JOURNAL OF ENGINEERING annals OF FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 1, (ISSN 1584 – 2665)

# FUZZY APPROXIMATOR OF THE FORCE-LENGTH-PRESSURE RELATIONSHIP FOR A PNEUMATIC ARTIFICIAL MUSCLE

Alexander HOŠOVSKÝ

Faculty of Manufacturing Technologies,
Technical University of Košice, Prešov, SLOVAKIA

**ABSTRACT:**
This paper deals with a design of fuzzy system capable of approximating nonlinear relationship between generated force of the pneumatic artificial muscle, its length and pressure based on the manufacturer-supplied curves. The main idea was to use an universal approximating capabilities of fuzzy systems and provide a model intended for integration within the overall model of PAM-based position servosystem. This two-variable function had to be approximated within the whole universes of discourse, calling for some kind of interpolation between the discrete values of pressure as manufacturer-supplied curves were represented in 2-D form. ANFIS editor in Matlab was used for fuzzy approximator training.
**KEYWORDS:**
fuzzy approximator, approximation error, pneumatic artificial muscle, approximation

## 1. INTRODUCTION

Due to their properties somewhat comparable to the skeletal muscles of humans, pneumatic artificial muscles are rather interesting type of actuator especially for biorobotics and prosthetics applications (still it is possible to use them in industrial applications as well). The uniqueness of these properties introduces a new dimension in aforementioned areas and it allows for widening the application spectrum. Nevertheless, the technology of construction makes the easiness of control design quite prohibitive whether be it due to the inherently nonlinear character of used elastic materials or well-known complexities associated with the description of physical laws governing the phenomenons in pneumatic systems. All these intricacies naturally make the modelling of PAM-based systems much more challenging. As fuzzy systems are considered to be universal approximates, it looks as an ideal solution to approximating certain nonlinear functions describing the relationship between various pneumatic artificial muscles parameters. This approximator shall be used as a part of model derived for a PAM-based position servo system.

## 2. METHODOLOGY

When devising a model of pneumatic artificial muscle, one has to deal with relationship between geometric properties of the muscle as well as the physical phenomenons taking place within the muscle. Force-length relationship in older types of pneumatic artificial muscles was distinctive for its rather significant hysteresis being caused by their technology of construction (e.g. Shadow Air Muscles). Thanks to the advancements in this field it was possible to reduce this unwanted feature (e.g. FESTO) and it won't be addressed in modeling of this type of muscle. The simplest mathematical formula for a muscle force may be derived from the equality between the work elements carried out during volume expansion and length contraction. If the difference between ambient pressure and muscle pressure is $p = P - P_N$ and the volume increases by an infinitely small amount, then the following amount of work is done [2]

$$dW_d = pdV \tag{1}$$

Also the work done when contracting the muscle by an infinitely small length *dl* is (minus sign indicates the contraction)

$$dW_f = -Fdl \tag{2}$$

Then the force is

$$F = -p\frac{dV}{dl} \tag{3}$$

It is obvious from Equation (3) that the generated force is highest at the smallest contractions and falling to zero when the muscle volume is not changed with the contraction anymore. In this equation, the geometric parameters are not formulated expressively and the following alternative is often used instead:

$$F = \frac{P_g b^2}{4\pi n^2}\left(\frac{3L^2}{b^2} - 1\right) \tag{4}$$

where $b$ – thread length, $L$ – muscle length, $d$ – muscle diameter,
$\theta$ – weave angle, $P_g$ – muscle pressure

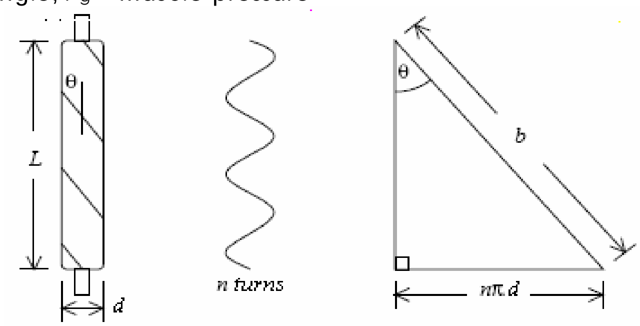


Figure 1. Geometric parameters of the pneumatic artificial muscle [1]

It is clear from (4) that the force depends on two variables, namely pressure and muscle length (in case the thread length is assumed constant). Equation (4) describes a relationship between the force and two other variables in a simplified manner as it does not take all the factors affecting this relationship into account.

### 2.1 Static fuzzy model

It is this universal approximation capability that the solution for approximation of force-length relationship was sought in the field of fuzzy systems. Let's assume a general function g that maps the values of independent variable from the set X into the set of dependent variable Y,

$$g : X \to Y \tag{5}$$

where in general $X \subset R^n$ and $Y \subset R$. The goal is to design a fuzzy system that provides mapping

$$f : \overline{X} \to \overline{Y} \tag{6}$$

where $\overline{X} \subset X$ and $\overline{Y} \subset Y$. Then for the original function *g(x)* we have

$$g(x) = f(x \mid \theta) + e(x) \tag{7}$$

where $f(x \mid \theta)$ is a term of approximated function depending on adjustable parameter $\theta$ and *e(x)* is an approximation error [6].

The problem is to minimize the approximation error expressed in *e(x)* by adjusting the parameter $\theta$. Fuzzy systems are capable of approximating virtually any function with approximation error expressed as follows (assuming N-dimensional input vector) [3]:

$$\left\| g(x) - f(x) \right\|_\infty \leq \left\| \frac{\partial f}{\partial x_1} \right\|_\infty e_1 + \left\| \frac{\partial f}{\partial x_2} \right\|_\infty e_2 + \ldots + \left\| \frac{\partial f}{\partial x_N} \right\|_\infty e_N \qquad (8)$$

from where it is obvious that the approximation error directly depends on

1. function gradient $\left\| \frac{\partial g}{\partial x_i} \right\|_\infty$
2. distance between the centres of two adjacent fuzzy sets $e_i = \left| m^i_{j+1} - m^i_j \right|$, where $m^i_j$ is a centre of $j$-th fuzzy set for $i$-th input

In our case, the function $g(x)$ is a relationship between the force, pressure and length. Due to the technical limitations it was not possible to acquire this relationship experimentally, so the characteristics supplied by a manufacturer were used. The depicted curves (Fig.2) correspond to the type of muscles used in a real position servosystem (FESTO MAS-20).

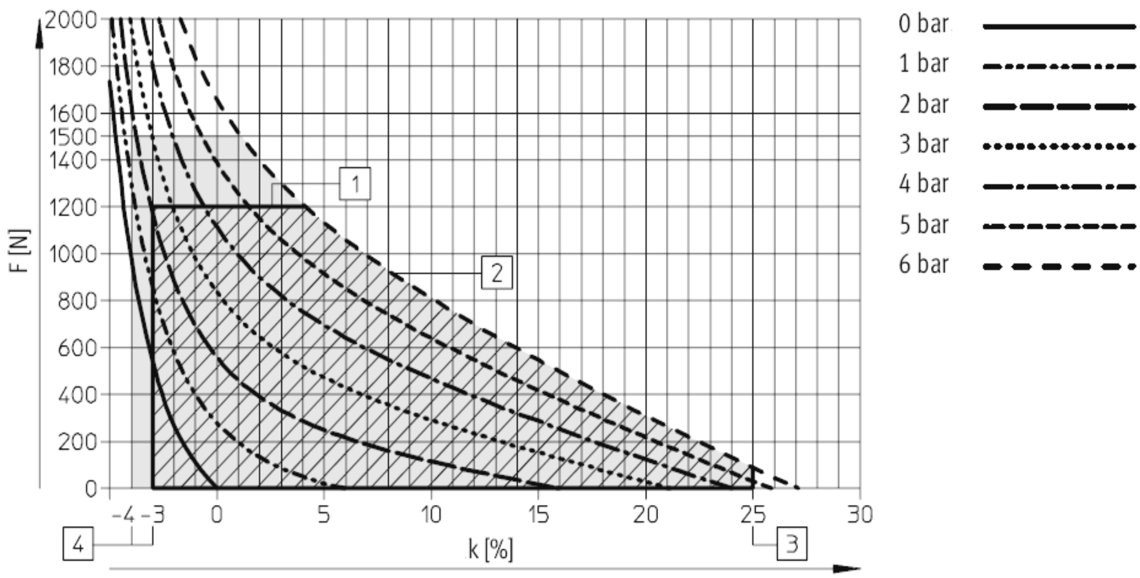


Figure 2. Force-length curves for FESTO MAS-20 [4]

### 2.2. Total fuzzy approximator

From Fig.2 it is clear that the depicted force-length curves were taken at 7 discrete values of muscle pressure. It is, of course, necessary to get a value of force for any combination of contraction and muscle pressure, so it must be determined even for the pressure in the interval between given discrete pressure values. There would be seven particular fuzzy approximators, each providing a mapping of muscle contraction to muscle force as an approximation to depicted relationship for each of the seven discrete values of muscle pressure. These particular fuzzy approximators are contained within the structure of total fuzzy approximator providing a general mapping of contraction and pressure crisp values from respective universes of discourse into the crisp value of force. The total fuzzy approximator works as a fuzzy system itself with 7 fuzzy sets defined on the universe of discourse ( $p \in [0,6]$ (bar) ) (denoted *A0, ..., A6*) representing the membership functions for activation of each of the particular fuzzy approximators. The membership functions used in a model are conventional triangular functions having intersections between two adjacent fuzzy sets at $\mu_{FA} = 0{,}5$. It is clear that the maximum number of simultaneously activated rules is two (in Fig.3 this situation is depicted for a value $p$ = 3,25 bar meaning that the rules No.4 and 5 are active). The activation of appropriate rules determines the condition for activation of particular fuzzy approximators corresponding to these fuzzy sets (the switch is shown at the input of every particular approximator to indicate this).

The structure of TFA as a fuzzy system is shown in Fig. 4. In this figure, only two particular fuzzy approximators are depicted (namely FA3 and FA4 corresponding to rules No.4 and 5 which are active for the crisp value $p$ = 3,25 bar). The output of each of these approximators is a crisp value of force determined on the basis of contraction value as an input to every particular fuzzy approximator for a constant pressure (in this case 3 and 4 bars).

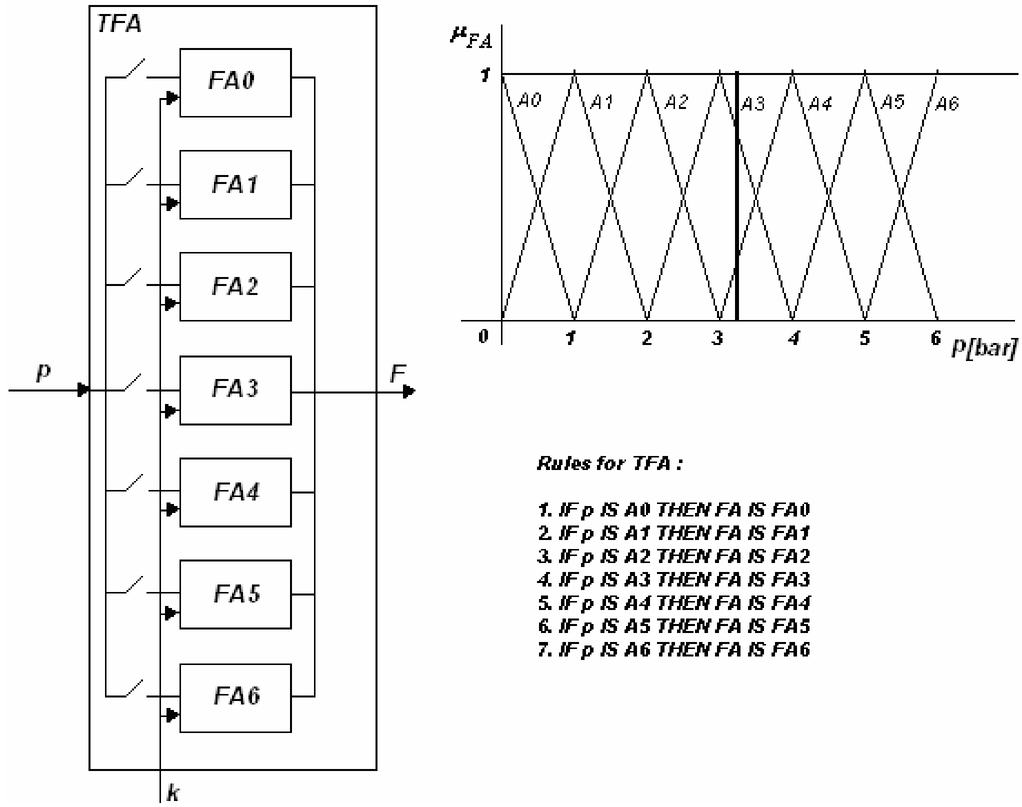


Figure 3. The structure of total fuzzy approximator, membership functions distributed over the universe of discourse and the rule base

The force values are consequently fed into the defuzzification block where they act as singletons in the calculation of total force according to the “weight-average” formula (9):

$$F = \frac{\sum_{l=1}^{L} F_{FA}^{l} \mu_{FA}^{l}}{\sum_{l=1}^{L} \mu_{FA}^{l}} \tag{9}$$

where $L$ is a number of rules for TFA, $\mu_{FA}^{l}$ is a membership function for $l$-th rule, $F_{FA}^{l}$ is a crisp value of force corresponding to the output of fuzzy approximator activated by $l$-th rule.

### 2.3 Particular fuzzy approximators

The particular fuzzy approximators were constructed using Fuzzy Logic Toolbox as a part of Matlab. Using well-designed GUI of ANFIS editor it is possible to design fuzzy systems after loading an input data in the form of discrete points of approximated function. This data set then serves as a training set for fuzzy approximator optimization. In our case 29 discrete points were taken for every force-length curve forming 7 training sets for each of the particular approximators (Fig.5).

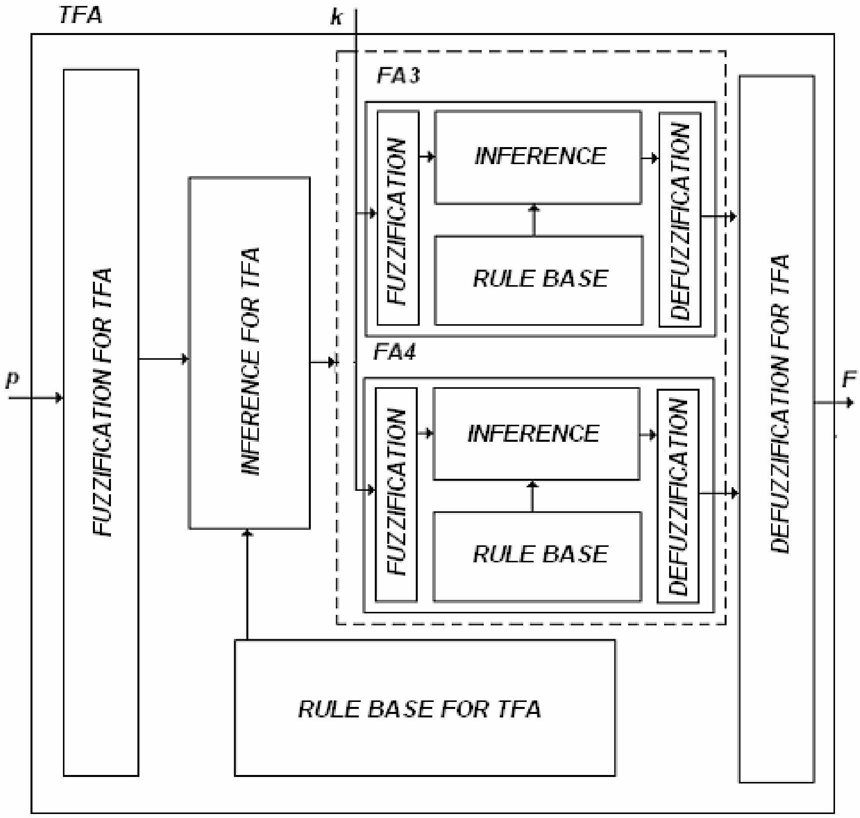


Figure 4. Internal structure of the fuzzy approximator with two active particular fuzzy approximators

The approximation process in ANFIS editor can be carried out using any of eight basic membership function shapes and constant or linear consequences (ANFIS uses only Takagi-Sugeno fuzzy systems). After some experimentation, it was found out that for the given number of data points the lowest approximation error was achieved using eleven membership functions of triangular shape evenly distributed over input universe of discourse with linear consequences. ANFIS optimization algorithms used for the calculation and adjustments of parameters in order to achieve the approximation error minimization combine the strengths of recursive least squares and gradient descent methods. The least squares method is used for determining the initial values of *a* and *b* coefficients in the formula for consequences calculation

$$f(x^i) = \frac{\sum_{l=1}^{L} (a_1^l x_1^i + b^l)\mu_l(x^i)}{\sum_{l=1}^{L} \mu_l(x^i)} \tag{10}$$

where for SISO system we have only one input $x_1^i$ with its *i*-th value, $a^l$ and $b^l$ being adjustable parameters and $\mu_l(x^i)$ indicates membership function of *i*-th input value. Having calculated *a* and *b* coefficients, the next adjustable parameter becomes the position of membership function centres in the input universe of discourse. These are determined by means of gradient descent method, i.e. according to the steepest descent of the function relating approximation error and adjustable parameter vector

$$\frac{\partial J}{\partial \theta} = \sum_{i=1}^{N} e^i \frac{\partial f(x^i, \theta)}{\partial \theta} \tag{11}$$

where $e^i$ means the approximation error and $\theta$ is adjustable parameter vector [3].

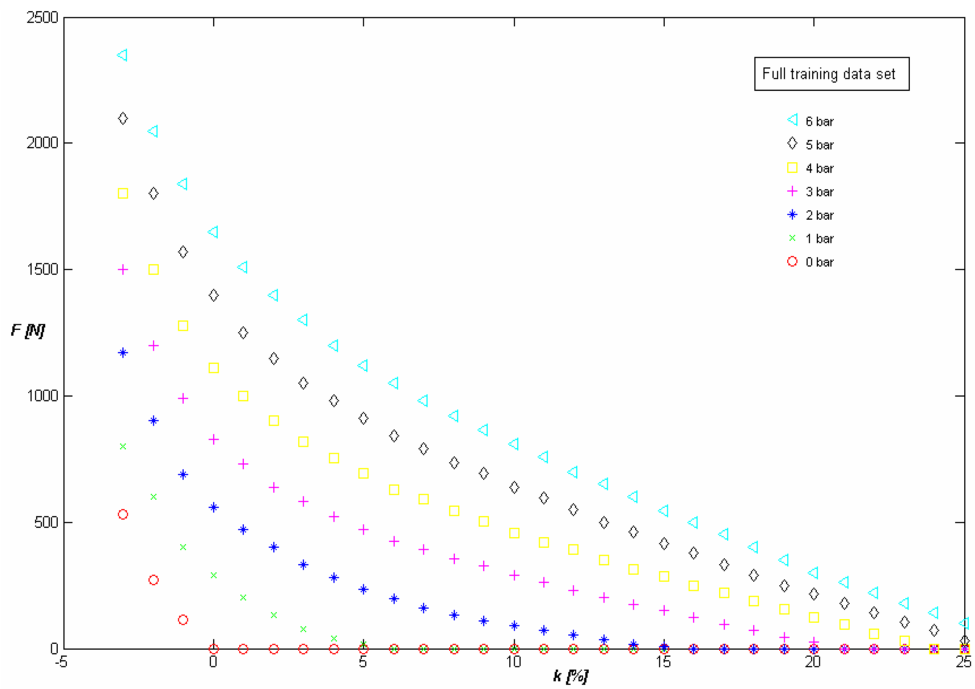


Figure 5. Training data set for ANFIS

## 3. RESULTS

TFA was designed completely in Simulink using 7 fuzzy controllers and appropriate logic functions. In Fig.6 the correlation between manufacturer-supplied force-length curves and fuzzy system approximation is depicted. It is quite clear that the curves are approximated very well for the whole range of muscle contractions. There is slight increase in approximation error for $p$ = 2 bar curve that might have resulted from the input data inaccuracies. All particular fuzzy approximators were trained in 100 epochs and the resulting approximation errors were as follows (increasing the number of epochs over number 100 at the given number of input data points didn't make any difference in resulting approximation errrors):

$$e_0 = 0.126871,\ e_1 = 0.04103,\ e_2 = 0.034274,$$
$$e_3 = 0.046127,\ e_4 = 0.05053,\ e_5 = 0.024701,\ e_6 = 0.045144$$

The results of fuzzy approximator training are clearly in accordance with (8) as the function gradient of 0 bar curve is higher when compared to the other. Input data points were chosen so that the training set provided sufficient excitation in the whole universe of discourse. If there is need to improve the accuracy, it would be needed to input a higher number of data points for training so that the generality of resulting fuzzy approximator is better.

Since the force depends on two variables, it is best to depict it in 3-D space (Fig. 7). It is clearly visible (also from the membership function shapes as well as the defuzzification formula) that the fuzzy system performs linear interpolation between two forces generated by two active particular fuzzy approximators. Even though that this might be looked upon as a simplification, the experiments carried out in [5] imply that the nonlinear term in relationship between generated force and muscle pressure at constant contraction (izometric conditions) may be neglected when working in nonextreme parts of operation envelope (out of this envelope, the effect of a muscle as nonideal cylinder becomes more pronounced). As it is not intended to use the mentioned artificial muscles in a real servosystem out of their operation envelope, this model is considered to be adequate.

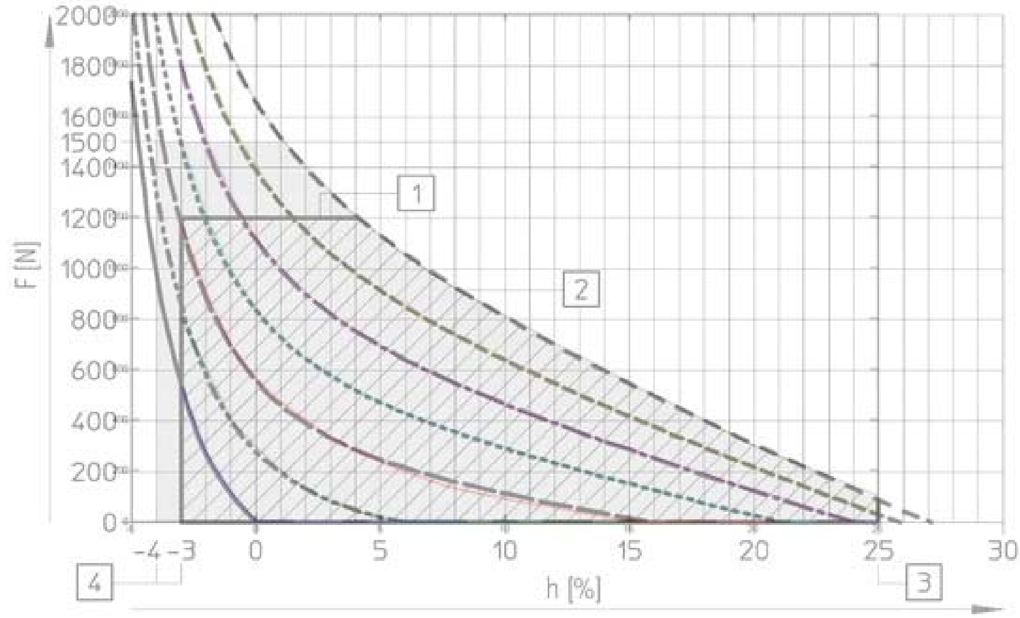


Figure 6. The correlation between manufacturer-supplied force-length curves and fuzzy system approximation

## 4. CONCLUSION

In this paper, the fuzzy approximator of relationship between generated force, pressure and length in a pneumatic artificial muscle used in position servosystem was derived. This approximator shall be a part of the servosystem model used for a fuzzy control design. It confirms the usefulness of idea of fuzzy systems as universal approximators. It was designed using an assumption that the relationship between generated force and muscle pressure at constant contraction might be considered linear when operating the muscle in usual range of working parameters. Using ANFIS editor in Matlab, it is possible to train a fuzzy system with very good approximation capability and generality provided that sufficient number of input data points are available.

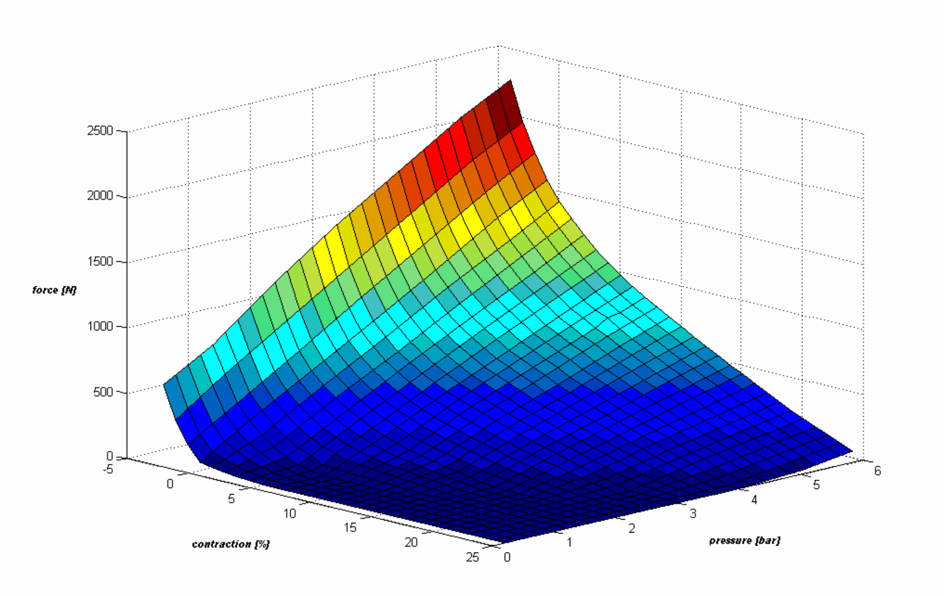


Figure 7. TFA fuzzy surface (force as a function of pressure and contraction)

The resulting approximation error as well the flexibility of fuzzy systems supports the idea of using them in modeling the real system. While this part shall serve as a static model of pneumatic artificial muscles, it is necessary to derive a dynamic model as well in which a nonlinear identification by means of fuzzy system might be used. Later on, it may become clear what other modifications of this approximator would be needed.

**REFERENCES / BIBLIOGRAPHY**

[1.] COLBRUNN W., Robb – NELSON M., Gabriel – QUINN D., Roger: Modeling of Braided Pneumatic Actuators for Robotic Control, Intelligent Robots and Systems, 2001 Proceedings, Volume 4, pp.1964-1970

[2.] DAERDEN, Frank: Concept and Realization of Pleated Pneumatic Artificial Muscles and their Use as Compliant Actuation Elements, PhD. thesis, 1999

[3.] ESPINOSA J. *et al.*: Fuzzy Logic, Identification and Predictive Control, Springer 2005. 263 s. ISBN 1-85233-828-8

[4.] FESTO company, www.festo.com

[5.] HANNAFORD, B., CHOU Ch.P.: Measurement and Modelling of McKibben Pneumatic Artificial muscles, IEEE Transaction on Robotics and Automation, 1996, Volume 12, Issue 1, ISSN: 1042-296X

[6.] PASSINO M.K., YURKOVICH S.: Fuzzy Control, Addison-Wesley 1998. 502 s. ISBN 0–201–18074–X