JOURNAL OF ENGINEERING
annals OF
FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 2, (ISSN 1584 – 2665)

PERFORMANCE-BASED DESIGN APPLIED FOR A BEAM SUBJECTED TO COMBINED STRESS

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ABSTRACT:
Performance-Based Design (PBD) is based on the theory of probability connected with statistics. PBD is also based on the performance requirements (PR) which are usually defined as a synthesis of functionality, all-in cost, safety etc. PR can be expressed as an acceptable level of damage (i.e. acceptable probability of possible failure). In presented example (i.e. a shaft of unknown circular shape is exposed to bending moment, normal force and torque, which are given by bounded histograms) is used Simulation-Based Reliability Assessment Method method (direct Monte Carlo Method, AntHill software). The task is to calculate the nominal value of diameter which is given by normal bounded distribution. The acceptable level of damage is related to the yield stress. The calculation of the diameter (i.e. solution of the inverse problem of theory of probability), must be solved via iterative approaches. To get the solution of this type of inverse problem is much difficult than the solution of the classical problem of theory of probability.
KEYWORDS:
beam, combined loading, probability analysis, Simulation-Based Reliability Assessment (SBRA) method, Monte Carlo simulations, Performance-Based Design

1. INTRODUCTIVE NOTES

In the last several decades, the science and engineering community has progressively ventured outside of traditional boundaries in terms of materials, loads, configurations etc. for structural systems in mechanics. Consequently, a new designer's approach called Performance-Based Design (PBD) can be defined as: "Design specifically intended to limit the consequences of one or more perils to defined acceptable levels". PBD is based on the theory of probability and depends on many inter-connected issues including classification of constructed systems, definition of performance, tools for measuring performance, quantitative indices that may serve as assurance of performance, and especially, how to describe and measure performance especially under various levels of uncertainty which is connected with statistics. However, comprehensive approach of PBD is still in its infancy.

PBD is based on performance requirements which are usually defined as a synthesis of functionality, all-in cost, safety etc., see Fig.1 and 2. Performance requirements can be expressed as an acceptable level of damage, which is defined by acceptable probability of possible failure P_{ACCEPT}. For more details see reference (Hamburger, 1999).

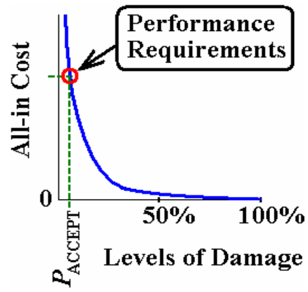

FIGURE 1. STRUCTURE OF PBD

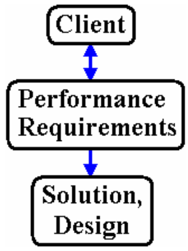

FIGURE 2. STRUCTURE OF PBD.

2. GIVEN EXAMPLE - SHAFT SUBJECTED TO COMBINED STRESS

A shaft of unknown circular diameter D (see Fig.3), is exposed to bending moment $M_o = 581515.3^{+802256.6}_{-344596.4}$ Nmm, normal force $N = 157493.7^{+217277.8}_{-93328.2}$ N and torque $M_k = 367485.4^{+506981.6}_{-217765.8}$ Nmm, which are given by truncated histograms, see Fig.4 to 6. Yield stress of material is $R_e = 338.3^{+161.7}_{-90.3}$ MPa, see truncated histogram in Fig.7. Calculate the value of diameter D which is given by normal truncated distribution ±1% (i.e. $D^{+1\%}_{-1\%}$) with accuracy 0.1mm. The acceptable level of damage is $P_{ACCEPT} = 0.0005 = 0.05\%$ (standard reliability level) is related to yield stress. In other words, 0.05% of all loading states can result in yielding.

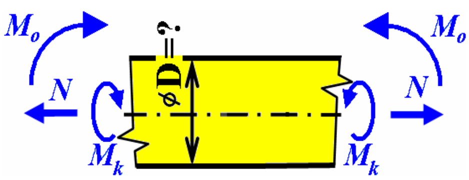

FIGURE 3. SHAFT WITH UNKNOWN DIAMETER IS SUBJECTED TO COMBINED LOADING

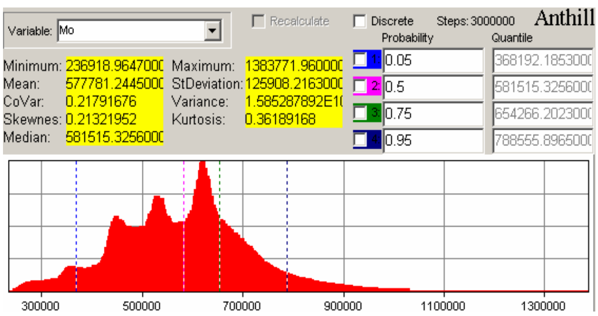

FIGURE 4. HISTOGRAM OF BENDING MOMENT M_o

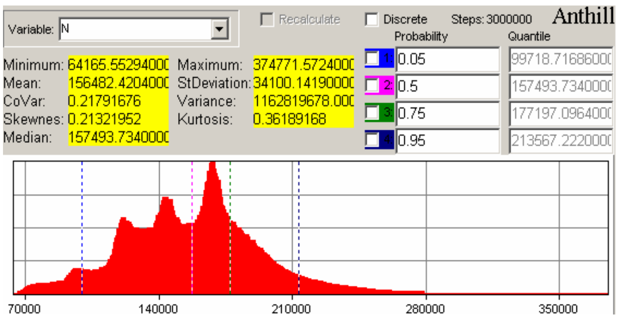

FIGURE 5. HISTOGRAM OF NORMAL FORCE N.

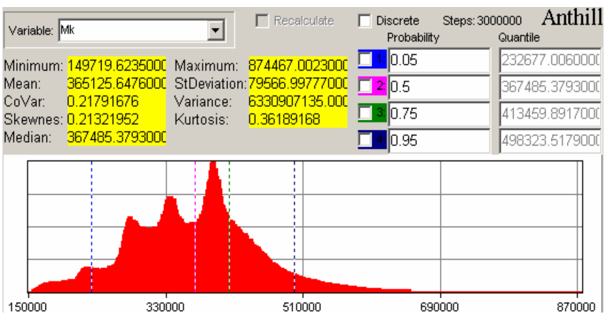

FIGURE 6. HISTOGRAM OF TORQUE M_k.

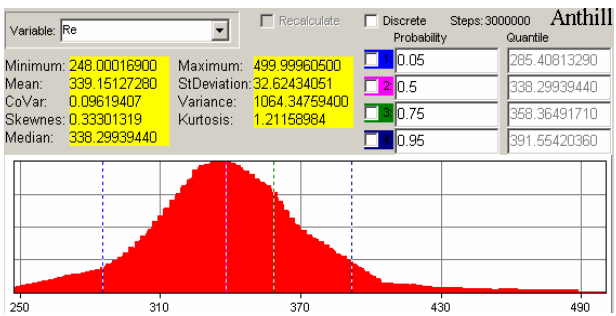

FIGURE 7. HISTOGRAM OF YIELD STRESS R_e.

In the following example is used SBRA method (Simulation-Based Reliability Assessment, direct Monte-Carlo method, AntHill software), see [2] and [3].

3. SOLVED EXAMPLE - PBD APPLIED FOR A SHAFT SUBJECTED TO COMBINED STRESS

According to the theory of small deformations (see [4]) can be written:

$$\sigma = \frac{4N}{\pi D^2} + \frac{32M_o}{\pi D^3} = \frac{4}{\pi D^2}\left(N + \frac{8M_o}{D}\right), \quad \tau = \frac{16M_k}{\pi D^3}, \tag{1}$$

where σ /MPa/ is maximal normal stress and τ /MPa/ is maximal shear stress.

Hence, for equivalent von Mises stress σ_{HMH} /MPa/ can be written:

$$\sigma_{HMH} = \sqrt{\sigma^2 + 3\tau^2} = \frac{4}{\pi D^2}\sqrt{\left(N + \frac{8M_o}{D}\right)^2 + \frac{48M_k^2}{D^2}}. \tag{2}$$

Factor of safety (i.e. probability of situations when $R_e < \sigma_{HMH}$) is defined as:

$$FS = P(R_e - \sigma_{HMH} < 0), \tag{3}$$

where operator "P" means probability.

Hence, when $FS \geq 0$, it is evident that yield limit is not reached (i.e. in the shaft are not any plastic deformations).

The goal is to calculate diameter D which satisfy condition:

$$FS \leq P_{ACCEPT}. \tag{4}$$

However, it is necessary to applied iteration methods, because from eq. (2) is not possible to express directly the unknown parameter D. Hence, iteration loop with application of secant method can be used, see Fig.8.

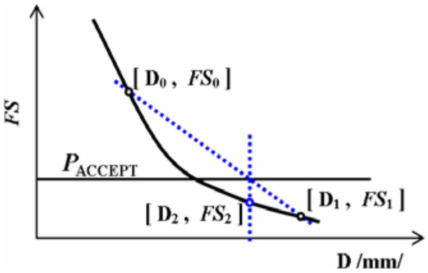

FIGURE 8. SECANT METHOD (calculation of D_2)

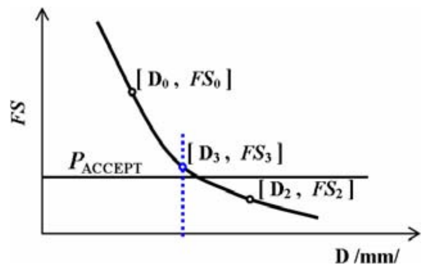

FIGURE 9. BISECTION METHOD (calculation of D_3)

For chosen initial conditions (diameters): $D_0 = 30 \pm 0.3$ mm and $D_1 = 48 \pm 0.48$ mm (truncated normal distributions $\pm 1\%$) is possible to calculate (via SBRA method for 10^6 Monte Carlo simulations) the values of FS_0 and FS_1:

$$D_0 = 30 \pm 0.3\ \text{mm}, \quad FS_0 = 0.869929 \geq P_{ACCEPT},$$

$$D_1 = 48 \pm 0.48\ \text{mm}, \quad FS_1 = 0.000032 \leq P_{ACCEPT}.$$

Hence, $FS_0 \geq P_{ACCEPT}$ and $FS_1 \leq P_{ACCEPT}$. It is evident that the required diameter D must be in interval: $D \in (D_0; D_1) = (30 \pm 0.3; 48 \pm 0.48)$ mm.

From Fig.8, can be derived new approximation of diameter (i.e. D_2) via secant method:

$$D_2 = f(P_{ACCEPT}) = \frac{D_1(P_{ACCEPT} - FS_0) + D_0(FS_1 - P_{ACCEPT})}{FS_1 - FS_0} = 47.9 \pm 0.48\ \text{mm}.$$

From the results of AntHill software follows:

$$D_2 = 47.9 \pm 0.48\ \text{mm}, \quad FS_2 = 0.000035 \leq P_{ACCEPT},$$

$$D \in (D_0; D_2) = (30 \pm 0.3; 47.9 \pm 0.48)\ \text{mm}.$$

Next approximation of D (i.e. D_3) can be also calculated via bisection method, see Fig.9. Hence:

$$D_3 = \frac{D_0 + D_2}{2} = 38.95 \pm 0.39\ \text{mm}, \quad FS_3 = 0.048731 \geq P_{ACCEPT},$$

$$D \in (D_3\,; D_2) = (38.95 \pm 0.39;\, 47.90 \pm 0.48)\ \mathrm{mm}\,.$$

Next applications of bisection method give:

$$D_4 = \frac{D_3 + D_2}{2} = 43.42 \pm 0.43\ \mathrm{mm}\ , \quad FS_4 = 0.002057 \geq P_{ACCEPT},$$

$$D \in (D_4\,; D_2) = (43.42 \pm 0.43;\, 47.90 \pm 0.48)\ \mathrm{mm}\,,$$

$$D_5 = \frac{D_4 + D_2}{2} = 45.66 \pm 0.46\ \mathrm{mm}\ , \quad FS_5 = 0.000292 \leq P_{ACCEPT},$$

$$D \in (D_4\,; D_5) = (43.42 \pm 0.43;\, 45.66 \pm 0.46)\ \mathrm{mm}\,,$$

$$D_6 = \frac{D_4 + D_5}{2} = 44.54 \pm 0.45\ \mathrm{mm}\ , \quad FS_6 = 0.000820 \geq P_{ACCEPT},$$

$$D \in (D_6\,; D_5) = (44.54 \pm 0.45;\, 45.66 \pm 0.46)\ \mathrm{mm}\,.$$

Because the values of FS_5 and FS_6 are very close to given value of P_{ACCEPT}, it is wise to increase the number of Monte Carlo simulations to 3×10^6. Hence, the calculated values of FS will be more accurate.

Next application of bisection method gives:

$$D_7 = \frac{D_6 + D_5}{2} = 45.1 \pm 0.45\ \mathrm{mm}\ , \quad FS_7 = 0.000499 \leq P_{ACCEPT}\ ,$$

$$D \in (D_6\,; D_7) = (44.54 \pm 0.45;\, 45.1 \pm 0.45)\ \mathrm{mm}\,.$$

Next application of secant method gives:

$$D_8 = f(P_{ACCEPT}) = \frac{D_7(P_{ACCEPT} - FS_6) + D_6(FS_7 - P_{ACCEPT})}{FS_1 - FS_6} = 45.098 \pm 0.45\ \mathrm{mm}\,,$$

$$FS_8 = 0.000544 \geq P_{DOV}\ , \quad D_8 = 45 \pm 0.45\ \mathrm{mm}\ ,$$

$$D \in (D_8\,; D_7) = (45.0 \pm 0.45;\, 45.1 \pm 0.45)\ \mathrm{mm}\ .$$

Because $D_8 \cong D_7$ and $FS_7 \cong P_{ACCEPT}$. (with defined accuracy 0.1 mm), the diameter is:

$$D = D_7 = \underline{\underline{45.1 \pm 0.45\ \mathrm{mm}}}\,,$$

see histograms shown in Fig.10 and 11.

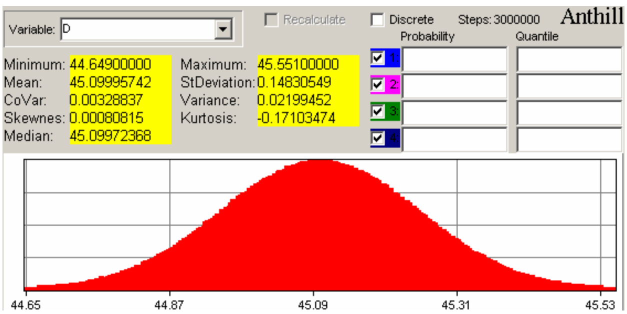

FIGURE 10. FINAL DIAMETER. $D = 45.1 \pm 0.45\ \mathrm{mm}$.

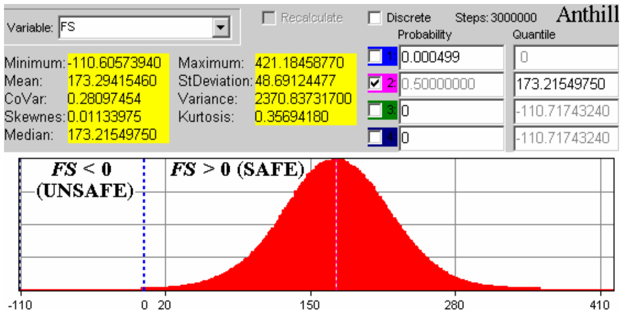

FIGURE 11. FACTOR OF SAFETY FS CALCULATED FOR $D = 45.1 \pm 0.45\ \mathrm{mm}$.

Hence the diameter $D = 45.1 \pm 0.45\ \mathrm{mm}$ is calculated with the acceptable level of damage $P_{ACCEPT} = 0.0005 = 0.05\%$.

The results of the presented iteration loop as a function $D = f(FS)$ is shown in Fig.12.

Histogram of calculated stress $\sigma_{HMH} = 166.89^{+259.90}_{-99.31}$ MPa and 2D histogram of R_e v.s. σ_{HMH} are presented in Fig.13 and 14.

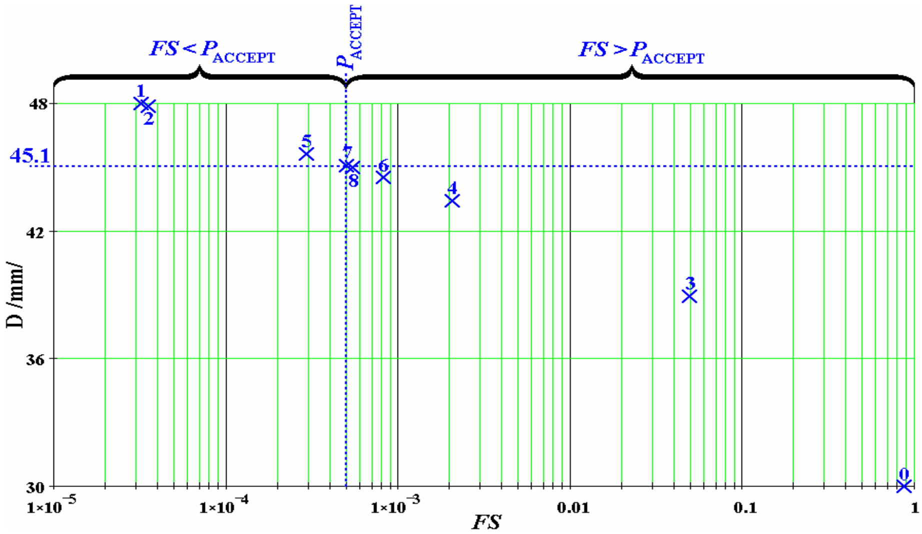

FIGURE 12. DIAGRAM OF CONVERGENCE FOR CALCULATED DIAMETER D (RESULTS OF ITERATIVE PROCEDURES)

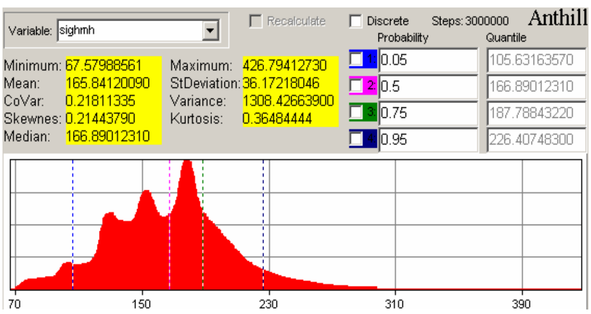

FIGURE 13. MAXIMUM STRESS σ_{HMH} (HISTOGRAM CALCULATED FOR $D = 45.1 \pm 0.45$ mm).

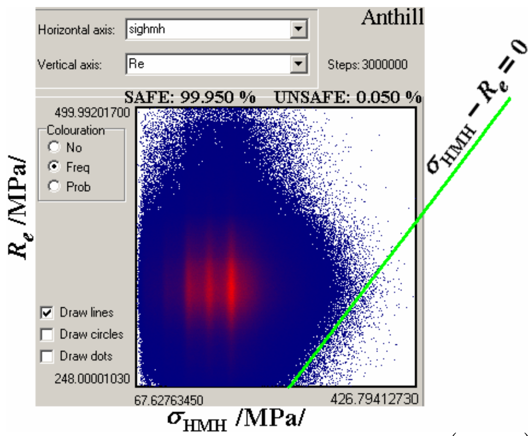

FIGURE 14. 2D HISTOGRAM $R_e = f(\sigma_{HMH})$ CALCULATED FOR $D = 45.1 \pm 0.45$ mm .

4. CONCLUSIONS

Performance-Based Design as a new and modern trend in mechanics is based on theory of probability and stochastic methods.

The calculation of the diameter D, with given acceptable probability of damage level P_{ACCEPT} (i.e. solution of the inverse problem of theory of probability), is solved via iterative approaches (secant method, bisection method). Hence, to get the solution of this type of inverse problem is much difficult than the classical problem of theory of probability (iterative approach with usually more than 10^7 simulations.

Instead of secant method or bisection method can be used also another methods such as Regula-Falsi Method etc.

The whole iterative procedures and Monte Carlo simulations can be speed-up by application of parallel computers. However, on the present days, it is impossible to solve the large problems of mechanics via PBD. The reason of this is the low rate of present-day computers.

Another applications of SBRA method are presented in [2], [3] and [5].

This work has been supported by the Czech project FRVŠ 534/2008 F1b.

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