JOURNAL OF ENGINEERING annals OF FACULTY OF ENGINEERING HUNEDOARA
TOME VI (year 2008), FASCICULE 3, (ISSN 1584 – 2673)

HEAT AND MASS TRANSFER EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH UNIFORM MAGNETIC FIELD

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ABSTRACT
An exact solution of unsteady flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion has been presented in the presence magnetic field. The plate temperature is raised to T_w and species concentration level near the plate is made to rise linearly with time. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration profiles are studied for different physical parameters like magnetic field parameter, thermal Grashof number, mass Grashof number, Schmidt number, time and a. It is observed that the velocity decreases with increasing magnetic field parameter.
Keywords: accelerated, isothermal, vertical plate, exponential, heat transfer, mass diffusion, magnetic field.

1. INTRODUCTION

A few representative fields of interest in which combined heat and mass transfer plays an important role, are filtration processes, the drying of porous materials in textile industries and the saturation of porous materials by chemicals, nuclear reactors, spacecraft design, solar energy collectors, design of chemical processing equipment and pollution of the environment.

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semiconducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals. The effects of transversely applied magnetic field, on the flow of an electrically conduction fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [6].

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al* [7]. The dimensionless governing equations were solved using Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [5]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [2]. Gupta [1] studied flow of an electrically conducting fluid near an uniformly accelerated vertical plate in the presence of uniform magnetic field. Raptis *et al* [3] discussed MHD flow past an accelerated vertical plate with variable suction and heat flux. Singh [4] analyzed MHD flow past an exponentially accelerated vertical plate with uniform temperature.

It is proposed to study the effects of on unsteady MHD flow past an exponentially accelerated infinite isothermal vertical plate in the presence of variable mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. MATHEMATICAL FORMULATION

Here the flow of a viscous incompressible fluid past an infinite isothermal vertical plate with variable mass diffusion in the presence of magnetic field has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis taken normal to the plate. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. At time $t' \le 0$ the plate and fluid are at the same temperature T_∞. At time t' > 0, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and the mass is diffused from the plate to the fluid is raised linearly with time. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho} u \tag{1}$$

$$\rho C_P \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned} &u = 0,\ T = T_\infty,\ C' = C'_\infty \text{ for all } y, t' \le 0 \quad u = u_0 \exp(a't'),\ T = T_w,\ C' = C'_\infty + (C'_w - C'_\infty)At' \\ &\text{at } y = 0 \quad u \to 0\ \ T \to T_\infty,\ C' \to C'_\infty \text{ as } y \to \infty \end{aligned} \tag{4}$$

where $A = \frac{u^2{}_0}{\nu}$. On introducing the following non-dimensional quantities:

$$\begin{gathered} U = \frac{u}{u_0},\quad t = \frac{t' u^2{}_0}{\nu},\quad Y = \frac{y u_0}{\nu},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0{}^3},\quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\quad Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u^3{}_0}, \\ \Pr = \frac{\mu C_p}{k},\quad a = \frac{a'\nu}{u_0{}^2},\quad Sc = \frac{\nu}{D},\quad M = \frac{\sigma B_0{}^2 \nu}{\rho u_0{}^2} \end{gathered} \tag{5}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0 \\ t > 0:\ &U = \exp(at), \quad \theta = 1, \quad C = t \quad \text{at } Y = 0 \\ &U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } Y \to \infty \end{aligned} \tag{9}$$

3. SOLUTION PROCEDURE

The dimensionless governing equations (6) and (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc(\eta\sqrt{\Pr}) \tag{10}$$

$$C = t\left[(1+2\,\eta^2\,Sc)\;erfc(\eta\sqrt{Sc}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Sc}\exp(-\eta^2 Sc)\right] \tag{11}$$

$$\begin{aligned}
U = {} & \frac{\exp(at)}{2}\Big[\exp(2\eta\sqrt{(M+a)t}\,)erfc(\eta+\sqrt{(M+a)t}\,) \\
& + \exp(-2\eta\sqrt{(M+a)t}\,)erfc(\eta-\sqrt{(M+a)t}\,)\ \Big] \\
& + ((d+e(1+ct))\ \Big[\exp(2\eta\sqrt{Mt}\,)erfc(\eta+\sqrt{Mt}\,)+\exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt})\ \Big] \\
& - \frac{ec\eta\sqrt{t}}{\sqrt{M}}\Big[\exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt}) - \exp(2\eta\sqrt{Mt})erfc(\eta+\sqrt{Mt})\Big] \\
& - d\exp(bt)\ \Big[\exp(2\eta\sqrt{(M+b)t})erfc(\eta+\sqrt{(M+b)t}) \\
& + \exp(-2\eta\sqrt{(M+b)t})erfc(\eta-\sqrt{(M+b)t})\ \Big] - 2d\,erfc(\eta\sqrt{\Pr}) - 2e\,erfc(\eta\sqrt{Sc}) \\
& - 2ec\,t\left[(1+2\,\eta^2\,Sc)\;erfc(\eta\sqrt{Sc}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Sc}\exp(-\eta^2 Sc)\right] \\
& - e\exp(ct)\ \Big[\exp(2\eta\sqrt{(M+c)t})erfc(\eta+\sqrt{(M+c)t})\ + \exp(-2\eta\sqrt{(M+c)t})erfc(\eta-\sqrt{(M+c)t})\ \Big] \\
& + d\exp(bt)\ \Big[\exp(2\eta\sqrt{\Pr bt}\,)\ erfc(\eta\sqrt{\Pr}+\sqrt{bt})\ + \exp(-2\eta\sqrt{\Pr bt})erfc(\eta\sqrt{\Pr}-\sqrt{bt})\ \Big] \\
& + e\exp(ct)\ \Big[\exp(2\eta\sqrt{Sc\,ct}\,)\ erfc(\eta\sqrt{Sc}+\sqrt{ct}) + \exp(-2\eta\sqrt{Sc\,ct})erfc(\eta\sqrt{Sc}-\sqrt{ct})\ \Big]
\end{aligned} \quad [12]$$

where,

$$b = \frac{M}{\Pr - 1},\quad c = \frac{M}{Sc-1},\quad d = \frac{Gr}{2b(1-\Pr)},\quad e = \frac{Gc}{2c^2(1-Sc)}\ \text{ and }\ \eta = \frac{Y}{2\sqrt{t}}.$$

4. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters M, a, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of Prandtl number *Pr* is chosen such that they represent water (*Pr* = 7.0). The numerical values of the velocity are computed for different physical parameters like magnetic filed parameter, a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different (*a* = 0.2 ,0.5 ,0.9), *Gr* = *G c* = 5, *M* = 0.2, *Sc* = 2.01 at *t* = 0.2 are studied and presented in figure 1. It is observed that the velocity increases with increasing values of Figure 2. illustrates the effects of the Magnetic field parameter on the velocity when (*M* = 2, 5, 10), *a* = 0.5, *Gr* = *G c* = 5, *S c* = 2.01 and *t* = 0.2. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective low.

The velocity profiles for different values of time (*t* = 0.2, 0.4, 0.6), *M* = 0.2, *a* = 0.5, *Gr* = *Gc* = 5 and *Sc* = 2.01 are shown in figure 3. It is clear that the velocity increases with decreasing values of the time. Figure 4 demonstrates the effects of different thermal Grashof number (*Gr* = 2, 5, 10) and mass Grashof number (*Gc* = 5, 10) on the velocity when *M* = 0.2, *a* = 0.5, *Sc* = 2.01 and *t* = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

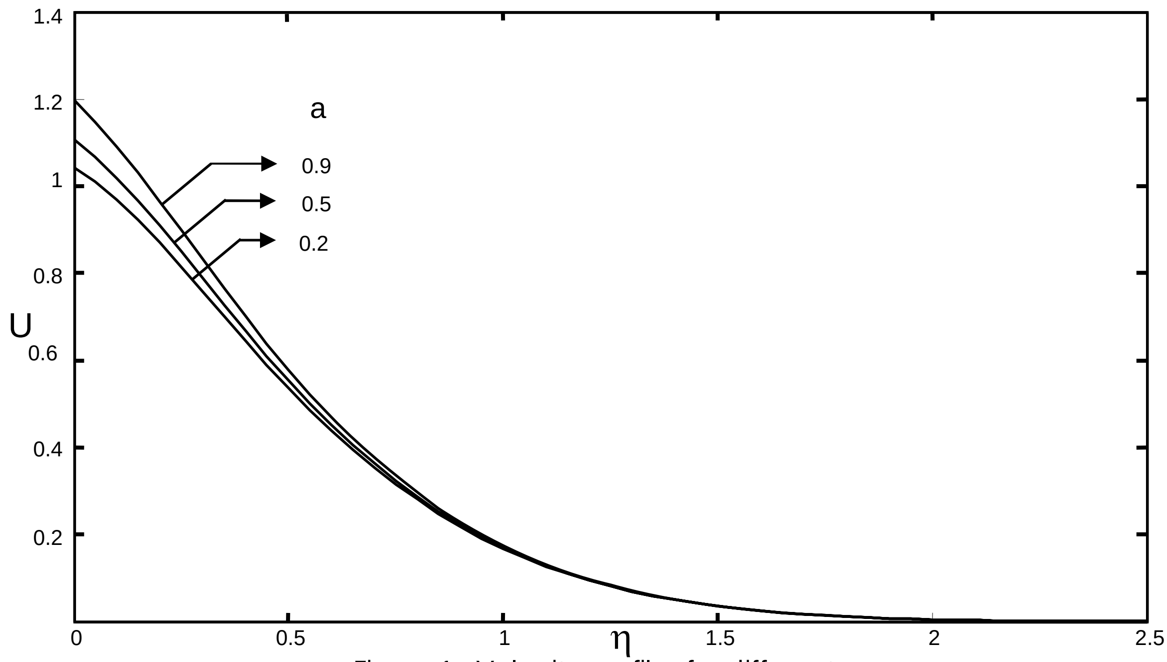

Figure 1. Velocity profiles for different a

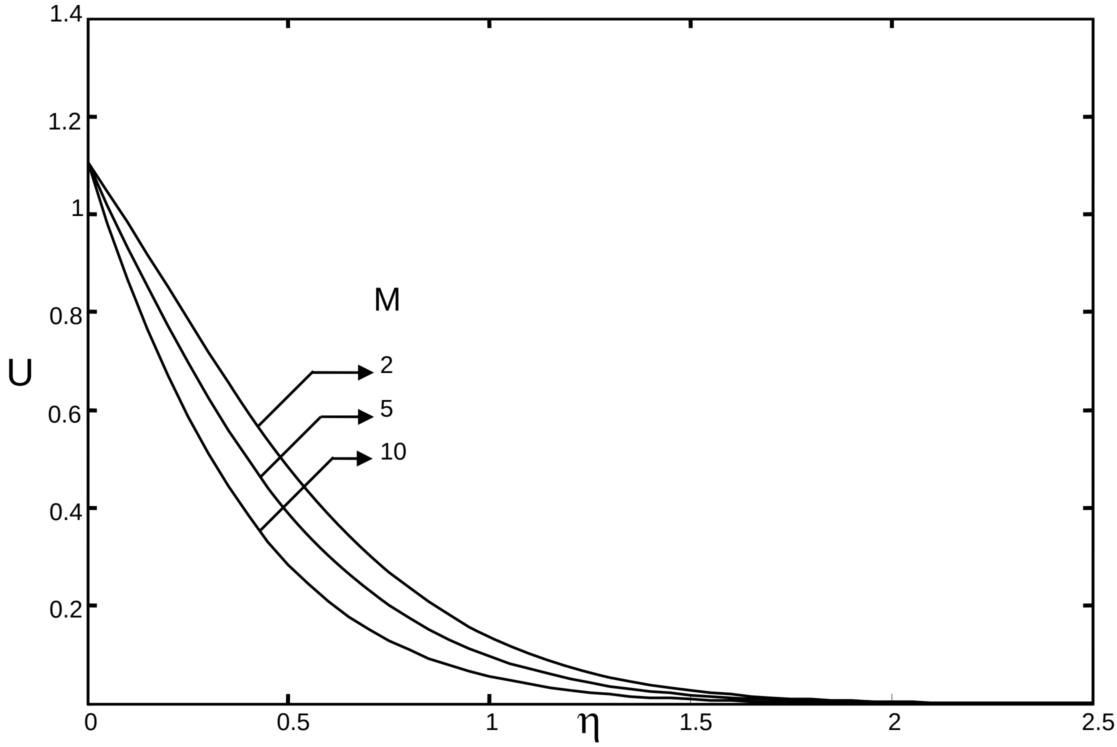

Figure 2. Velocity profiles for different M

The numerical values of the concentration profiles are computed from equation (11) and the numerical values are plotted in figure 5 for different values of Schmidt number *Sc* = (0.3, 0.6, 0.78, 2.01) and time *t* = 0.2. The effect of Schmidt number is very important in concentration field. It is observed that the concentration increases with time and decreases with increasing Schmidt number. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream.

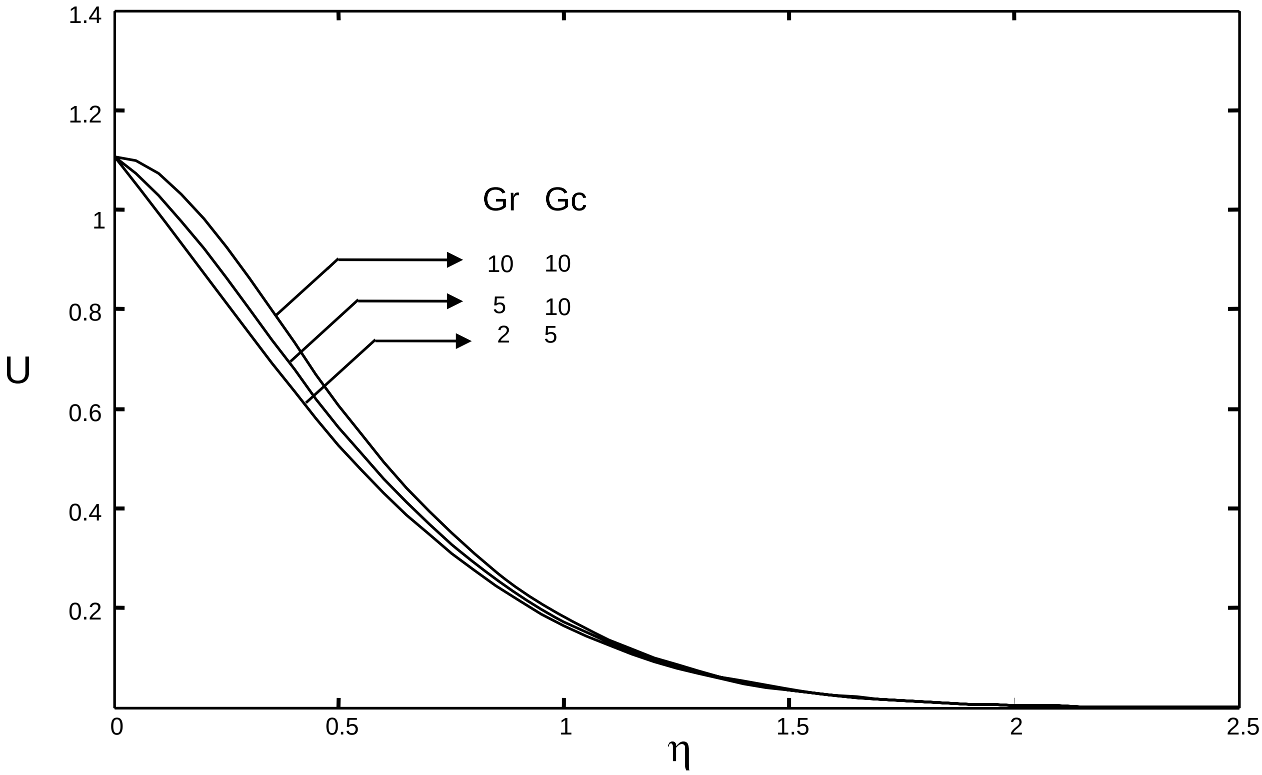

Figure 4. Velocity profiles for different Gr and Gc

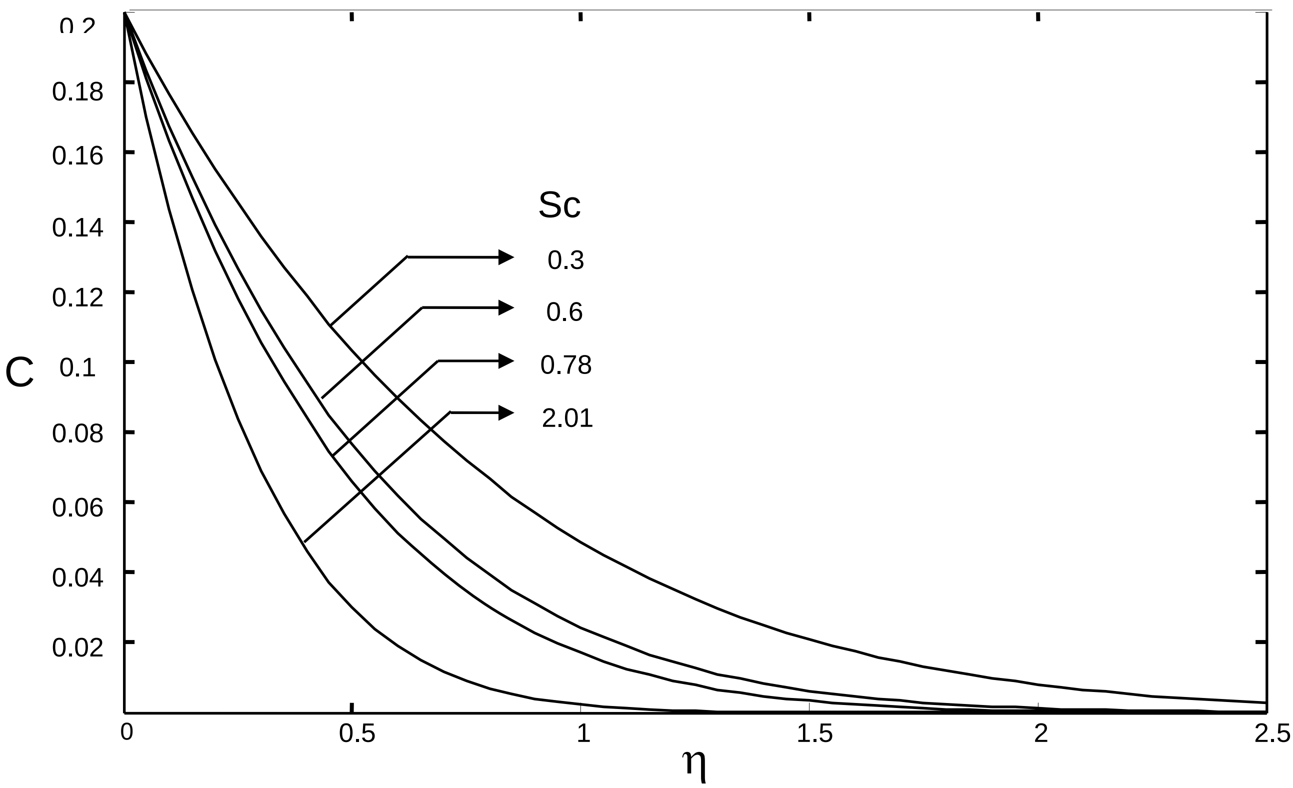

Figure 5. Concentration profiles for different Sc

5. CONCLUSIONS

An exact analysis of hydromagnetic flow past an exponentially accelerated infinite isothermal vertical plate in the presence of variable mass diffusion have been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, *a* and *t* are studied graphically. The conclusions of the study are as follows:

- The velocity decreases with increasing magnetic field parameter *M.*
- The velocity increases with increasing *a*, *Gr*, *Gc* and *t.*
- The concentration level near the plate decreases with increasing values of the Schmidt number *Sc.*

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