_{ij} are the thermal conductivities, C_e and τ_0 are respectively the specific heat at constant strain, and thermal relaxation time, σ_{ij} and e_{ij} are the stress and strain tensor respectively; β_{ij} are thermal moduli; α_{ij} is the thermal expansion tensor; T is temperature; and the fourth order tensor of the elasticity C_{ijkl} satisfies the (Green) symmetry conditions:

$$c_{ijkl}=c_{klij}=c_{ijlk}=c_{jikl}\text{, and }\alpha_{ij}=\alpha_{ji},\ \beta_{ij}=\beta_{ji},\ K_{ij}=K_{ji} \tag{2.4}$$

Now specializing the Eq. (2.1) to Eq. (2.3) for temperature T (x, y, z, t) and the displacement vector $\mathbf{u}$ (x, y, z, t) = (u, 0, w) in the absence of the body forces and heat source heat conducting transversely isotropic elastic half-space, at reference temperature T_0.

$$\left(C_{11}\frac{\partial^2}{\partial x^2}+C_{44}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)u+(C_{13}+C_{44})\frac{\partial^2 w}{\partial x\partial z}=\beta_1\frac{\partial T}{\partial x}, \tag{2.5}$$

$$\left(C_{44}\frac{\partial^2}{\partial x^2}+C_{33}\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}\right)w+(C_{13}+C_{44})\frac{\partial^2 u}{\partial x\partial z}=\beta_3\frac{\partial T}{\partial z}, \tag{2.6}$$

$$K_1 \frac{\partial^2 T}{\partial x^2} + K_3 \frac{\partial^2 T}{\partial z^2} - \rho C_e \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T = T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\beta_1 \frac{\partial u}{\partial x} + \beta_3 \frac{\partial w}{\partial z}\right), \quad (2.7)$$

where

$$\beta_1 = (C_{11} + C_{12})\,\alpha_1 + C_{13}\,\alpha_3;\ \ \beta_3 = 2\,C_{13}\,\alpha_1 + C_{33}\,\alpha_3, \quad (2.8)$$

C_{ij} being the elastic parameters, C_e and τ_o are the specific heat at constant strain and thermal relaxation time respectively. K_3, K_1 and α_3, α_1 are the coefficients of the thermal conductivities and linear thermal expansions respectively, along and perpendicular to axis of symmetry.

Using the dimensionless quantities (Appendix-A) into equations and the plane of isotropy is perpendicular to z-axis, which is normal into the half-space. The disturbance in the beginning undisturbed elastic thermoelastic solid is caused by abruptly applied thermal line load on the free surface. Thermal Load applied is symmetrically with respect to the y-axis. Considering fixed coordinate system Oxyz with origin being any point of the plane boundary z = 0. The boundary conditions (on suppressing the primes throughout)

$$\tau_{zz} = 0;\ \tau_{xz} = 0;\ \frac{\partial T}{\partial z} = Q_0\,\delta(x)\,f(t),$$

at the surface z = 0, become

$$(c_3 - c_2)\frac{\partial u}{\partial x} + c_1 \frac{\partial w}{\partial z} - \bar{\beta} T = 0, \quad (2.9)$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad (2.10)$$

$$\frac{\partial T}{\partial z} = Q_0^*\,\delta(x)\,f(t), \quad (2.11)$$

where $Q_0^* = v_1\, Q_0 / T_o$. The condition at infinity requires that the solutions be bounded as z becomes large. Finally, the initial conditions are such that the medium is at rest for t < 0.

If we take

$$C_{11} = C_{33} = \lambda + 2\,\mu;\ C_{44} = 2\mu;\ C_{13} = \lambda,$$
$$K_3 = K_1 = K;\ \alpha_1 = \alpha_3 = \alpha_t;\ \beta_1 = \beta_3 = (3\lambda + 2\,\mu)\alpha_t, \quad (2.12)$$

then equations (2.1) to (2.3) reduce to the corresponding form for an isotropic body, with Lamé's parameter λ, μ; thermal conductivity K and the coefficients of linear thermal expansion α_t.

3. ANALYSIS

To obtain the solution of the problem following Nayfeh and Nasser [14] and Sharma [17], following we apply the Laplace transform with respect to time and the exponential Fourier transform with respect to the *x*-co-ordinate to the system of equations (2.7) to (2.11). The appropriate solution of the resulting equation is then constructed and subsequently inverted. The Laplace and the exponential Fourier transforms are defined respectively as

$$L\left[\varphi(x,t)\right] = \int_0^\infty \varphi(x,t) e^{-pt}\, dt = \bar{\phi}(x,p), \quad (3.1)$$

$$F\left[\bar{\phi}(x,p)\right] = \int_{-\infty}^{\infty} \bar{\phi}(x,p) e^{iqx}\, dx = \hat{\phi}(q,p).$$

With this, a formal solution of equations (2.7) to (2.11) is given by

$$(u, w, T) = L^{-1}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} \sum_{k=1}^{3} (a_{1k}, a_{2k}, a_{3k}) e^{-(m_k z - iqx)} dq\right\}, \quad (3.2)$$

where L^{-1} designate the inverse Laplace transform and where we have set

$$a_{1j} = \frac{iq\,\bar{f}\,Q_0^*}{c_1\,c_2\Delta} M_{1j} R_{1j}, \quad (3.3)$$

$$a_{2j} = \frac{\bar{f}\ Q_0^*}{c_1\ c_2 \Delta}\ M_{1j}\, m_j\, S_j, \tag{3.4}$$

$$a_{3j} = \frac{-\bar{f}\ Q_0^*}{\Delta}\ M_{1j}\,(m_j^2 - m_{10}^2)\,(m_j^2 - m_{20}^2), \qquad j = 1, 2, 3, \tag{3.5}$$

$$M_{1i} = \begin{vmatrix} m_k d_k & l_k \\ m_j d_j & l_j \end{vmatrix} \quad i \neq j \neq k = 1, 2, 3. \text{ Taking } i, j \text{ and } k \text{ in the cyclic order.} \tag{3.6}$$

$R_{1j} = (c_1 - c_3\bar{\beta})\, m_i^2 - c_2\, q^2 - p^2,$

$S_j = c_2\, \bar{\beta}\, m_j^2 - c_2 q^2 - p^2, \quad j = 1, 2, 3$

$$\Delta = \sum_{j=1}^{3} m_j (m_j^2 - m_{10}^2)(m_j^2 - m_{20}^2) M_{1j}\ . \tag{3.7}$$

$$l_j = [c_1 c_2\, \bar{\beta}\, (m_{10}^2 + m_{20}^2) + c_1((c_3 - \bar{\beta})q^2 - \bar{\beta}\, p^2) - (c_3 - c_2)(c_1 - c_3\bar{\beta})q^2]m_j^2 - c_1 c_2\, \bar{\beta}\, m_{10}^2\, m_{20}^2 + (c_3 - c_2)\, q^2\,(c_2\, q^2 + p^2) \tag{3.8}$$

$$d_j = (c_1 - c_3\, \bar{\beta} + c_2\bar{\beta})m_j^2 - (c_3 - c_2 - \bar{\beta})q^2 - (1 + \bar{\beta})p^2, \qquad j = 1,2,3, \tag{3.9}$$

$$m_1^2 + m_2^2 + m_3^2 = m_{10}^2 + m_{20}^2 + m_{30}^2 + \varepsilon_1\, \bar{\beta}\, \tau p^2 / \bar{k}\, c_1,$$

$$m_1^2\, m_2^2 + m_2^2\, m_3^2 + m_3^2\, m_1^2 = m_{10}^2\, m_{20}^2 + m_{20}^2\, m_{30}^2 + m_{30}^2\, m_{10}^2 + \frac{\varepsilon_1 \tau p^2}{\bar{k} c_1 c_2}\{(c_1 - 2c_3\, \bar{\beta} + \bar{\beta}^2)\, q^2 + p^2\},$$

$$m_1^2\, m_2^2\, m_3^2 = m_{10}^2\, m_{20}^2\, m_{30}^2 + \varepsilon_1 \frac{\tau\,(c_2 q^2 + p^2)\, p^2 q^2}{\bar{k} c_1\ c_2}, \tag{3.10}$$

$$m_{10}^2 + m_{20}^2 = (Pq^2 + J\, p^2)/c_1 c_2, \tag{3.11a}$$

$$m_{10}^2\, m_{20}^2 = (q^2 + p^2)\,(c_2\, q^2 + p^2)/c_1 c_2, \tag{3.11b}$$

$$m_{30}^2 = (q^2 + \tau\, p^2)/\,\bar{k}, \tag{3.11c}$$

$$P = c_1 + c_2^2 - c_3^2;\ J = c_1 + c_2,\ \tau = \tau_o + 1/p,\ \bar{f}(p) = \int_0^\infty f(t)\, e^{-pt}\, dt\ . \tag{3.11d}$$

Also stresses are given as

$$\tau_{xx} = L^{-1}\left\{-\frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{k=1}^{3}(iqa_{1k} + (c_3 - c_2)m_k a_{2k} + a_{3k})\exp[-(m_k z - iqx)]dq\right\},$$

$$\tau_{zz} = L^{-1}\left\{-\frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{k=1}^{3} m_k (c_3 - c_2)\, a_{2k} + ic_1\, q\, a_{2k} + \bar{\beta}\, a_{3k}\ )\exp[-(m_k z - iqx)]dq\right\},$$

$$\tau_{xz} = L^{-1}\left\{-\frac{c_2}{2\pi}\int_{-\infty}^{\infty}\sum_{k=1}^{3}(m_k a_{1k} + iqa_{2k})\exp[-(m_k z - iqx)]dq\right\}\ . \tag{3.12}$$

To obtain solutions that are bounded as $z \to \infty$, we require that m_k have positive real parts. We observe that equation (3.9) pertain to the coupled dilatational, distortional and thermal waves. To find explicit expression for m_1, m_2 and m_3, we seek solutions of (3.9) for small values of the thermoelastic coupling ε_1, coupling coefficient between the field of temperature and the field of strain, for the plane state of strain. Since coupling coefficient is a physical characteristic of the material, so the effect of damping and dispersion of thermoelastic waves depends exclusively on the value of this coefficient. The coupling term is generally small for all materials and therefore higher order terms holding it can be neglected. Neglecting the coupling term simplifies the analysis without noticeable effect on the frequency spectrum as we saw earlier. If we look at the transforms we observe that the expression for m_1, m_2, m_3 and denominators of a_{jk}, are of higher orders, so the application of inverse transforms is complicated and impractical. On the other hand, by neglecting the terms holding ε_1, inverse transforms can be found from the tables of Laplace transforms, in that case, however, the sense of the solution will be lost, because the effect of coupling of the two physical fields will be neglected. We proceed to a solution by linearizing the term holding ε_1, under the assumption that the value of the term the term holding ε_1, is smaller compared to other terms, the expression can be reduced. Therefore following Nayfeh and Nasser [14], assuming that ε_1 is sufficiently small, after the first order of approximation in ε_1, we have

$$m_j^2 = m_{j0}^2 + \varepsilon_1 m_{j1}^2 + \cdots, \qquad j = 1,\ 2,\ 3, \tag{3.13}$$

where m_{j0}^2 are given by (3.10), and

$$m_{j1}{}^2 = \frac{\tau p^2[(c_2 q^2 + p^2)q^2 - m_{jo}{}^2\{(c_1 - 2c_3\bar{\beta} + \bar{\beta}^2)q^2 + \bar{\beta}p^2 - c_2\bar{\beta}^2 m_{jo}{}^2\}]}{\bar{k}c_1 c_2 (m_{jo}{}^2 - m_{io}{}^2)(m_{jo}{}^2 - m_{ko}{}^2)}. \tag{3.14}$$

$i \neq j \neq k$ = 1, 2, 3.

If the temperature and strain fields are not coupled with each other then the thermoelastic coupling constant ε_1 is identically zero. In this case m_{30} splits from m_{10} and m_{20}. From (3.10c) m_{30} correspond to the thermal waves, whereas m_{10} and m_{20} corresponds to the coupled longitudinal and transverse elastic waves, and also from m_{10} and m_{20} it is clear that elastic waves are not affected by thermal variations and thermal relaxation time but are affected by due to the anisotropy of the medium. If the strain and temperature fields are coupled with each other then from (3.12) we see from that m_1, m_2 and m_3, get modified due to the thermo-mechanical coupling effects and anisotropy of the medium under study. Equations (3.3) to (3.5) with the help of equations (3.12) yield

$$a_{1j} = \frac{iQ_0^*\bar{f}}{c_1 c_2 \Delta'}\Big[\Big\{\left(c_1 - c_3\bar{\beta}\right)m_{j0}^2 - c_2 q^2 - p^2\Big\} g_{1j}$$
$$+\varepsilon_1\Big\{(c_1 - c_3\bar{\beta})m_{j1}^2 g_{1j} + \Big\{(c_1 - c_3\bar{\beta})m_{j0}^2 - c_2 q^2 - p^2\Big\}\Big(f_{1j} - g_{1j}\,{}^{\Delta''}\!/_{\Delta'}\Big)\Big\}\Big], \tag{3.15}$$

$$a_{2j} = \frac{Q_0^*\bar{f}}{c_1 c_2 \Delta'}\Big[\left(c_2\bar{\beta}m_{j0}^2 - c_2 q^2 - p^2\right)m_{j0} g_{1j} + \varepsilon_1\Big\{\Big[\left(c_1\bar{\beta}m_{j0}^2 - c_2 q^2 - p^2\right)\left(m_{j1}^2/2m_{j0} - m_{j0}\,{}^{\Delta''}\!/_{\Delta'}\right)$$
$$+ c_2\bar{\beta}m_{j1}^2 m_{j0}\Big]g_{j1} + \left(c_2\bar{\beta}m_{j0}^2 - c_2 q^2 - p^2\right)m_{j0} f_{1j}\Big\}\Big], \qquad j = 1, 2, 3 \tag{3.16}$$

$$a_{31} = \frac{-Q_o^*\bar{f}}{\Delta'}\varepsilon_1\left(m_{10}^2 - m_{20}^2\right)m_{11}^2 g_{11}, \tag{3.17}$$

$$a_{32} = \frac{-Q_o^*\bar{f}}{\Delta'}\varepsilon_1\left(m_{20}^2 - m_{10}^2\right)m_{21}^2 g_{12}, \tag{3.18}$$

$$a_{33} = \frac{-Q_o^*\bar{f}}{\Delta'}\Big[\left(m_{30}^2 - m_{10}^2\right)\left(m_{30}^2 - m_{20}^2\right)g_{13} + \left(m_{30}^2 - m_{10}^2\right)\left(m_{30}^2 - m_{20}^2\right)\left(f_{13} - g_{13}\,{}^{\Delta''}\!/_{\Delta'}\right)$$
$$+ \left(\left(m_{30}^2 - m_{10}^2\right) + \left(m_{30}^2 - m_{30}^2\right)\right)m_{31}^2 g_{13}\Big], \tag{3.19}$$

where

$$\Delta' = m_{30}\left(m_{30}^2 - m_{10}^2\right)\left(m_{30}^2 - m_{20}^2\right)g_{13},$$

$$\Delta'' = m_{10}\left(m_{10}^2 - m_{20}^2\right)m_{11}^2\, g_{11} + m_{20}\left(m_{20}^2 - m_{10}^2\right)m_{21}^2\, g_{12}$$
$$+ m_{30}\Big\{\left(m_{30}^2 - m_{10}^2\right)\left(m_{30}^2 - m_{20}^2\right)f_{13} + \left(2\,m_{30}^2 - m_{10}^2 - m_{20}^2\right)m_{31}^2\, g_{13}\Big\} \tag{3.20}$$

$$g_{1i} = \begin{vmatrix} m_{k0}D_{k0} & L_{k0} \\ m_{j0}D_{j0} & L_{j0} \end{vmatrix} \quad i \neq j \neq k = 1, 2, 3. \text{ Taking } i, j \text{ and } k \text{ in the cyclic order.}$$

$$f_{1i} = \begin{vmatrix} m_{k0}D_{k0}(\eta) & L_{k1}(\eta) \\ m_{j0}D_{j0}(\eta) & L_{j1}(\eta) \end{vmatrix} + \begin{vmatrix} m_{k0}D_{k1}(\eta) & L_{k0}(\eta) \\ m_{j0}D_{j1}(\eta) & L_{j0}(\eta) \end{vmatrix} + \begin{vmatrix} D_{k0}(\eta)L_{j0}(\eta) & m_{j1}^2/2m_{j0} \\ D_{j0}(\eta)L_{k0}(\eta) & m_{k1}^2/2m_{k0} \end{vmatrix} \tag{3.21}$$

$i \neq j \neq k$ = 1, 2, 3. Taking i, j and k in the cyclic order.

$$L_{j0} = U\, m^2_{jo} - c_1 c_2\, \bar{\beta}\, m_{10}^2\, m_{20}^2 + (c_3 - c_2)\, q^2\, (c_2 q^2 + p^2),$$

$$L_{j1} = U\, m_{ji}^2$$

$$D_{j0} = (c_1 - c_3 + c_2)\, m_{jo}^2 + (c_3 - c_2 - \bar{\beta})\, q^2 - (1+) p^2 \tag{3.22}$$

$$D_{j1} = (c_1 - c_3\, \bar{\beta} + c_2 \bar{\beta})\, m_{j1}^2,\ j = 1,2,3.$$

$$U = c_1 c_2\, \bar{\beta}\, (m_{10}^2 + m_{20}^2) + c_1 ((c_3 - \bar{\beta}) q^2 - \bar{\beta} p^2) - (c_3 - c_2)(c_1 - c_3 \bar{\beta})\, q^2$$

4. INVERSION OF TRANSFORMS

Now Using Cagniard-De-Hoop method to evaluate the right hand side of equation (3.12), each integral in (3. 12) is recast into the Laplace transform of a known function, and thus allowing us to write down the inverse transform by inspections. Mathematically this procedure is based on a rather elementary observation that

$$L^{-1}\left\{\frac{p^n}{2\pi}\int_{t_o}^{\infty} f(t)\, e^{pt} dt - p^{n-1} f(0) - p^{n-2} f'(0) \ldots - f^{(n-1)}(0)\right\} = \frac{d^n f(t)}{dt^n} H(t - t_o), \tag{4.1}$$

and $$L^{-1}\left\{\frac{1}{2\pi p^n}\int_{t_o}^{\infty} f(t)\, e^{-pt}\, dt\right\} = \int_1 \int_2 \int_3 \ldots \int_n f(\bar{t})\, H(\bar{t} - t_o)\, d\bar{t}, \qquad n = 0, 1, 2, \ldots \tag{4.2}$$

To apply this technique a complete description is given in De-Hoop [16], Cagniard [17] and Fung [20]. Following this technique, Laplace transform parameter p is to be isolated as required in (4.1) and (4.2). Due to existence of damping term in the temperature field equation (2.9), isolation of p is impossible Nayfeh and Nasser [14] and Sharma [17]. However, this isolation of p may be achieved for small time, i.e. if we assume p to be large. Hence, an expansion in the inverse power of p followed by the change of variable $q = p$, reduces m_{ko} and m_{k1}^2 to

$$m_{10} = p\alpha_{10}, \qquad m_{20} = p\alpha_{20}, \qquad m_{30} = p\alpha_{30} + 1/2\, \bar{k}\ \alpha_{30} \tag{4.3}$$

$$m_{j1}^2 = p^2\left[\alpha_{j1}^2 + \alpha^{*2}_{j1} / p\right] \qquad j = 1, 2, 3. \tag{4.4}$$

$$\alpha_{10}^2, \alpha_{20}^2 = \left[P\eta^2 + J \pm \left\{(P\eta^2 + J)^2 - 4c_1c_2(\eta^2+1)(c_2\eta^2+1)\right\}^{1/2}\right] / 2c_1c_2 \tag{4.5}$$

$$\alpha_{30}^2 = (\eta^2 + \tau_o) / \bar{k} \tag{4.6}$$

$$\alpha_{ji}^2 = \tau_o \eta^2 (c_2 \eta^2 + 1) - \alpha_{jo}^2 \left\{(c_1 - 2c_3\bar{\beta} + \bar{\beta}^2)\eta^2 + \bar{\beta}^2 - c_2 \bar{\beta}^2 \alpha_{jo}^2\right\} / \alpha_{jik} \tag{4.7}$$

$$\alpha_{jik} = \bar{k} c_1 c_2 (\alpha_{jo}^2 - \alpha_{io}^2)(\alpha_{jo}^2 - \alpha_{ko}^2), \qquad i \neq j \neq k = 1,2,3 \tag{4.8}$$

$$\alpha_{11}^{*2} = \alpha_{11}^2\ \left(\tau_o + 1/\bar{k}\,(\alpha_{10}^2 - \alpha_{30}^2)\right) \tag{4.9}$$

$$\alpha_{21}^{*2} = \alpha_{21}^2\ \left(\tau_{o+1} / \bar{k}\,(\alpha_{20}^2 - \alpha_{30}^2)\right), \tag{4.10}$$

$$\alpha_{31}^{*2} = \alpha_{31}^2 \left\{\tau_o + \left((\alpha_{10}^2 + \alpha_{20}^2) - 2\,\alpha_{30}^2\right) c_1 c_2 / \alpha_{312}\right\}$$
$$- \tau_o \left\{(c_1 - 2c_3\bar{\beta} + \bar{\beta}^2)\eta^2 + \bar{\beta}^2 - 2c_2\, \bar{\beta}^2\, \alpha_{30}^2\right\} / \bar{k}\alpha_{312}. \tag{4.11}$$

We take $f(t) = H(t)$, the unit step function so that surface of the half space is subjected to a thermal source of magnitude Q_o^* and $\bar{f}(p) = \dfrac{1}{p}$. Substitution of equations (4.1) to (4.11) in equation (3.14) to (3.18) and then into equation (3.2) yields

$$(u, w, T) = L^{-1}\left(\sum_{i=1}^{3} \bar{u}_i, \sum_{i=1}^{3} \bar{w}_i, \sum_{i=1}^{3} \bar{T}_i\right) \tag{4.12}$$

$$\bar{u}_k = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{A_{1k}}{p}+\frac{B_{1k}}{p^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] = \frac{1}{\pi}\mathrm{Im}\int_0^{\infty}\left(\frac{A_{1k}}{p}+\frac{B_{1k}}{p^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] \quad (4.13)$$

$$\bar{w}_k = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{A_{2k}}{p}+\frac{B_{2k}}{p^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] = \frac{1}{\pi}\mathrm{Re}\int_0^{\infty}\left(\frac{A_{2k}}{\mathrm{p}}+\frac{B_{2k}}{\mathrm{p}^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] \quad (4.14)$$

$$\bar{T}_k = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{A_{3k}}{p}+\frac{B_{3k}}{p^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] = \frac{1}{\pi}\mathrm{Re}\int_0^{\infty}\left(\frac{A_{3k}}{p}+\frac{B_{3k}}{p^2}\right)\exp[-p(z\alpha_{k0}+i\eta x)d\eta] \quad (4.15)$$

where A_{ij} and B_{ij} can be determined in a straight forward manner.

5. INTEGRAL'S EVALUATION AND SINGULARITY

In the process of evaluating above integrals, from equations (4.3) to (4.6), we have discriminate in (4.5) for singularities

$$\left(P\eta^2+J\right)^2 - 4c_1c_2\left(\eta^2+1\right)\left(c_2\eta^2+1\right)\Big\}^{1/2} = 0 \quad (5.1)$$

and $\alpha_{k0}=0$, k =1, 2, 3, it follows that in calculating (4.12) to (4.15), taking η-as a complex variable and distort the path of integration in the η-plane, we have obtained the same poles and branch points Branch points:

$\eta = \pm\, i$, $\eta = \pm\, {}^{i}\!/\!\sqrt{c_2}$, $\eta = \pm\, i\,\sqrt{\tau_O}$, k =1, 2, 3 which are same as obtained by Nayfeh and Nasser [19] and Sharma [17], and for isotropic medium which reduce to,

$$\eta = \pm\, i,\ \eta = \pm\, {}^{iv_1}\!/_{v_2},\ \eta = \pm\, i\,\sqrt{\tau_0}\ , \quad (5.2)$$

where v_1 and v_2 are the velocities of dilatational and distortional waves. Again first equation (5.1) is a quadratic equation in η^2 and has real roots if discriminant of this equation is positive. Further, if

$$PJ > 2c_1c_2(c_2+1),\ P^2 > 4c_1c_2^2 \quad (5.3)$$

then equation (5.1) cannot have positive roots in η^2. Therefore assume that equation (5.1) is hold and its discriminant is positive, thus the quartic equation has only pure imaginary pure roots.

Other singular points of the integrands are their poles, which are given by

$$(\alpha_{10}^2-\alpha_{20}^2)(\alpha_{20}^2-\alpha_{30}^2)(\alpha_{30}^2-\alpha_{10}^2)=0\ , \quad (5.4)$$

$$\alpha_{k0}=0\ , \quad (5.5)$$

$$\text{and}\quad \Delta'(\eta)=0 \quad (5.6)$$

The equation (5.4) provides $\alpha_{10}^2=\alpha_{20}^2=\alpha_{30}^2$. This does not hold true as Re $(\alpha_{k0}) \geq 0$ and $\alpha_{10}\neq\alpha_{20}\neq\alpha_{30}$, therefore this yields no singularities. The poles of (5.5) coincide with branch points (5.2). Now to find poles given by (5.6), on taking i/V, rationalize and simplifying it reduces to Eq. (45) of Verma 20] giving phase velocity for isothermal Rayleigh waves in a transversely isotropic half-space in thermoelasticity. It can easily verified (see Abubakar [01]) that under the assumption $P > J\,c_2$, only one root of the resulting equation (see Eq. (45) Verma [20] of satisfy (5.6) on the upper leaf of the Riemann surface and that is the root which lies in the range $0 < V^2 < c_2$. Let it is V_R^2, where V_R is the Rayleigh waves velocity in uncoupled theory of thermoelasticity, which are same as obtained by Verma [20]. Thus under the assumption made, the singularities of integrands (4.13)-(4.15), which lie on the upper leaf of the Riemann surface are

$$\eta = \pm\, i,\ \eta = \pm\, {}^{i}\!/\!\sqrt{c_2},\ \eta = \pm\, i\,\sqrt{\tau_O}\ ,\ \eta = \pm\, i\eta_0,\ \eta = \pm\pm\, \mathrm{i}/V_R. \quad (5.7)$$

In the special case of $\tau_0 < 1$ and V_R^2 = 0.1834 for zinc crystal. The path of integration is long the real axis. To make the functions of single valued in the complex plane of integration, we make a cut joining the singularities $i/\sqrt{c_2}$ and $-i/\sqrt{c_2}$ in the -plane. First we consider one of the integrals (4.12) - (4.15), say

$$\bar{u}_1\,(x,z,p) = \frac{1}{\pi}\ \mathrm{Im} \int_{z/\sqrt{c_2}}^{\infty} \left(\frac{A_{11}}{p} + \frac{B_{11}}{p^2}\right)\frac{d\eta}{dt}.e^{-pt}\,dt\ . \qquad (5.8)$$

Using equations (4.1) and (4.2), we get

$$u_1(x,z,t) = \mathrm{Re}\left[\int_0^t A_{11} H\left(\bar{t} - z/\sqrt{c_2}\right)\left(\frac{\partial\eta}{\partial\bar{t}}\right)d\bar{t} + \int_0^t dt \int_0^{\bar{t}} B_{11} H\left(t_1 - z/\sqrt{c_2}\right)\left(\frac{\partial\eta}{\partial t}\right)dt\right]. \qquad (5.9)$$

Similarly

$$u_2(x,z,t) = \mathrm{Re}\left[\left\{\int_0^t A_{12} H\left(\bar{t} - z/\sqrt{c_1}\right)\left(\frac{\partial\eta_2}{\partial\bar{t}}\right)d\bar{t} + \int_0^t\left\{\int_0^{\bar{t}} B_{12} H\left(t_1 - z/\sqrt{c_1}\right)\left(\frac{\partial\eta_2}{\partial t_1}\right)dt_1\right.\right\}dt\right], \qquad (5.10)$$

$$u_3(x,z,t) = \mathrm{Re}\left[\left\{\int_0^t A_{13}\ \mathrm{H}\ \left(\bar{t} - z\sqrt{(\tau_o/\bar{k})}\right)\left(\frac{\partial\eta_3}{\partial\bar{t}}\right)d\bar{t} + \int_0^t\left\{\int_0^{\bar{t}} \mathrm{B}_{13}\mathrm{H}\left(t_1 - z\sqrt{(\tau_o/\bar{k})}\right)\left(\frac{\partial\eta_3}{\partial t_1}\right)dt_1\right.\right\}dt\right]. \qquad (5.11)$$

Thus, we have

$$u(x,y,t) = \sum_{k=1}^{3}\mathrm{Re}\left[\int_0^t H\left(\bar{t} + s_k z\right)\left(\frac{\partial\eta_k}{\partial\bar{t}}\right)d\,\bar{t} + \int_0^t\left\{\int_0^{\bar{t}} B_{1k} H\left(t_1 - s_k z\right)\left(\frac{\partial\eta_k}{\partial t_1}\right)dt_1\right\}dt\right], \qquad (5.12)$$

where $s_1 = {}^{1}\!/\!\sqrt{c_2}$, $s_2 = {}^{1}\!/\!\sqrt{c_1}$, $s_3 = \sqrt{\tau_o/\bar{k}}$ are the slowness of the transverse dilatational and the thermal waves, respectively.

Similarly

$$w(x,z,t) = \sum_{k=1}^{3}\mathrm{Re}\left[\int_0^t A_{2k} H\left(\bar{t} - s_k z\right)\left(\frac{\partial\eta_k}{\partial\bar{t}}\right)d\,\bar{t} + \int_0^t\left\{\int_0^{\bar{t}} B_{2k} H\left(t_1 - s_k z\right)\left(\frac{\partial\eta_k}{\partial t_1}\right)dt_1\right\}dt\right], \qquad (5.13)$$

$$T(x,z,t) = \sum_{k=1}^{3}\mathrm{Re}\left[A_{3k} H\left(\bar{t} - s_k z\right)\left(\frac{\partial\eta_k}{\partial\,t}\right) + \int_0^t B_{3k} H\ \left(\bar{t} - s_k z\right)\left(\frac{\partial\eta_k}{\partial\,\bar{t}}\right)dt\right], \qquad (5.14)$$

where η_k, k =1, 2, 3 can be determined from $t = \alpha_{k0} z + i\eta_k x$. When the thermoelastic coupling constant ε_1 vanishes, then temperature field also vanishes.

6. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the results obtained theoretically, in the above sections, for temperature and stresses are computed numerically for a single crystal of zinc for which the physical data is given as

$c_{11} = 1.628 \times 10^{11}\ Nm^{-2}$, $\rho = 7.14 \times 10^3\ kmg^{-3}$, $w^* = 5.01 \times 10^{11} s^{-1}$, $\varepsilon_1 = 0.0\,221$,
$c_1 = 0.385$, $c_2 = 0.2385$, $c_3 = 0.549$, $\bar{k} = 1.0$, $\bar{\beta} = 0.9$, $\tau_0 = 0.02$, $T_0 = 296^0 K$,

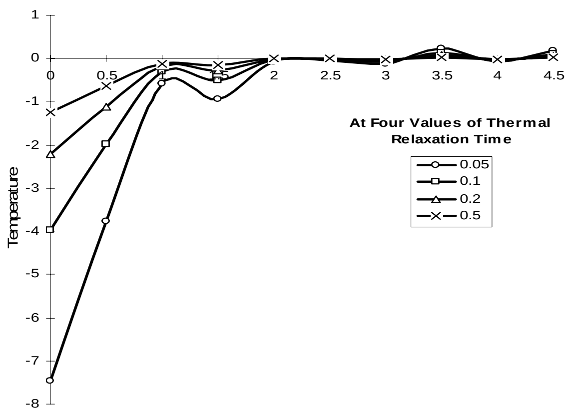
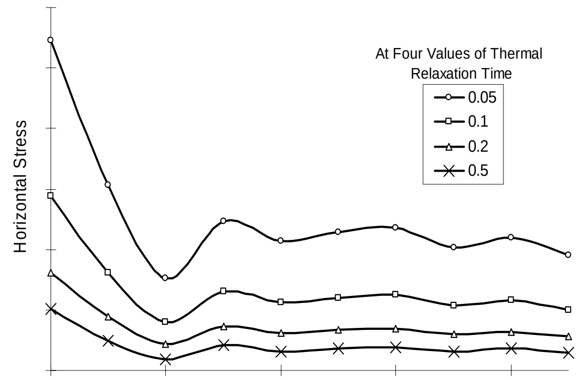


Figure:1 Variation of surface temperature with distance and time



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The computations were carried out for four values of relaxation time namely τ = 0.05, 0.1, 0.2, 0.5 at the surface z = 0. The results for temperature (T), horizontal stress (τ_{xx}), vertical stress (τ_{zz}) and shear stress (τ_{xz}) with respect to distance are shown in Figures 1, 2, 3 and 4 respectively.

From the figures it is observed that the vertical and shear stresses at the surface are positive and decreases in magnitude with the passage of time whereas the horizontal stress varies from negative value to positive one with the passage of time. The temperature also increases from negative value to positive value with the passage of time. Also the variations of all these quantities are more prominent at small times and decrease with passage of time, which established the fact that the second sound effect is short lived. All these quantities vanish when it move away from the heat source, at certain distance at all times, which shows the existence of the wave fronts and ascertain the fact that generalized theory of thermoelasticity admits finite velocity of heat.

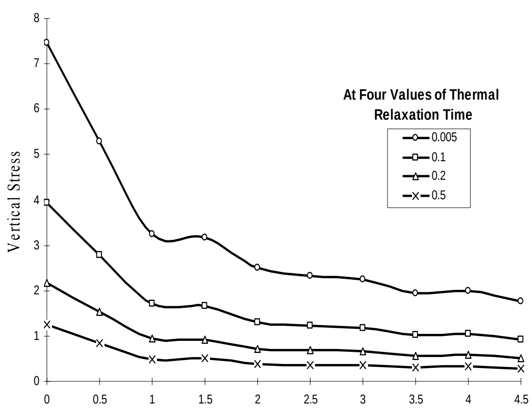

Figure:3 Variation of Vertical Stress with distance and time

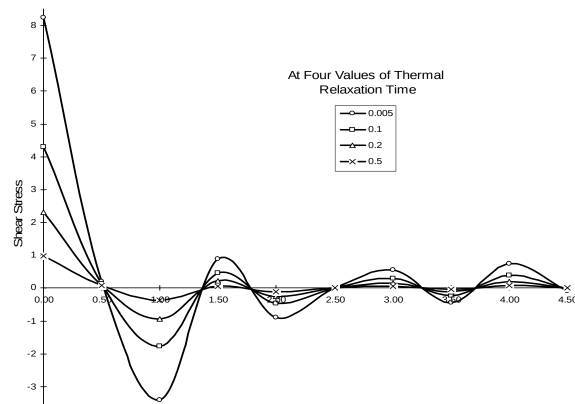

Figure:4 Variation of Shear Stress with distance and time

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Appendix-A $x' = \dfrac{v_1}{k_1}x;\ z' = \dfrac{v_1}{k_1}z;\ t' = \dfrac{v_1^2}{k_1}t;\ \tau_0' = \dfrac{v_1^2}{k_1}\tau_0;\ u' = \dfrac{\rho v_1^3}{k_1\beta_1 T_0}u;\ z' = \dfrac{\rho v_1^3}{k_1\beta_1 T_0}z, T' = \dfrac{T}{T_0};$

$$\bar{k} = \frac{K_3}{K_1}; \bar{\beta} = \frac{\beta_3}{\beta_1};\quad c_1 = \frac{C_{33}}{C_{11}}; c_2 = \frac{C_{44}}{C_{11}};\ c_3 = \frac{(C_{13}+C_{44})}{C_{11}}, \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_e v_1^2},$$

where

$k_1 = \frac{K_1}{\rho C_e}$ and $v_1 = (\frac{C_{11}}{\rho})^{\frac{1}{2}}$ are the thermal diffusivity and the velocity of compressional waves in the x-direction, respectively. Here ε_1 is the thermoelastic coupling constant.

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