JOURNAL OF ENGINEERING
annals OF
FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 1, (ISSN 1584 – 2665)

HEAT AND MASS TRANSFER EFFECTS ON FREE CONVECTIVE FLOW PAST A SEMI-INFINITE VERTICAL PLATE

Govindasamy PALANI, Kwang-Yong KIM

DEPARTMENT OF MECHANICAL ENGINEERING,
INHA UNIVERSITY, INCHEON, REPUBLIC OF KOREA

Abstract
Heat and Mass transfer effects on free convective flow past a semi-infinite vertical plate are considered here-with. The governing boundary layer equations for the above problem are set up and non-dimensionalised. The non-dimensional governing equations are solved by an implicit finite difference scheme of Crank-Nicolson Method, which is fast convergent and unconditionally stable. Numerical results are obtained and representative set of these results is displayed graphically on the velocity, temperature and concentration profiles. The local and average skin friction, Nusselt number and Sherwood Number are also displayed graphically.
Keyword: Finite-difference method, Nusselt number, vertical plate, heat and mass transfer

1. INTRODUCTION

In recent years, the problem of two-dimensional free convective flow past a semi-infinite plate with different boundary conditions has attracted the attention of many researchers. This is connected with a wide range of natural occurring phenomena and practical applications. Simultaneous heat and mass transfer in natural convection flows on a vertical plate has a wide range of applications in the field of science and technology.

Polhausen [1] first studied the steady free-convective flow past a semi-infinite vertical plate by integral method. Similarity solution to this problem was given by Ostrach [2]. Siegel [3] was the first to study the transient free-convective flow past a semi-infinite vertical plate by integral method. These problems are concerned thermal convection only. But in nature along with the free convection currents caused by the temperature difference, the flow is affected by the differences in concentration on material constitution. For example, in atmospheric flows there exists a difference in the H_2O concentration and hence the flow is affected by such concentration difference. Hence steady free convective flow with mass transfer was studied by Gebhart and Pera [4]. The effects of mass transfer on transient free convective flow past a semi-infinite vertical isothermal plate were studied by Callahan and Marner [5] by explicit finite difference method. Soundalgekar and Ganesan [6] studied the same problem by an implicit finite difference method. Soundalgekar and Ganesan [7] analysed the problem of transient free convection with mass transfer on a vertical plate with constant heat flux by using an implicit finite difference scheme. In nature, the mass also may be diffused at a constant rate from the surface. However, the problem of natural convection flow over a vertical plate with heat and mass flux did not receive the attention of any researcher. Hence, in this problem it is proposed to solve the problem of transient unsteady free convection flow past a semi-infinite vertical plate with heat and mass flux.

2. FORMULATION OF THE PROBLEM

We considered a two-dimensional unsteady flow of a viscous incompressible fluid past a semi-infinite vertical plate with constant heat and mass flux.. We assume that the concentration C' of the diffusing species in the binary mixture is to be very small in comparison with other chemical species, which are present, and hence we neglect Soret and Duffor effects. It is also assumed that the effects of viscous dissipation are negligible in the energy equation. Initially, it is also assumed that the plate and the fluid are of the same temperature and concentration. At time $t' \geq 0$ heat and mass is assumed to be supplied at a constant rate at the plate and is maintained at the same value. The x-axis is measured along the plate and y-axis is measured normal to the plate. Then under the usual Boussinesq's approximation, the boundary layer flow is governed by the following equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2} \quad (4)$$

Initial and Boundary conditions are as follows:

$$\begin{aligned}
&t' \leq 0: u=0,\ v=0,\ T'=T'_\infty,\ C'=C'_\infty \\
&t' > 0: u=0, v=0,\ \frac{\partial T'}{\partial y}=-\frac{q_w}{k},\ \frac{\partial C'}{\partial y}=-\frac{q^*_w}{D} \quad \text{at } y=0 \\
&u=0,\ T'=T'_\infty,\ C'=C'_\infty \quad \text{at } x=0 \\
&u\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \quad \text{as } y\to\infty
\end{aligned} \quad (5)$$

Introducing the following non-dimensional quantities

$$\begin{aligned}
&X=\frac{x}{L},\quad Y=\frac{y}{L}Gr^{1/4},\quad U=\frac{uL}{\upsilon}Gr^{-1/2},\ V=\frac{vL}{\upsilon}Gr^{-1/4}, \\
&t=\frac{\upsilon t'}{L^2}Gr^{1/2},\quad T=\frac{T'-T'_\infty}{[q_w L/k]}Gr^{1/4},\quad C=\frac{(C'-C'_\infty)}{[q^*_w L/D]}Gr^{1/4}, \\
&Gr=\frac{g\beta L^4 q_w}{\nu^2 k},\quad Gc=\frac{g\beta^* L^4 q^*_w}{\nu^2 D} \\
&Pr=\frac{\nu}{\alpha},\quad Sc=\frac{\nu}{D},\quad N=\frac{Gc}{Gr},
\end{aligned} \quad (6)$$

Governing equation reduces to the following form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (7)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=TGr^{-1/4}+NCGr^{-1/4}+\frac{\partial^2 U}{\partial Y^2} \quad (8)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \quad (9)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (10)$$

The corresponding initial and boundary conditions in non-dimensional quantities are given by

$$\begin{aligned}
&t \leq 0: U=0,\ V=0,\ T=0,\ C=0 \\
&t>0: U=0,\ V=0,\ \frac{\partial T}{\partial Y}=-1,\ \frac{\partial C}{\partial Y}=-1 \quad \text{at } Y=0 \\
&U=0,\ T=0,\ C=0 \quad \text{at } X=0 \\
&U\to 0,\ T\to 0,\ C\to 0 \quad \text{as } Y\to\infty
\end{aligned} \quad (11)$$

3. NUMERICAL TECHNIQUE

An implicit finite difference scheme of Crank-Nicolson type has been used to solve the governing non-dimensional equations (7) – (10) under the initial and boundary conditions (11). The method of solving the above finite difference equations using Crank-Nicolson type has been discussed by Soundalgekar and Ganesan [6]. The region of integration is considered as a rectangle with sides X_{max} (= 1.0) and Y_{max} (= 22.0) where Y_{max} corresponds to $Y = \infty$ which lies very well outside the momentum, thermal and concentration boundary layers. Appropriate mesh sizes $\Delta X = 0.05$, $\Delta Y = 0.25$ and time step $\Delta t = 0.01$ are considered for calculations. Computations are repeated until the steady state is reached. The steady-state solution is assumed to have been reached when the absolute difference between values of velocity U as well as temperature T and concentration C at two consecutive time steps are less than 10^{-5} at all grid points. The Crank-Nicolson implicit finite difference scheme is always stable and convergent.

4. RESULTS AND DISCUSSION

The effects of the Grashof number and buoyancy ratio parameter on the transient velocity, temperature and concentration profiles are shown in Figures 1,2 and 3 respectively. The velocity increases steadily with time reaches a temporal maximum and consequently it reaches the steady state. However, time required to reach the steady state depends upon the value of Grashof number and buoyancy ratio parameter N. An increase in N leads to an increase the velocity, whereas the velocity decreases with the increasing value of the Grashof number. This indicates that the buoyancy force due to concentration dominates in the region near the plate over thermal buoyancy force on velocity. The buoyancy forces due to temperature and concentration are oppose in nature when N is negative. Due to the interaction of these two opposing force, time taken to reach the steady state is more. It is observed that the temperature and concentration increases with increasing Grashof number, whereas the effect of Grashof number is nil when t is small. Temperature and concentration field decreases with increasing value of buoyancy ratio parameter N.

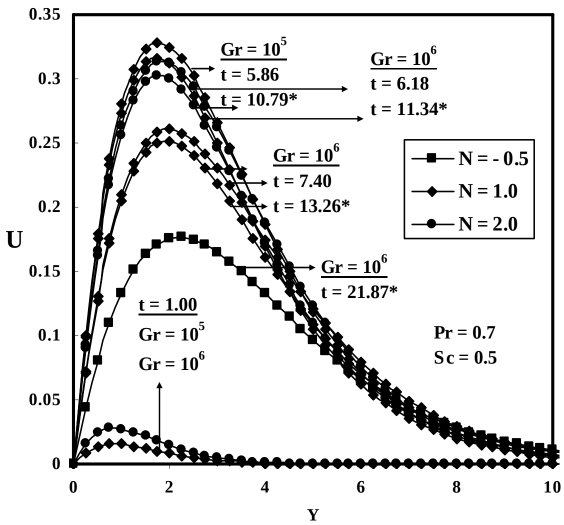

Fig.1 Transient velocity profiles at X=1.0 for different Gr and N (* - steady state)

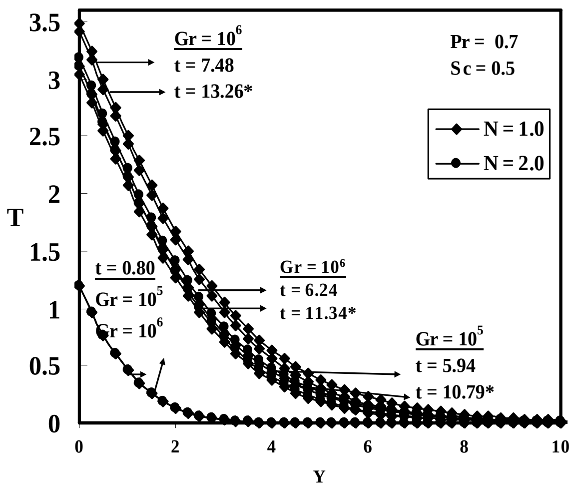

Fig.2 Transient temperature profiles at X=1.0 for different Gr and N (* - steady state)

The steady state velocity, temperature and concentration profiles are shown at the Upper edge of the plate viz., at X=1.0 for different value of Prandtl number and Schmidt number are shown in Figures 4, 5 and 6 respectively. The velocity is more when light species concentration is present than that when the heavy species concentration is present. Time taken to reach the steady state depends upon the value of Sc. The velocity of air (Pr = 0.7) is always greater than the water (Pr = 7.0) is justified. Temperature increases with Schmidt number and concentration increases with Prandtl number. This is quite expected. Thermal boundary layer decreases for the larger value of Pr. Steady state concentration profiles attained at an early state for the lower value of Pr.

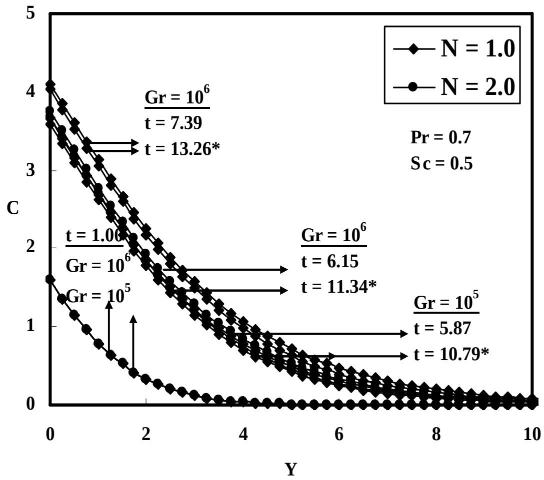

Fig.3 Transient concentration profiles at X=1.0 for different Gr and N (* - steady state)

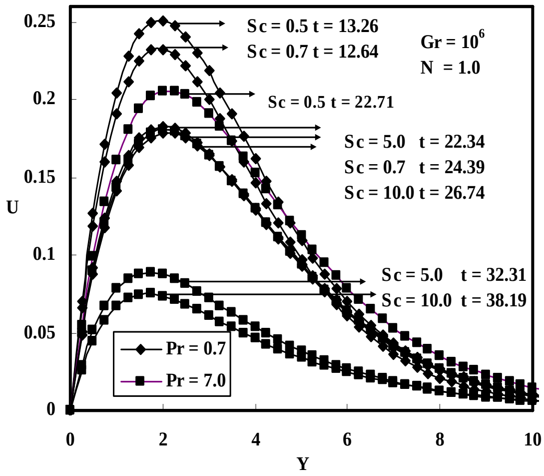

Fig.4 Steady state velocity profiles at X=1.0 for different Pr and Sc

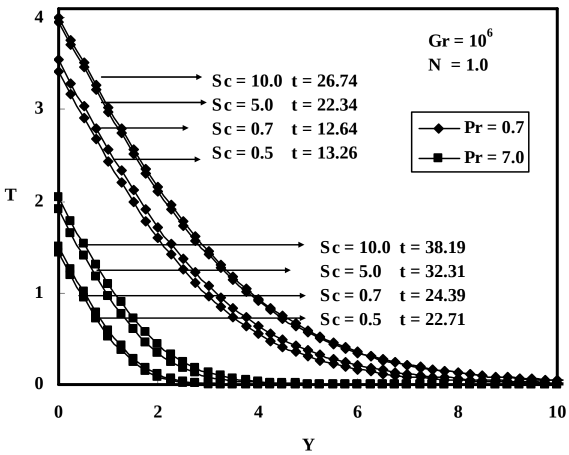

Fig.5 Steady state temperature at X=1.0 for different Pr and Sc

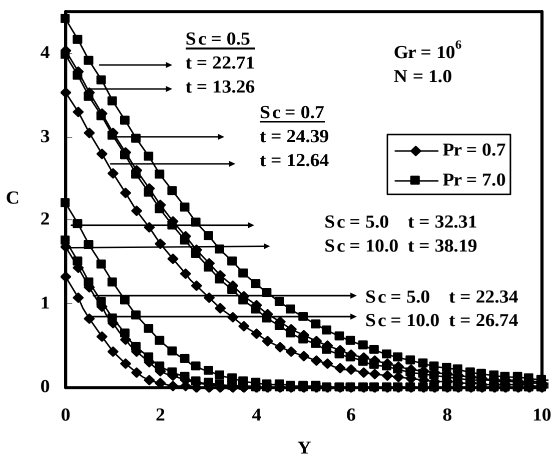

Fig.6 Steady state concentration profiles at X=1.0 for different Pr and Sc

We now study local and average skin friction, local and average Nusselt number, local and average Sherwood number. In non-dimensional quantities, they are given by,

$$\tau_X = Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{12}$$

$$\bar{\tau} = Gr^{3/4}\int_0^1\left[\frac{\partial U}{\partial Y}\right]_{Y=0} dX \tag{13}$$

$$Nu_X = -X\,Gr^{1/4}\left(\frac{\partial T}{\partial Y}\right)_{Y=0} \tag{14}$$

$$\overline{Nu} = -Gr^{1/4}\int_0^1\left[\left(\frac{\partial T}{\partial Y}\right)_{Y=0}\right] dX \tag{15}$$

$$Sh_X = -X\,Gr^{1/4}\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{16}$$

$$\overline{Sh} = -Gr^{1/4}\int_0^1\left[\left(\frac{\partial C}{\partial Y}\right)_{Y=0}\right] dX \tag{17}$$

The derivatives involved in equations (12) to (17) are evaluated by using a five-point approximation formula and then the integrals are evaluated by Newton-Cotes closed integration formula.

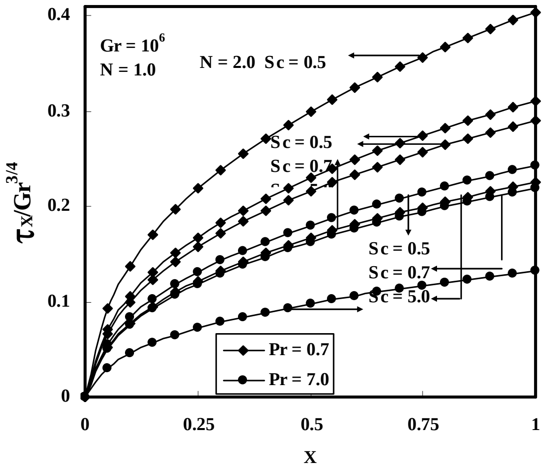

Fig.7 Local skin friction

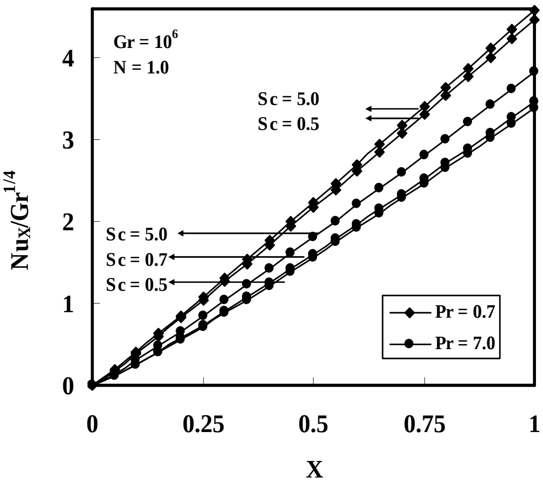

Fig.8 Local Nusselt number

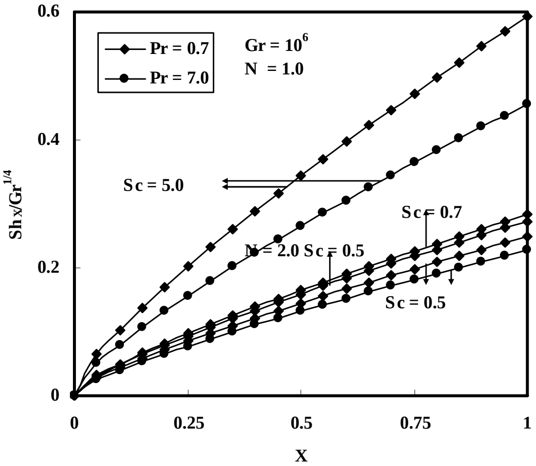

Fig.9 Local Sherwood Number

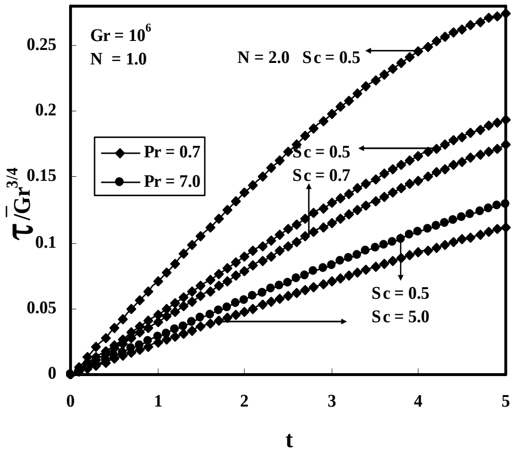

Fig.10 Average skin friction

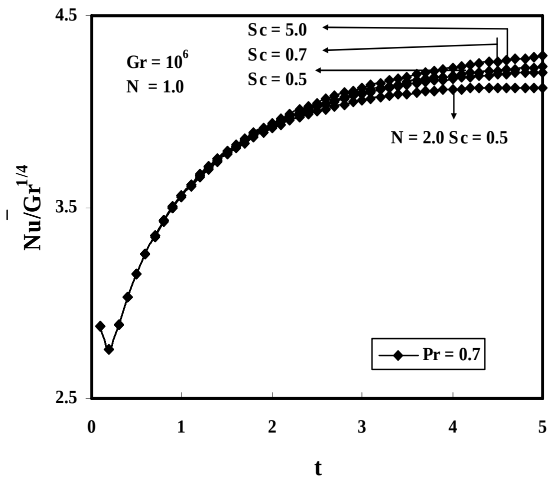

Fig.11 Average Nusselt number

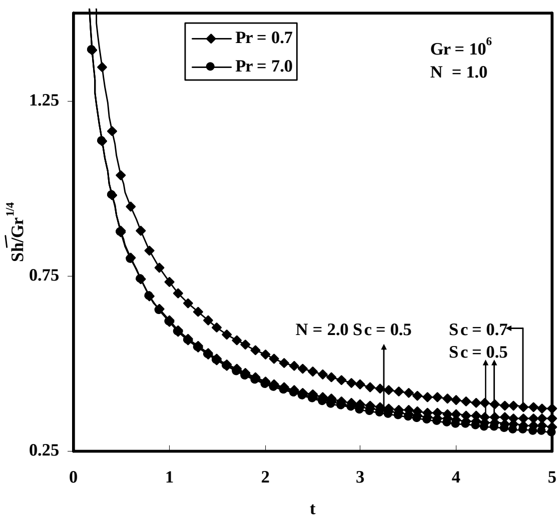

Fig.12 Average Sherwood Number

Local values of skin friction, Nusselt number and Sherwood number are plotted in Figures 7, 8 and 9 respectively. We observe from the figure 7 that an increase in Sc leads to a decrease in local shear stress when N is constant but it increases with increasing N. Local skin friction is not significantly affected by the presence of species concentration near the leading edge of the plate, since the leading edge the heat and mass transfer process are by pure conduction and pure mass diffusion and hence the velocity field is not affected by convection of heat and mass. We observe from figure 8 that an increase in Sc leads to an increase in Local Nusselt number whereas an increase in Pr leads to a decrease in Local Nusselt number.. At small values of X, i.e., near the leading edge of the plate, Sc does not affect the local Nusselt number as there exist pure diffusion and conduction. We conclude from figure 9, that an increase in Sc or N leads to an increase in local Sherwood Number.. At small values of X, due to pure conduction and diffusion, local Sherwood Number is not affected by both Sc and N.

Average values of skin friction, Nusselt number and Sherwood number are plotted for various parameters in Figures 10, 11 and 12 respectively.

Average skin friction increases at small values of t whereas an at large value of t, the average skin-friction remains independent of t, i.e., the average skin-friction depends on time t only when t is small. Average skin friction gets reduced with increasing value of Sc, but it get increased with N through out the transient period and steady state level. Also we conclude that average skin friction decreases as Pr increases. From Figure 11, we conclude that the average Nusselt number decreases sharply at small values of time t, being unaffected by Sc or N, but at large values of t, it is independent of time. Average Nusselt number gets decreased with decreasing value of Sc but it gets increased with decreasing value of N in the transient period and steady state level. We observe from figure 12 that the behaviour of average Sherwood number is the same as local Sherwood number with respect to Sc, Pr and N. But average Sherwood number is independent of time when t is large.

Nomenclature

C' species concentration
C dimensionless species concentration
D coefficient of diffusion in the mixture
Gc mass Grashof number
Gr Grashof number
g acceleration due to gravity
$\overline{Nu}$ dimensionless average Nusselt number
Nu_x dimensionless local Nusselt number
Pr Prandtl number
q_w heat flux per unit area
q_w^* mass flux per unit area
Sc Schmidt number
$\overline{Sh}$ dimensionless average Sherwood number
Sh_x dimensionless local Sherwood number
T' temperature
T dimensionless temperature
t' time
t dimensionless time
u,v velocity components in x, y directions
U, V dimensionless velocity components in X, Y directions respectively
x spatial coordinate along the plate
X dimensionless spatial coordinate
y spatial coordinate along upward normal to the plate
Y dimensionless spatial coordinate along upward normal to the plate

Greek symbols

α thermal diffusivity
β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration
ν kinematic viscosity
τ_X dimensionless local skin friction
$\bar{\tau}$ dimensionless average skin friction

Subscripts

w conditions on the wall
∞ free stream condition

REFERENCE

[1] Polhausen E., "Der Wärmeaustausch Zwischen festen Körpern und Flussigkeiten mit Kleiner Reibung und Kleiner Wärmeleitung", ZAMM, Vol.1, pp.115-121.

[2] Ostrach S, "An analysis of laminar free convection flow and heat transfer about a flat plate parallel to the direction of the generating body force", NACA, Report TR-1111, pp.63-79.

[3] Siegel R, "Transient free convection from a vertical plate', Trans. Amer. Soc. Mech. Engg., 1958, Vol.80, pp.347-359.

[4] Gebhart B and Pera L, "The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion", Int. J. Heat Mass Transfer, 1971, Vol.14, pp.2025-2050.

[5] Callahan G.D. and Marner W.J., "Transient free convection with mass transfer on an isothermal vertical flat plate", Int. J. Heat Mass Transfer, 1976, Vol.19, pp.165-174.

[6] Soundalgekar V.M. and Ganesan P., "Finite difference analysis of transient free convection with mass transfer on an isothermal vertical flat plate", Int. J. Engg. Sci., Vol.19, pp.757-770.

[7] Soundalgekar V.M. and Ganesan P., "Transient free convection with mass transfer on a vertical plate with constant heat flux", Energy Research, 1985, Vol.9, pp.1-17.