JOURNAL OF ENGINEERING
annals OF
FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 1, (ISSN 1584 – 2665)

THE USE OF SAVITZKY-GOLAY FILTER TO DENOISING MOROCCAN BOUGUER GRAVITY ANOMALY MAP

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Abstract
Noise in geophysical maps is common. Map denoising and filtering attempts to locally preprocess these maps primarily to suppress map noise by making use of the redundancy in the map data. The Savitzky-Golay smoothing and differentiation filter optimally fits a set of data points to a polynomial in the least-squares sense. The Savitzky-Golay filtering method has been used to filtering and denoising Moroccan Bouguer gravity data map. Data processing procedures as the Savitzky-Golay filter transform was found to be consistently useful The Savitsky-Golay analysis surface of Moroccan Bouguer gravity anomaly map as obtained by the above procedure in the study area provided a direct image for an interpretation of the Moroccan gravity survey. The overall effect is that of scanning and denoising the anomalous bodies resulting from the Moroccan Bouguer gravity anomaly map.
Key words: gravity, Bouguer, Savitzky-Golay, denoising, Morocco.

1. INTRODUCTION

Geophysical data as gravity data collected in the survey are often contaminated with noise and artifacts coming from various sources. High-frequency noise varying in intensity and frequency often contaminates the data and needs to be filtered. Unfortunately, this type of noise may have a frequency content similar to that of the signal.The presence of noise in data distorts the characteristics of the geophysical signal resulting in poor quality of any subsequent processing. Consequently the first step in any processing of such geophysical data is the “cleaning up” of the noise in a way that preserves the signal sharp variations. So geophysical maps usually contain a number of features as anomalies and structures [1][2] which are superposed on each other. For instance, a gravity map may be composed of regional, local, and micro-anomalies. The aim of an interpretation of such maps is to extract as much useful information as possible from the data. Since one type of anomaly often masks another, the need arises to separate the various features from each other. One of the main purposes of geophysical mapping is the identification of units that can be related to the unknown geology. Gravity maps are most useful tools presently available, although other techniques such as conductivity mapping [3] or remote sensing [4] are very helpful in locating lithologic boundaries. The interpretation which makes extensive use of enhanced maps of gravity data often involves initial steps to eliminate or attenuate unwanted field components in order to isolate the desired anomaly (e.g., residual-regional separations). These initial filtering operations include the radial weights methods [5], least squares minimisation

[6], the Fast Fourier Transform methods [7] and recursion filters [8] and rational approximation techniques [9]. The premise of gravity data smoothing is that one is measuring a variable that is both slowly varying and also corrupted by random noise. Then it can sometimes be useful to replace each gravity data point by some kind of local average of surrounding data points. Since nearly points measure very nearly the same underlying value, averaging can reduce the level of noise without (much) biasing the value obtained. Rather than having their properties defined in the Fourier domain, and then translated to the time domain, Savitzky-Golay filter [10] derive directly from a particular formulation of the data smoothing problem in the time domain. So denoisng Moroccan Bouguer gravity anomaly using Savitsky-Golay filtering method appear as an new original method. The theory has acquired the status of a unifying theory underlying many of the methods used in physics and signal processing.

The present paper deals with denoising and filtering Moroccan Bouguer gravity anomaly map using the Savitsky-Golay filtering method. The results show a high significant suppression of the noise and a very good smoothing and recovery of the Bouguer gravity anomaly signal. So the Savitsky-Golay filtering method processing is thought to be a good method to geophysical anomaly filtering and optimizing. This modern signal processing approach is used as an alternative to classical gravity anomaly separation methods.

2. MOROCCAN GEOGRAPHICAL AND GEOLOGICAL SETTINGS

Morocco (figure 1) is located at the north-western point of the African continent. Of a surface of 710.000 km², it is bordered in the west by the Atlantic Ocean (2934 km of coasts), in north by the Mediterranean (512 km of coasts) and is not separated from Spain that by the 14 km of the Straits of Gibraltar. It has terrestrial borders common with Algeria (1350 km) to the east, Mauritania (650 km) in the south. In north, a chain relatively low, Rif, borders the Mediterranean shore. A mountainous corridor separates it from another chain, the Means-Atlas, which stretches the North-East in south-west.

Figure 1. Map of the situation of Morocco

More to the south, the chain of the High-Atlas. The plates of Meseta constitute Atlantic Morocco and arid plates are present in the East. Five major geological domains are distinguished [11], these are (figure 2):

The Anti Atlas and Saharian domain comprised of gently deformed Paleozoic sealed to the Northwest African Craton during the Hercynian orogeny.

The Mesetian domain characterized by deformed Paleozoic unconformably overlain by gently folded Meso-Cenosoic deposits.

The Atlasic domain corresponding to the inversion of the Atlasic through which was filled by thick Mesozoic sediments.

The Atlantic passive margin consisting of thick Mesozoic sediments locally deformed by salt tectonics and gravity induced imbrications with important turbidite sheet deposits basinwards.

The Rif domain a thrust fold belt where the deformation of the Meso-Cenozoic series decreases southward.

The post Precambrian structural evolution of Morocco was marked by two major compressional events (Hercynian and Alpine orogenies) separated by an extensional period related to the Atlantic opening:

The Hercynian orogeny: the Upper Devonian-Carboniferous compression resulted in folding and faulting of the Paleozoic series. The major remnants of the hercynian fold belt crop out in the Anti Atlas and the Moroccan meseta.

The Triassic-Liassic extension: the opening of the Atlantic was preceded by Triassic-Lower Jurassic rifting which was followed by massive regional subsidence during the Jurassic and much of the Lower Cretaceous in the Atlantic passive margin. This extension was also responsible of the opening of the Atlas troughs.

The Alpine orogeny: during the Upper Eocene-Oligocene, the Atlas Mesozoic troughs were inverted to form the High and Middle Atlas. The late collision of Africa with Europe during the Neogene resulted in the formation of the Rif which is a segment of the Western Mediterranean Alpine fold belt.

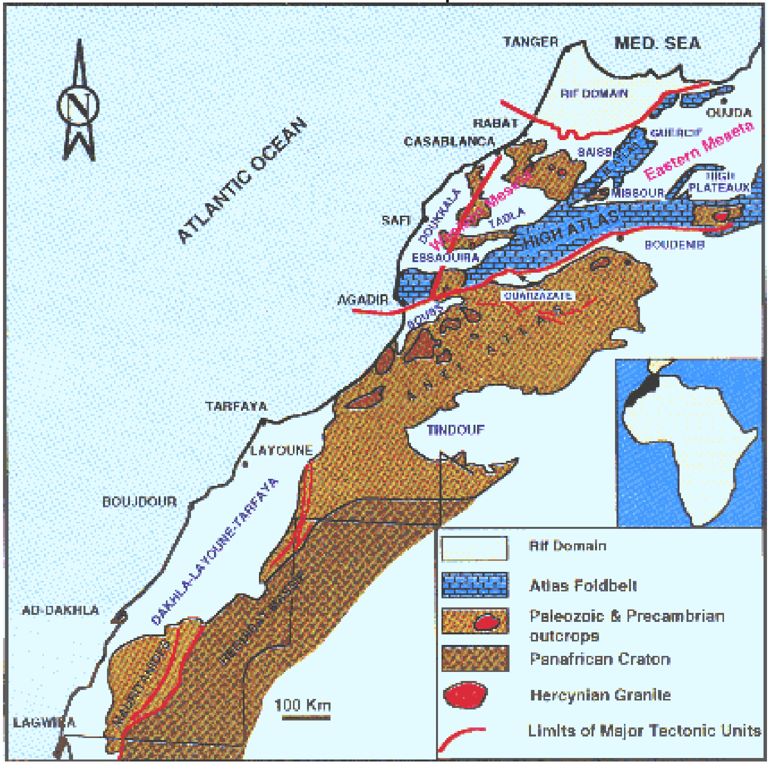

Figure 2. Map of the Moroccan main geological domains

3. THE BOUGUER GRAVITY ANOMALY

The studied zone is limited between longitudes West 1° to 11°, and latitudes North 28° to 36°. The gravity data references used were obtained from the "Bureau Géodésique International"**.** The gravity data used for this study have been obtained from publicly available worldwide databases and from local sources, and have been complemented with new measurements acquired in the High Atlas. Moroccan Bouguer gravity data (figure 3) come from the Moroccan gridded data sets [12].

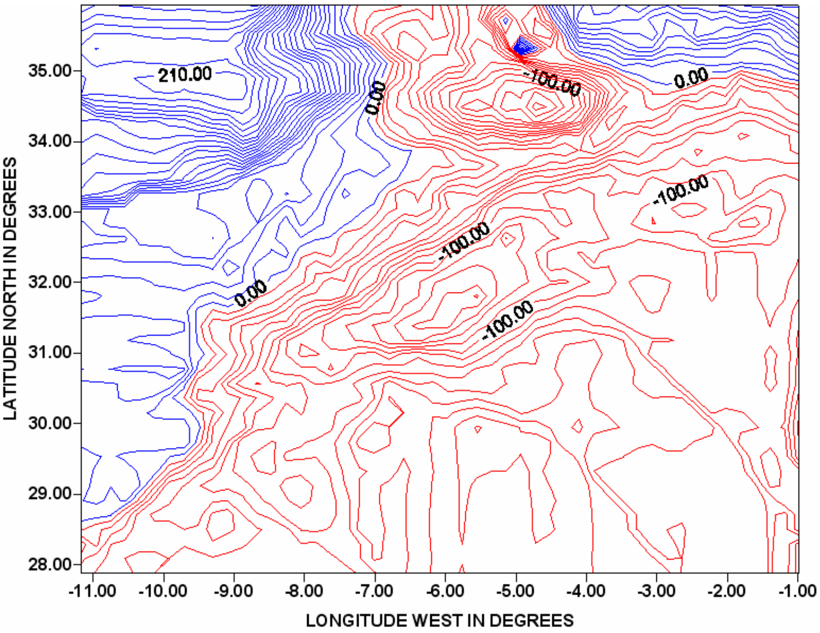

Figure 3. Map of the Moroccan Bouguer gravity data (interval contour : $10\ mgal$)

Additional gravity measurements have been acquired during 2002 and 2003 using a Scintrex CG 3 microgravimeter. All measurements were brought back to the level of reference of the international network of gravimetric standardization of 1971. The theoretical values of gravity were calculated using the gravimetric formula of the geodetic System of reference (I.A.G, 1971). The Moroccan Bouguer anomaly was calculated using a vertical gradient of the gravity of $0.3086\, mgal.m^{-1}$ and a density of $2.67\, g.cm^{-3}$ for crustal lithologies. If θ represents the geographical latitude of the station in degrees of a point given to the surface of the Earth, the theoretical value of gravity g_T in this point is provided by the following international gravimetric formula:

$$g_T = 978031.85\left(1 + 0.005278895 \sin^2 \theta + 0.000023462 \sin^4 2\theta\right)$$

Bouguer anomalies (Δg_B) for each station were calculated using the following expression:

$$\Delta g_B = g_{obs} + \left(0.3086 - 2\pi G \rho\right) H - g_T$$

where g_{obs} is the observed gravity, H is the orthometric altitude in meters, ρ is the average density of the crust ($2.67\, g.cm^{-3}$) and G the universal gravitational constant which value is $6.673 x 10^{-11}\, N.m^2.kg^{-2}$. We applied this method to the gravimetric map of the area of Morocco. Topography and free air gravity data are taken from the TOPEX gravity data set [13].

The Moroccan Bouguer gravity map obtained after all these corrections (figure 4) was build starting from 5075 data station references and 4661 data measurements which made it possible to calculate a regular square grid of step 450m with about 1 $mgal$ precision [14]. This process gives gravity information better than the pre-existing data sets, where the spacing between samples was around 5 km in the plains and much larger in mountain areas. The Bouguer anomaly reflects the lateral variations of the density of the rocks.

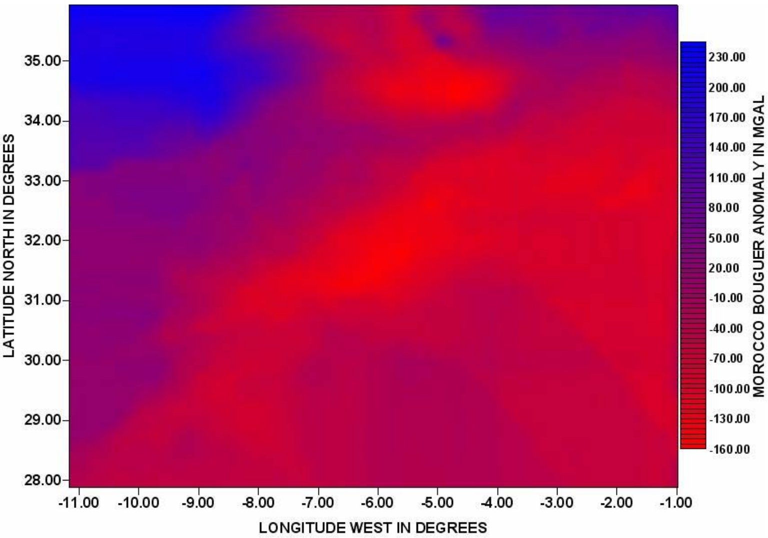

Figure 4. Moroccan Bouguer gravity anomaly data map

Gravity data collected are often contaminated with noise and artifacts coming from various sources. The presence of noise distorts the characteristics of the geophysical signal resulting in poor quality of any subsequent processing. Consequently the first step in any processing of such geophysical data is the “cleaning up” of the noise in a way that preserves the signal sharp variations. The Savitzky-Golay filtering method has become a powerful signal and image processing tool which has found applications in many scientific areas. This method is a widely used technique that is applicable to the filtering geophysical data.

The present paper deals with filtering Moroccan Bouguer gravity data map using the Savitzky-Golay filtering method. The results show a significant suppression of the noise and a very good recovery of the Bouguer anomalies signal. So the Savitzky-Golay processing is thought to be a good method to geophysical anomaly filtering.

4. THE SAVITZKY-GOLAY FILTER

The Savitzky-Golay smoothing filter was introduced for smoothing data and for computing the numerical derivatives. The smoothing points are found by replacing each data point with the value of its fitted polynomial. The process of Savitzky-Golay is to find the coefficients of the polynomial which are linear with respect to the data values. Therefore the problem is reduced to finding the coefficients for fictitious data and applying this linear filter over the complete data. The size of the smoothing window is given as $N x N$ where N is odd, and the order of the polynomial to fit is k where $N \geq k+1$.

The Savitzky-Golay smoothing applied to our Bouguer gravity data consisted to consider the Bouguer gravity anomaly as a polynomial function defined as:

$$\Delta g_B(x_i,y_i)=a_{00}+a_{10}x_i+a_{01}y_i+a_{20}x_i^2+a_{11}x_iy_i+a_{02}y_i^2+\ldots\ldots+a_{0k}y_i^k \qquad (1)$$

where x_i and y_i represent the easting (*longitude*) and northing (*latitude*) coordinates of a gridded point of the Bouguer gravity anomaly map. We then want to fit a polynomial of type in Eq. 1 to the data. By solving the least squares we can find the polynomial coefficients. We start with the general equation, $A.\vec{a}=\Delta\vec{g}_B$ where a is the vector of polynomial coefficients $a=(a_{00} \quad a_{01} \quad a_{10}\ldots.a_{0k})^T$. We can then compute the coefficient matrix as follows, $(A^T.A).a=(A^T.\Delta\vec{g}_B)$, which in least squares can be written as $a=(A^T.A)^{-1}.(A^T.\Delta\vec{g}_B)$. Due to the linear-squares fitting being linear to the values of the Bouguer gravity data, the coefficients can be independent of data.

The general coefficient matrix becomes $C=(A^T.A)^{-1}A^T$. C can be reassembled back into a traditional looking filter of size $N\,x\,N$ to achieve the reconstruction of the filtered geophysical signal corresponding to Bouguer gravity anomaly.

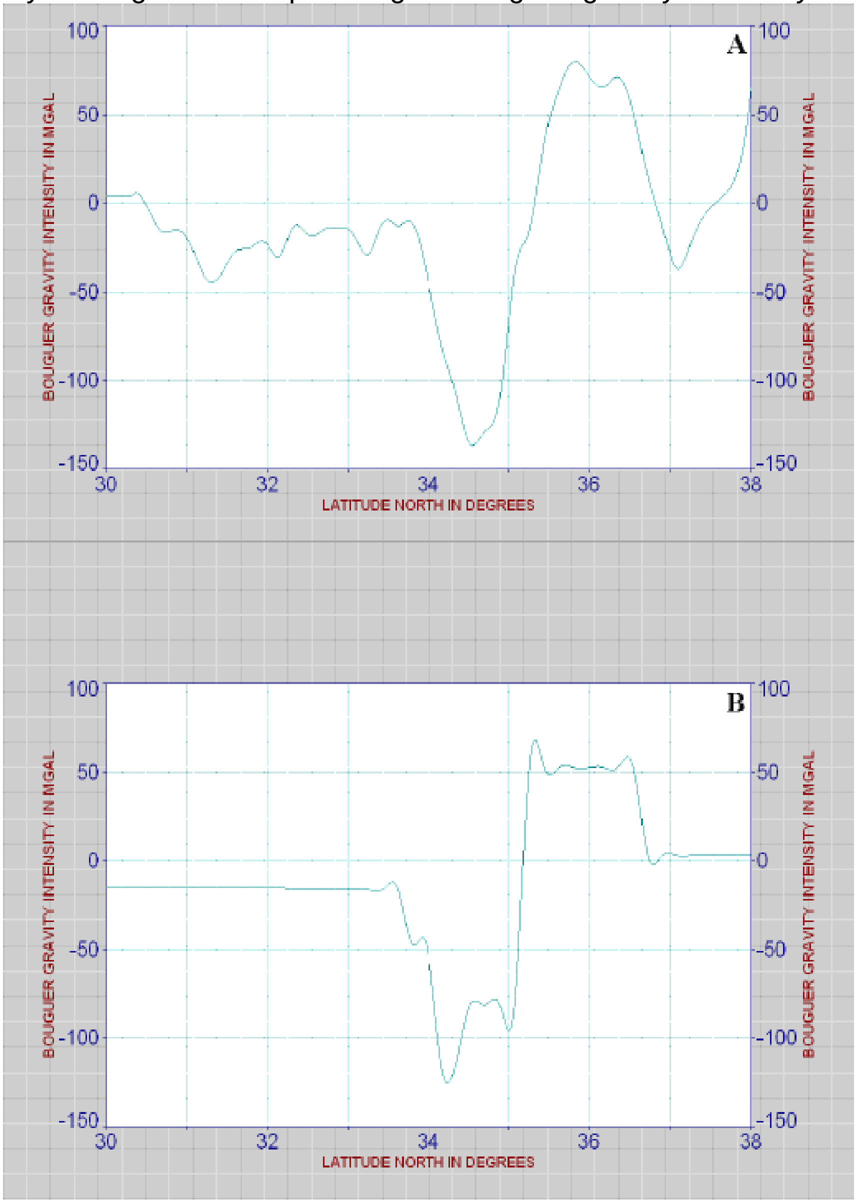

Figure.5. Example of real Bouguer gravity profile data of the survey (A) and the corresponding Savitsky-Golay denoised output (B)

The advantage of the Savitzky-golay filter has the ability to preserve higher moments in the Bouguer gravity data and thus reduce smoothing on peak heights. It is a powerfful tool particulary suitable in denoising, filtering and analyzing problems and potential singularities [15]. Moreover this property is crucial for performing an efficient linear denoising Bouguer gravity anomaly map.

The Moroccan gravity data base is a compilation of Bouguer gravity profiles at a regular spacing. We calculated the output Savitzky-Golay filtered signal using using Origin Pro 8 routine [16] for each Bouguer gravity profile (figure 5). Then we deferred all the results to built a regular map which represent in fact Savitzky-Golay filtering and denoising map of the Moroccan Bouguer gravity data. Since a major potential application of filtering is in image processing, the Savitzky-Golay filter transform is a necessity to be applied as a detector, analyzer and filter of singularities like edges, contours or corners [17].

5. RESULTS & CONCLUSIONS

Figure 6 represents an indicator of the level of variation of the density contrast between the disturbed areas and the normal and stable areas. These method enable us to identify the Moroccan Bouguer anomaly zones which turned out to be strongly correlated with the disturbed zones. Moreover the level of the geological disturbance zones is very clearly shown. The level density contrast resulting from such method is also more defined in all the disturbed zones. The surface modeling of Moroccan Bouguer gravity anomaly map is obtained using Origin Pro 8 routine from our transformed data obtained using the Savitsky-Golay filter response.The Savitsky-Golay analysis surface of Moroccan Bouguer gravity anomaly zones as obtained by the above procedure in the study area provided a direct image for an interpretation of the Moroccan gravity survey. The overall effect is that of scanning and denoising the anomalous bodies. Comparatively to classical approaches used in filtering and denoising geophysical data map [18], the advantage of the Savitsky-Golay filtering method is doesn't introduce significant distorsion to the shape of the original Bouguer gravity anomaly signal. We have described an analytical procedure to denoise, to smooth and to filter the Bouguer gravity anomaly map. The results proved satisfying. Data processing procedures as the Savitzky-Golay filter transform was found to be consistently useful and the corresponding map may be used as auxiliary tools for geologist decision making under field conditions.

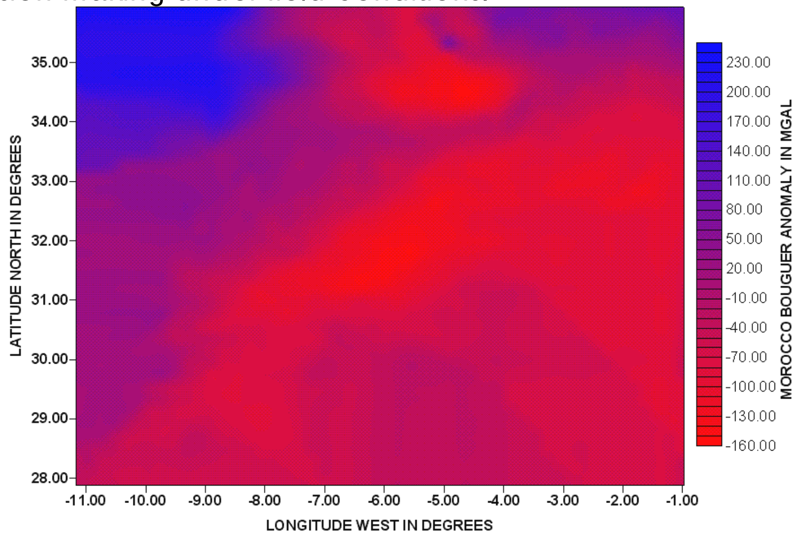

Figure 6. 2D Savitzky-Golay filtering output of Moroccan Bouguer gravity anomaly data map given in figure 4

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