JOURNAL OF ENGINEERING annals OF FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 2, (ISSN 1584 – 2665)

SQUEEZE FILM BASED ON MAGNETIC FLUID IN CURVED ROUGH CIRCULAR PLATES

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ABSTRACT:
An attempt has been made to investigate the behavior of a magnetic fluid based squeeze film between two curved rough circular plates where in, the curved upper plate lying along the surface determined by an exponential function approaches the stationary curved lower plate along the surface generated by a hyperbolic function. A magnetic fluid is used as a lubricant in the presence of an external magnetic field oblique to the radial axis. The roughness of the bearing surface is characterized by a stochastic random variable with non-zero mean, variance and skewness. The Reynolds equation governing the film pressure is averaged with respect to the random roughness parameter. The associated non-dimensional differential equation is solved with suitable boundary conditions in dimensionless form, in order to obtain the pressure distribution. This is then used to derive the expression for the load carrying capacity thereby, leading to the way for the calculation of response time. The results are presented graphically as well as in tabular form. The results establish that the performance of the bearing system improves significantly as compared to that of a bearing system working with a conventional lubricant. It is seen that the pressure, the load carrying capacity and the response time substantially increase with increasing magnetization parameter. This investigation tends to suggest that albeit the bearing suffers in general due to transverse surface roughness, there exist some scopes for obtaining better performance in the case of negatively skewed roughness, by suitably choosing the curvature parameters of both the plates. In addition, variance (-ve) adds to the positive effect introduced by negatively skewed roughness.

KEYWORDS:
Magnetic fluid, Squeeze film, Pressure distribution, Roughness, Reynolds equation, Load carrying capacity

1. THE INTRODUCTION

The behavior of squeeze film between various geometrical configurations of flat surfaces was analyzed by Archibald [1]. The squeeze film phenomena between curved plates considering curvature of the sine form and keeping minimum film thickness as constant was presented by Hays [2]. Murti [3] discussed the behavior of squeeze film between curved circular plates describing the film thickness by an exponential expression. The analysis was based on the assumption that the central film thickness, instead of minimum film thickness as assumed by Hays [2]; was kept constant. It was shown that the load carrying capacity rose sharply with the curvature in the case of concave pads. Gupta and Vora [4] dealt with the corresponding problem in the case of annular plates. In this investigation lower plate was taken to be flat. Ajwaliya [5] extended the analysis of this study by taking the lower plate also to be curved. Wu [6] and [7] investigated the squeeze film performance for two types of geometries namely, annular and rectangular wherein, one of the surface was porous faced. Prakash and Vij [8] studied several bearing configurations such as circular, annular, elliptical, rectangular and conical. They made a comparison between the squeeze film behavior of various geometries of equivalent surface area and concluded that the circular plates had the highest transient load carrying capacity.

All the above studies dealt with conventional lubricant. Verma [9] investigated the application of a magnetic fluid as lubricant. The magnetic fluid comprised of fine surfactant and magnetically passive solvent. Subsequently, the magnetic fluid based squeeze film behavior between porous annular disks was presented by Bhat and Deheri [10]. It was established that the application of magnetic fluid lubricant enhanced the performance of the squeeze film behavior. However, here the plates were considered to be flat. But in actual practice the flatness of the plate does not endure owing to elastic, thermal and uneven wear effects. With this end in view, Bhat and Deheri [11] analyzed the behavior of a magnetic fluid based squeeze film between curved circular plates. The magnetic fluid based squeeze film behavior between curved plates lying along the surfaces determined by exponential and hyperbolic function was subjected to investigation by Patel and Deheri [12] and [13] respectively. It was found that the application of magnetic fluid lubricant improved the performance of the bearing system.

It is a well-established fact that the bearing surfaces develop roughness particularly, after having some run-in and wear. The roughness appears to be random and disordered and does not seem to follow any particular structural pattern. The randomness and the multiple roughness scales both contribute to complexity of the geometrical structure of the surface. In inevitably, it is this complexity which contributes most of the problems in the investigation of friction and wear. The random character of the surface roughness was recognized by several investigators, who resorted to a stochastic approach in order to mathematically model the roughness of the bearing surfaces (Tzeng and Seibel [14], Christensen and Tonder [15], [16] and [17]). Tonder [18] theoretically analyzed the transition between surface distributed waviness and random roughness. Christensen and Tonder [15], [16] and [17] modified and developed the approach of Tzeng and Seibel [14] and proposed a comprehensive general analysis both for transverse as well as longitudinal surface roughness based on a general probability density function. The method adopted by Christensen and Tonder [15], [16] and [17] formed the basis of analyzing the effect of surface roughness on the performance of bearing system in a number of investigations (Ting [19], Prakash and Tiwari [20], Prajapati [21], Guha [22], Gupta and Deheri [23]). The probability density function for the random variable characterizing the surface roughness was assumed to be symmetric with mean of the random variable equal to zero, in these above analyses. However, in reality due to non-uniform rubbing of the surfaces, the distribution of surface roughness may indeed be asymmetrical. With this end in view, Andharia, Gupta and Deheri [24] discussed the effect of transverse surface roughness on the performance of a hydrodynamic squeeze film in a spherical bearing using general stochastic analysis. It was observed that the effect of transverse surface roughness on the performance of the bearing system was mostly adverse. Recently, Deheri, Patel and Abhangi [25] discussed the behavior of squeeze film performance between magnetic fluid based curved circular plates.

It has been proposed to study the magnetic fluid based squeeze film between curved transversely rough circular plates wherein the upper plate lies along the surface determined by an exponential function while the lower plate lies along a surface determined by a hyperbolic function.

2. THE ANALYSIS

The configuration of the bearing is shown in Figure 1. The bearing surfaces are assumed to be transversely rough. The thickness $h(x)$ of the lubricant film is taken as

$$h(x) = \bar{h}(x) + h_S$$

where $\bar{h}(x)$ is the mean film thickness while h_S is the deviation form the mean film thickness characterizing the random roughness of the bearing surfaces. The deviation h_S is considered to be stochastic in nature and described by the probability density function

$$f(h_S), -c \le h_S \le c$$

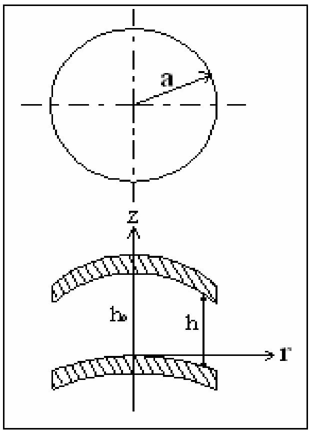


Figure 1: Configuration of the bearing system

where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry associated with random variable h_s are governed by the relations

$$\alpha = E(h_s), \qquad \text{and} \qquad \varepsilon = E[(h_s - \alpha)^3]$$
$$\sigma^2 = E[(h_s - \alpha)^2]$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} R f(h_s)\, dh_s$$

It has been assumed that the upper plate lying along a surface determined by

$$Z_U = h_0(\exp(-Br^2); \quad 0 \le r \le a$$

approaches with normal velocity $\dot{h}_0 = \frac{dh_0}{dt}$, to the lower plate lying along the surface $Z_l = h_0\left[\frac{1}{1+Cr} - 1\right]; \quad 0 \le r \le a$

where h_0 is the central distance between the plates, B and C are the curvature parameters of the corresponding plates. The central film thickness $h(r)$ then is defined by

$$h(r) = h_0\left[\exp(-Br^2) - \frac{1}{1+Cr} + 1\right]$$

Axially symmetric flow of the magnetic fluid between the plates is taken into consideration under an oblique magnetic field

$$\bar{H} = (H(r)\cos\varphi(r,z), 0, H(r)\sin\varphi(r,z))$$

whose magnitude H vanishes at $r = a$; for instance; $H^2 = ka(a-r), 0 \le r \le a$ where k is a suitably chosen constant so as to have a magnetic field of required strength, which suits the dimensions of both the sides. The direction of the magnetic field plays a significant role as $\bar{H}$ has to satisfy the equations

$$\nabla \cdot \bar{H} = 0, \quad \nabla \times \bar{H} = 0.$$

Therefore, $\bar{H}$ arises out of a potential function and the inclination angle φ of the magnetic field $\bar{H}$ with the lower plate is given by $\cot\varphi \frac{\partial\varphi}{\partial r} + \frac{\partial\varphi}{\partial z} = \frac{1}{2(a-r)}$

whose solution is determined from the equations

$$c_1^2 \operatorname{cosec}^2\varphi = a - r, \quad z = -2c_1\sqrt{(a - c_1^2 - r)}$$

where c_1 is a constant of integration.

The modified Reynolds equation governing the film pressure p turns out to be ([11, 13])

$$\frac{1}{r}\frac{d}{dr}\left[r g(h) \frac{d}{dr}\left(p - 0.5\,\mu_0 \bar{\mu} H^2\right)\right] = 12\,\mu\,\dot{h}_0$$

where $g(h) = h^3 + 3\sigma^2 h + 3h^2\alpha + 3h(\alpha^2 + \sigma^2) + 3\sigma^2\alpha + \varepsilon + \alpha^3$

Introducing the non-dimensional quantities

$$\bar{h} = \frac{h}{h_0}, \; R = \frac{r}{a}, \; \mu^* = \frac{-\mu_0\bar{\mu}k h_0^3}{\mu\dot{h}}, \; P = -\frac{h_0^3 p}{\mu a^2 \dot{h}_0},$$

$$\sigma^* = \frac{\sigma}{h_0}, \alpha^* = \frac{\alpha}{h_0}, \; \varepsilon^* = \frac{\varepsilon}{h_0^3}, \; \mathbf{B} = Ba^2, \; \mathbf{C} = Ca$$

and solving the associated Reynolds equation with the concerned boundary conditions

$$P(1)=0, \quad \frac{dP}{dR} = -\frac{\mu^*}{2} \text{ at } R=0$$

we get the non-dimensional pressure as

$$P = \frac{\mu^*}{2}(1-R) + 6\int_R^1 \frac{R}{G(h)} dR \tag{1}$$

where $G(h) = \bar{h}^3 + 3\bar{h}^2\alpha^* + 3\bar{h}(\sigma^{*2} + \alpha^{*2}) + \varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3}$.

The dimensionless load carrying capacity then is given by

$$\overline{W} = \frac{\mu^*}{12} + 3\int_0^1 \frac{R^3}{G(h)} dR \tag{2}$$

where the load carrying capacity W is determined by the relation $W = 2\pi\int_0^a rp(r)dr$

The response time in dimensionless form comes out to be

$$\Delta T = \frac{\Delta t W h_0^2}{\pi\mu a^4} = \overline{W}\int_{\bar{h}_1}^{\bar{h}_2} \frac{1}{G(h)} d\bar{h} \tag{3}$$

where $\bar{h}_1 = \frac{h_1}{h_0}$, $\bar{h}_2 = \frac{h_2}{h_0}$

3. THE RESULTS AND DISCUSSIONS

The dimensionless pressure P, load carrying capacity $\overline{W}$ and response time ΔT are determined by equations (1), (2) and (3) respectively. It is clear that these performance characteristics are dependent on various parameters such as μ^*, σ^*, ε^*, α^*, **B** and **C**. These parameters describe respectively the effect of magnetic fluid lubricant, transverse roughness and the curvature parameters.

The equation (1) shows that the pressure P is increased by $\frac{\mu^*}{2}(1-R)$ while the increase in load carrying capacity in $\overline{W}$ is $\mu^*/12$ which is the indication from equation (2). Taking the roughness parameters σ^*, ε^* and α^* to be zero one can get the performance of a magnetic fluid based squeeze film trapped between curved circular plates lying along the surfaces determined by exponential function and hyperbolic function. In addition, setting the magnetization parameter to be zero this investigation reduces essentially to the study of squeeze film behavior between the associated curved circular plates.

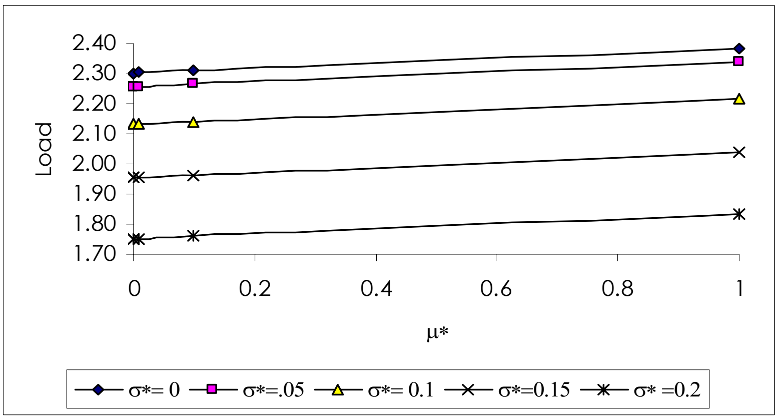


Figure 2: Variation of load carrying capacity with respect to μ^* and σ^*

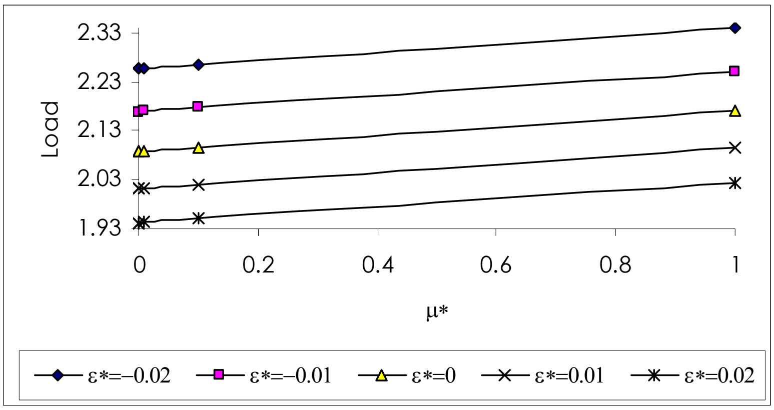


Figure 3: Variation of load carrying capacity with respect to $\mu*$ and $\varepsilon*$

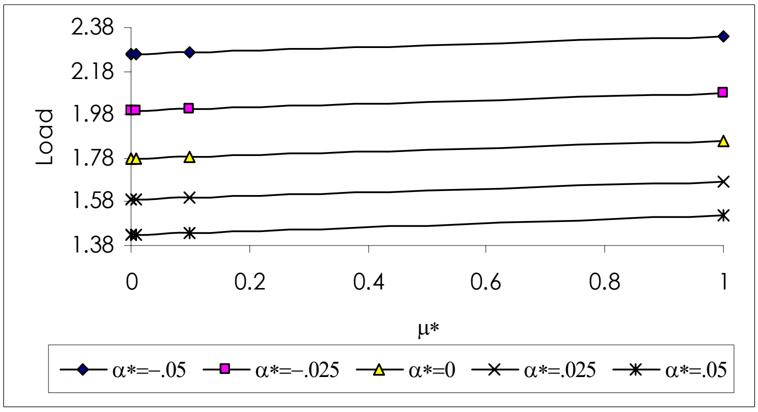


Figure 4: Variation of load carrying capacity with respect to $\mu*$ and $\alpha*$

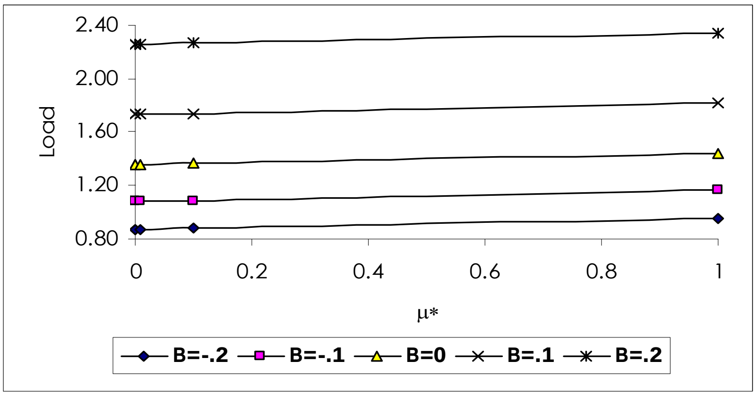


Figure 5: Variation of load carrying capacity with respect to $\mu*$ and B

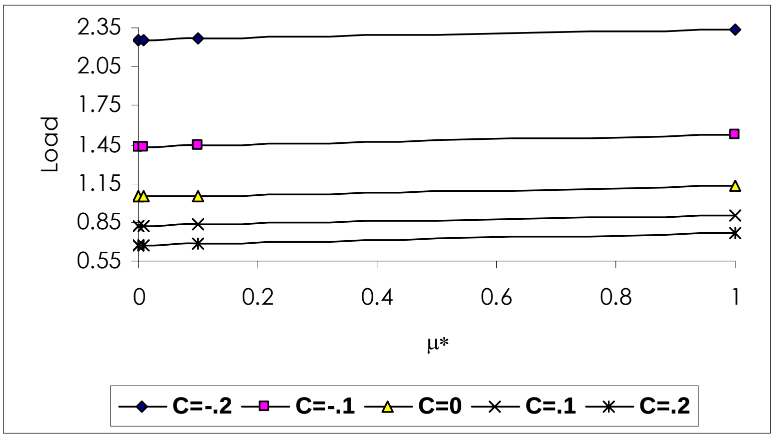
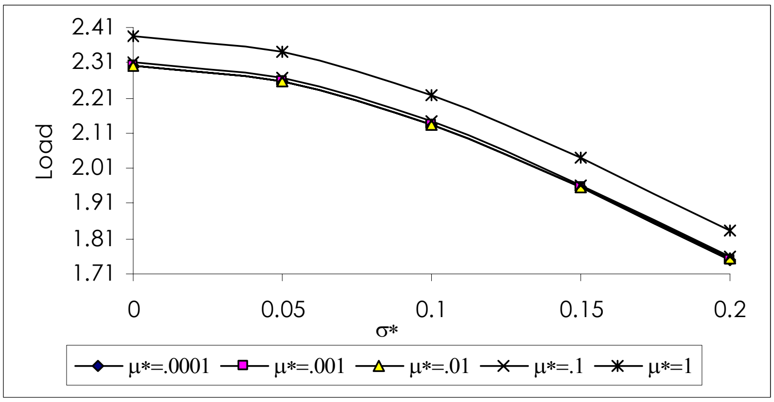
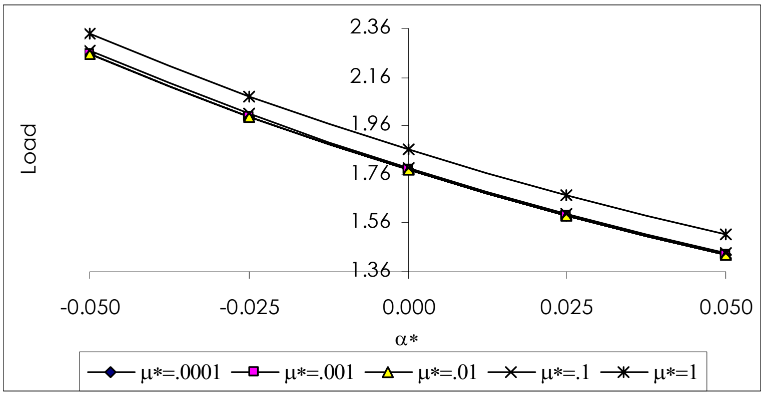


Figure 6: Variation of load carrying capacity with respect to $\mu*$ and C

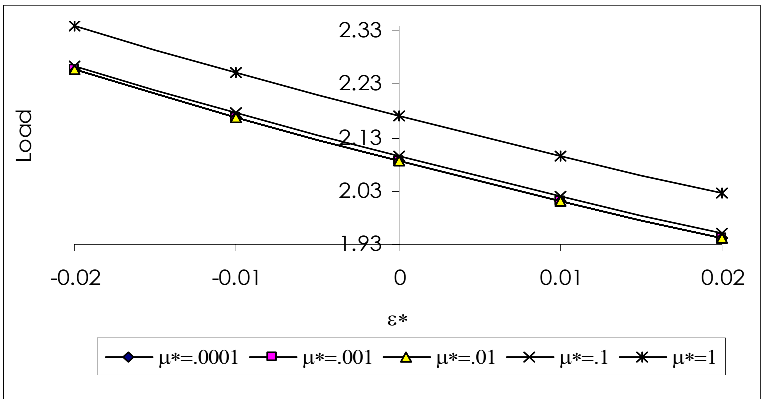
The variation of the load carrying capacity $\overline{W}$ with respect to the magnetization parameter $\mu*$ for various values of roughness parameters $\sigma*$, $\varepsilon*$ and $\alpha*$ and curvature parameters B and C respectively is presented in Figures 2-6. These figures suggest that the load carrying capacity increases considerably with respect to the magnetization parameter, although the effect of $\mu*$ is approximately negligible up to the value 0.1 as indicated in Figures A1-A5. It is interesting to note that among the roughness parameters the combined effect of magnetization parameter and skewness is more sharp.



A1: Load carrying capacity with respect to $\sigma*$ and $\mu*$

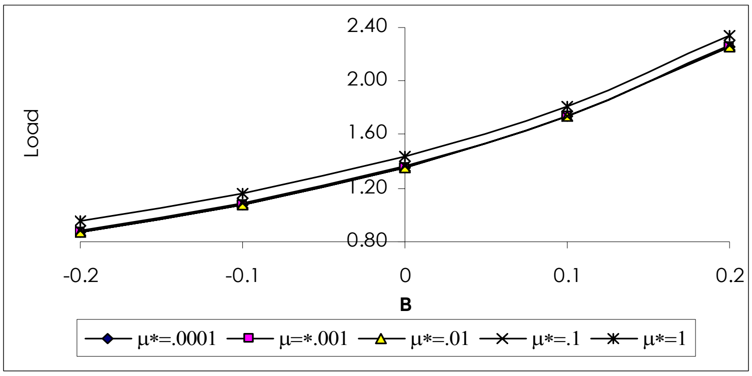


A2: Load carrying capacity with respect to $\alpha*$ and $\mu*$

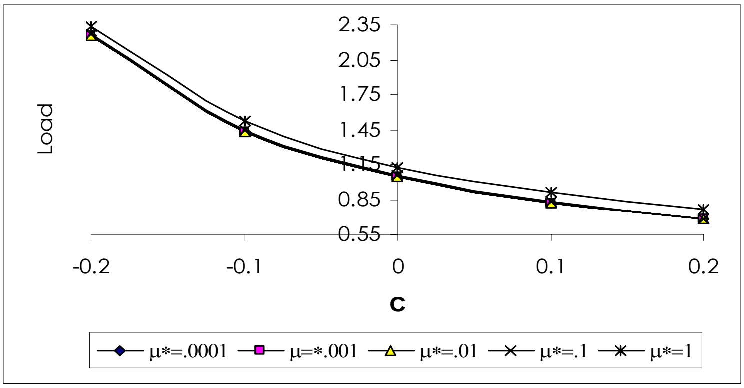


A3: Load carrying capacity with respect to $\varepsilon*$ and $\mu*$

Figures 7-9 show the effect of the standard deviation associated with roughness on the variation of load carrying capacity. It can be easily seen from these figures that the standard deviation has a noticeably adverse effect on the performance of the bearing system because load carrying capacity decreases considerably. The negative effect of $\sigma*$ is relatively less with respect to the lower plate curvature parameter C.



A4: Load carrying capacity with respect to **B** and $\mu*$



A5: Load carrying capacity with respect to **C** and $\mu*$

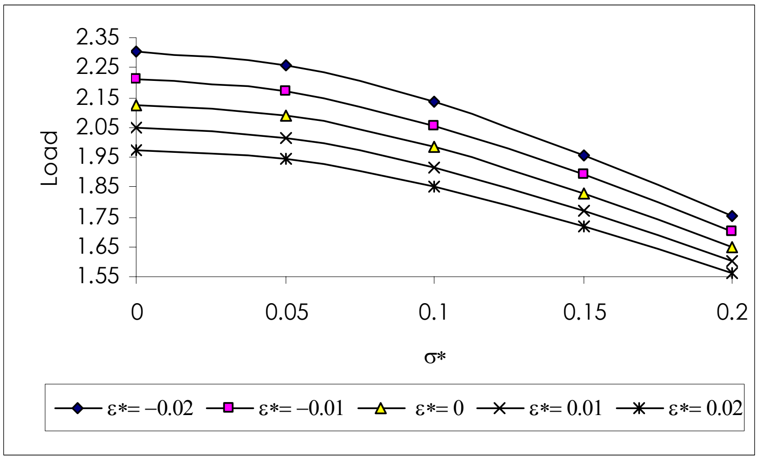


Figure 7: Variation of load carrying capacity with respect to $\sigma*$ and $\varepsilon*$

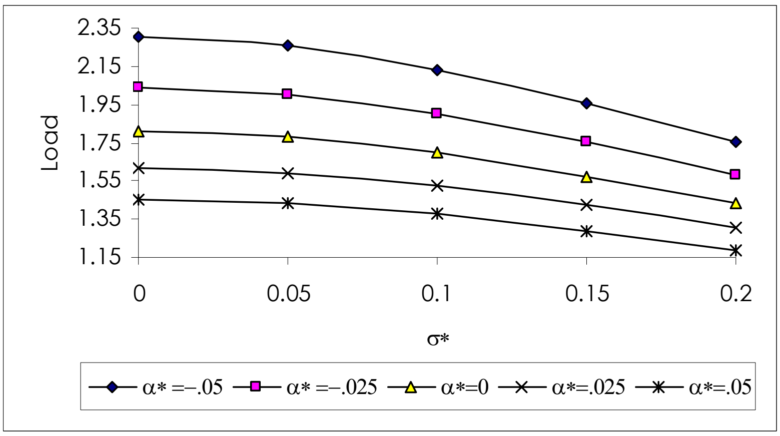


Figure 8: Variation of load carrying capacity with respect to $\sigma*$ and $\alpha*$

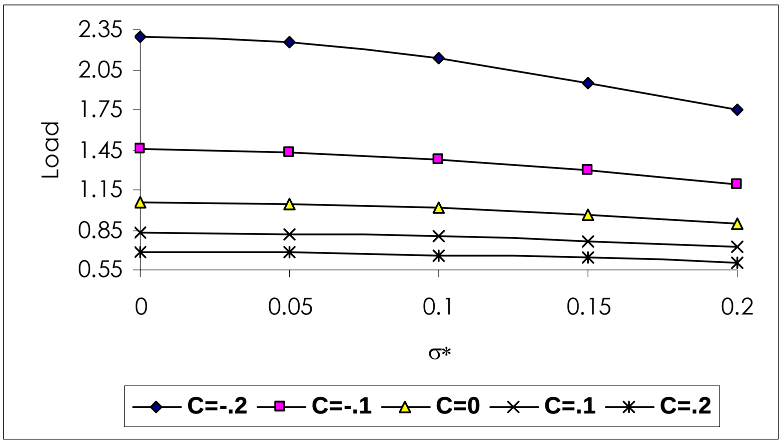


Figure 9: Variation of load carrying capacity with respect to σ^* and **C**

Figures 10-12 depict the effect of variance on the distribution of load carrying capacity. These figures make it clear that α^*(+ve) decreases the load carrying capacity while α^*(-ve) increases the load carrying capacity. Besides, it is revealed that the combined effect of the upper plate curvature parameter and the negative variance is significantly positive. The effect of skewness on the variation of load carrying capacity can be seen from Figures 13-14. As in the case of variance here also ε^*(+ve) decreases the load carrying capacity while $\overline{W}$ increases with respect to ε^*(-ve). Besides, the trends of B are almost opposite to the trends of C as can be seen from Figures 15-16.

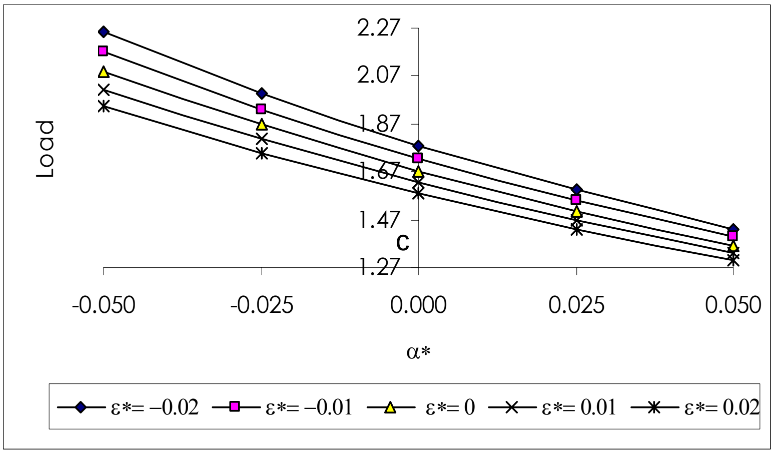


Figure 10: Variation of load carrying capacity with respect to α^* and ε^*

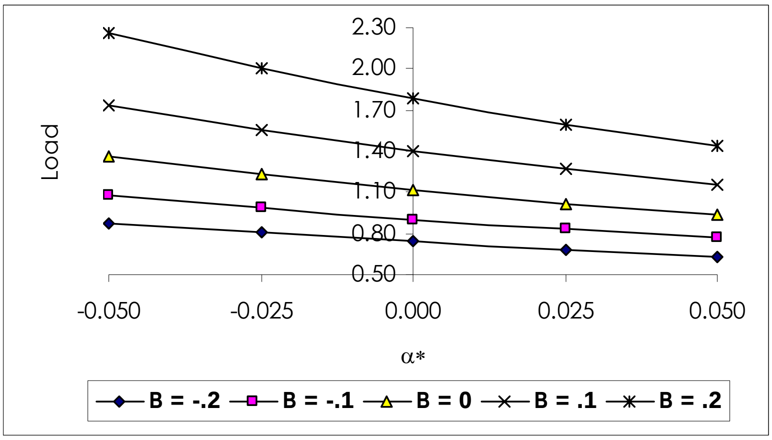


Figure 11: Variation of load carrying capacity with respect to α^* and B

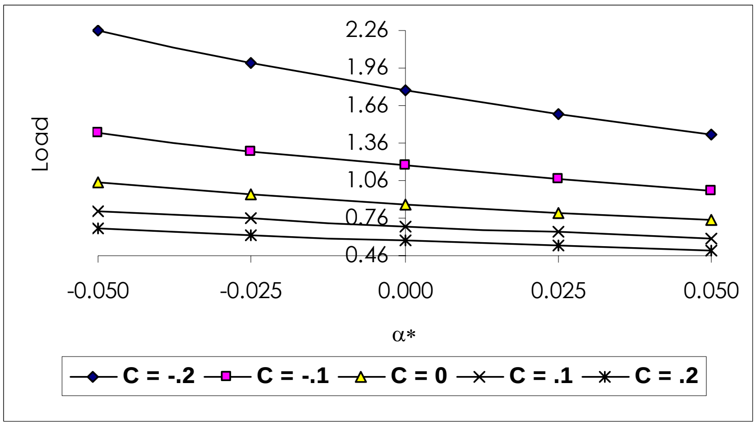


Figure 12: Variation of load carrying capacity with respect to α^* and C

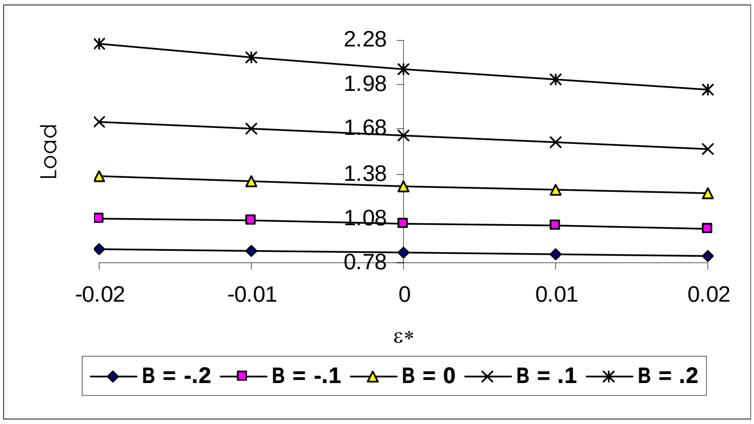


Figure 13: Variation of load carrying capacity with respect to ε^* and B

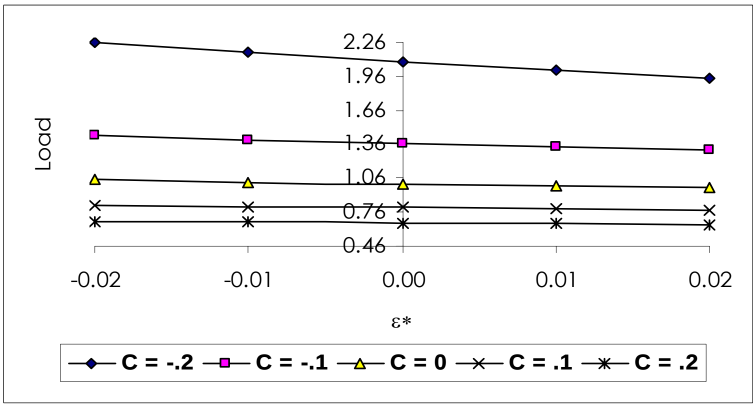


Figure 14: Variation of load carrying capacity with respect to ε^* and C

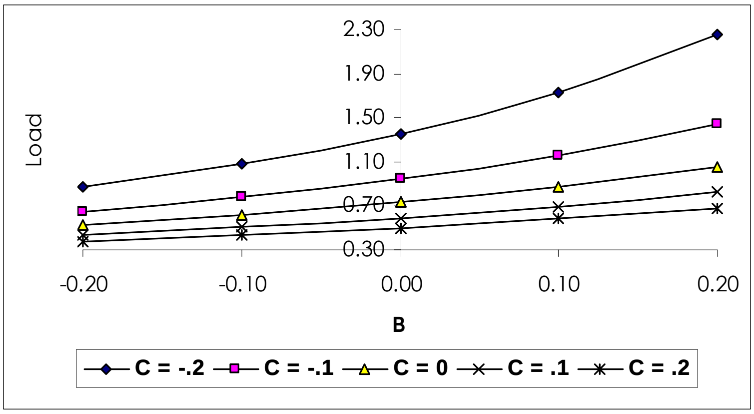


Figure 15: Variation of load carrying capacity with respect to B and C

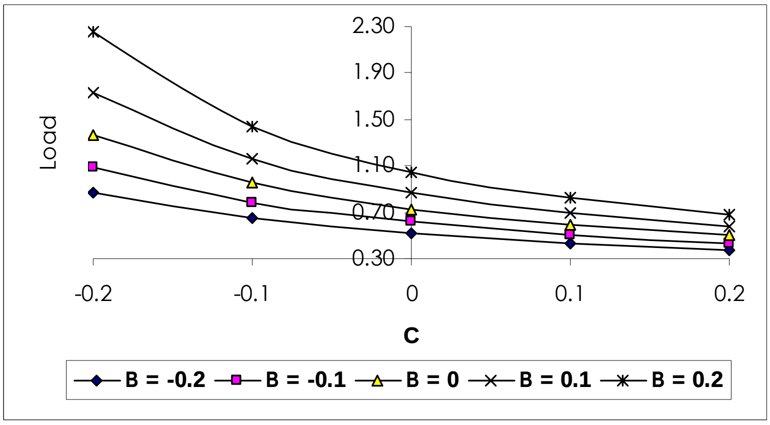


Figure 16: Variation of load carrying capacity with respect to C and B

Furthermore, the combined positive effect of $\varepsilon*$(-ve) and the lower plate curvature parameter C is relatively more as compared to the positive effect of $\varepsilon*$(-ve) and the upper plate curvature parameter B. Interestingly it is observed that the rate of increase in load carrying capacity with respect to the magnetization parameter is little-more with respect to the lower plate curvature parameter in comparison with the upper plate curvature parameter. Besides, it is noticed that the combined effect of negatively skewed roughness and negative variance is considerably positive. Lastly, it can be seen from tables that the trends of the response time ΔT are almost identical with that of the load carrying capacity.

Table 1: Variation of Response time with respect to $\varepsilon*$ and B

	B=-.2	B=-.1	B=0	B=.1	B=.2
$\varepsilon*$=-.2	0.244588	0.342275	0.491472	0.727823	1.120016
$\varepsilon*$=-.1	0.237833	0.331034	0.471692	0.690656	1.044464
$\varepsilon*$=0	0.231353	0.32034	0.453108	0.656403	0.976945
$\varepsilon*$=.1	0.225135	0.31016	0.435624	0.624754	0.916285
$\varepsilon*$=.2	0.219165	0.30046	0.419154	0.595441	0.861529

Table 2: Variation of Response time with respect to $\mu*$ and C

	C=-.2	C=-.1	C=0	C=.1	C=.2
$\mu*$=.0001	1.119607	0.521551	0.304562	0.201945	0.145310
$\mu*$=.001	1.119644	0.521579	0.304584	0.201963	0.145326
$\mu*$=.01	1.120016	0.521851	0.304803	0.202148	0.145487
$\mu*$=.1	1.123736	0.524577	0.306991	0.203994	0.147096
$\mu*$=1	1.160933	0.551840	0.328867	0.222457	0.163190

Table 3: Variation of Response time with respect to $\mu*$ and B

	B=-.2	B=-.1	B=0	B=.1	B=.2
$\mu*$=.0001	0.244357	0.342014	0.491174	0.727477	1.119607
$\mu*$=.001	0.244378	0.342038	0.491201	0.727508	1.119644
$\mu*$=.01	0.244588	0.342275	0.491472	0.727823	1.120016
$\mu*$=.1	0.246689	0.344647	0.494185	0.730971	1.123736
$\mu*$=1	0.267690	0.368369	0.521313	0.762452	1.160933

4. THE CONCLUSION

The article indicates that a proper choice of the magnetization parameter and both plates curvature parameters may result in a considerably better performance of the bearing system in the case of negatively skewed roughness, especially, when negative variance occurs. Therefore, the roughness must be accounted for while designing the bearing system.

NOMENCLATURE:

a = radius of the circular plate
p = lubricant pressure
B= curvature parameter of the upper plate
C= curvature parameter of the lower plate
H= magnitude of the magnetic field

$P = -\dfrac{h_0^3 p}{\mu \dot{h}_0 a^2}$ = dimensionless pressure

W= load carrying capacity

$\overline{W} = -\dfrac{h_0^3 W}{\mu \dot{h}_0 a^4}$ = dimensionless load carrying capacity

Δt = response time

$\Delta T = \dfrac{\Delta t W h_0^2}{\pi \mu a^4}$ = non-dimensional response time

α = mean of the stochastic film thickness
σ = standard deviation of the stochastic film thickness
σ^2 = variance
ε = measure of symmetry of the stochastic random variable
$\sigma^* = \sigma/h_0$
$\alpha^* = \alpha/h_0$
$\varepsilon^* = \varepsilon/h_0^3$
$R = r/a$
φ = inclination angle
μ = absolute viscosity of the lubricant
$\overline{\mu}$ = magnetic susceptibility
μ_0 = permeability of the free space

$\mu^* = \dfrac{-\mu_0 \overline{\mu} k h_0^3}{\mu \dot{h}_0}$ = magnetization parameter

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