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MHD EFFECTS ON FLOW PAST AN INFINITE OSCILLATING VERTICAL PLATE IN THE PRESENCE OF AN OPTICALLY THIN GRAY GAS

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ABSTRACT:
Theoretical solution of unsteady MHD flow past an infinite vertical oscillating plate with variable temperature and uniform mass diffusion in the presence of thermal radiation has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised linearly with respect to time and the concentration level near the plate is also raised to C'_w. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like phase angle, radiation parameter, magnetic field parameter, Schmidt number, thermal Grashof number, mass Grashof number and time are studied. It is observed that the velocity increases with decreasing phase angle ωt. It is also observed that the velocity decreases with increasing radiation parameter or magnetic field parameter.
Key words:
radiation, MHD, gray, oscillating, vertical plate, heat and mass transfer

1. INTRODUCTION

Magnetoconvection plays an important role in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry are examples of such engineering applications.

Das *et al*[1] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. England and Emery [2] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[3]. Raptis and Perdikis[4] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al* [7]. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate with uniform mass diffusion were studied by Soundalgekar *et al* [8]. The dimensionless governing equations were solved using Laplace transform technique.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [5]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [6]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. [9].

However the thermal radiation and MHD effects on infinite oscillating vertical plate with variable temperature and uniform mass diffusion, in the presence of thermal radiation and magnetic field is not studied in the literature. It is proposed to study MHD and thermal radiation effects on unsteady flow past an infinite oscillating vertical plate with variable temperature and uniform mass diffusion. The governing equations are solved using the Laplace transform technique.

2. MATHEMATICAL FORMULATION

Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised to linearly with respect to time and the concentration level near the plate is raised to C'_w. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{array}{llll} t' \le 0: & u = 0, & T = T_\infty, & C' = C'_\infty \quad \textit{for all} \quad y \\ t' > 0: & u = u_0 cos\omega' t', & T = T_\infty + (T_w - T_\infty)\, A\, t', & C' = C'_w \quad \textit{at} \quad y = 0 \\ & u = 0, & T \to T_\infty, & C' \to C'_\infty \quad \textit{as} \quad y \to \infty \end{array} \tag{4}$$

where $A = \frac{u_0^2}{\nu}$. The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3 (T_\infty - T) \tag{7}$$

On introducing the following dimensionless quantities:

$$U=\frac{u}{u_0},\ t=\frac{t'u_0^2}{\nu},\ Y=\frac{yu_0}{\nu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\quad Gr=\frac{g\beta\nu(T_w-T_\infty)}{u_0^3},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3},$$

$$Pr=\frac{\mu C_p}{k},\ Sc=\frac{\nu}{D},\ M=\frac{\sigma B_0^2\nu}{\rho u_0^2},\ R=\frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2},\ \omega=\frac{\omega'\nu}{u_0^2} \tag{8}$$

in equations (1) to (4), leads to $\dfrac{\partial U}{\partial t}=Gr\,\theta+Gc\,C+\dfrac{\partial^2 U}{\partial y^2}-M\,U$ (9)

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2}-\frac{R}{Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U=0, & \theta=0, & C=0, & for\ all & Y,t\le 0\\ t>0: & U=\cos\omega t, & \theta=t, & C=1, & at & Y=0\\ & U=0, & \theta\to 0, & C\to 0 & as & Y\to\infty\end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned}\theta=&\ \frac{t}{2}\left[\exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right)\right]\\ &-\frac{\eta Pr\sqrt{t}}{2\sqrt{R}}\left[\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right)-\exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right)\right]\end{aligned} \tag{13}$$

$$C=erfc(\eta\sqrt{Sc}) \tag{14}$$

$$\begin{aligned}U=&\ \frac{\exp(i\omega t)}{4}\left[\exp(2\eta\sqrt{i\omega t})erfc(\eta+\sqrt{i\omega t})+\exp(-2\eta\sqrt{i\omega t})erfc(\eta-\sqrt{i\omega t})\right]\\ &+\frac{\exp(-i\omega t)}{4}\left[\exp(2\eta\sqrt{-i\omega t})\,erfc(\eta+\sqrt{-i\omega t})+\exp(-2\eta\sqrt{-i\omega t})erfc(\eta-\sqrt{-i\omega t})\right]\\ &+\left(d(1+bt)+e\right)\left[\exp(2\eta\sqrt{Mt})\,erfc(\eta+\sqrt{Mt})+\exp(-2\eta\sqrt{Mt})\,erfc(\eta-\sqrt{Mt})\right]\\ &-\frac{db\,\eta\sqrt{t}}{\sqrt{M}}\left[\exp(-2\,\eta\sqrt{Mt})\,erfc(\eta-\sqrt{Mt})-\exp(2\,\eta\sqrt{Mt})\,erfc(\eta+\sqrt{Mt})\right]\\ &-d\,\exp(bt)\left[\begin{array}{l}\exp(2\,\eta\sqrt{(M+b)t})\,erfc(\eta+\sqrt{(M+b)t})\\ +\exp(-2\eta\sqrt{(M+b)t})\,erfc(\eta-\sqrt{(M+b)t})\end{array}\right]\\ &-e\,\exp(ct)\left[\exp(2\,\eta\sqrt{(M+c)t})\,erfc(\eta+\sqrt{(M+c)t})+\exp(-2\eta\sqrt{(M+c)t})\,erfc(\eta-\sqrt{(M+c)t})\right]\\ &-d(1+bt)\left[\exp(2\eta\sqrt{Rt})erfc(\eta\sqrt{Pr}+\sqrt{at})+\exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{Pr}-\sqrt{at})\right]\\ &+d\,\exp(bt)\left[\begin{array}{l}\exp(2\,\eta\sqrt{Pr(a+b)t})\,erfc(\eta\sqrt{Pr}+\sqrt{(a+b)t})\\ +\exp(-2\eta\sqrt{Pr(a+b)t})erfc(\eta\sqrt{Pr}-\sqrt{(a+b)t})\end{array}\right]\\ &-2e\,erfc(\eta\sqrt{Sc})\\ &-\frac{db\,\eta Pr\sqrt{t}}{\sqrt{R}}\left[\exp(-2\,\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr}-\sqrt{at})-\exp(2\,\eta\sqrt{Rt})\,erfc(\eta\sqrt{Pr}+\sqrt{at})\right]\\ &+e\,\exp(ct)\left[\exp(2\eta\sqrt{ct\,Sc})erfc(\eta\sqrt{Sc}+\sqrt{ct})+\exp(-2\eta\sqrt{ct\,Sc})\,erfc(\eta\sqrt{Sc}-\sqrt{ct})\right]\end{aligned} \tag{15}$$

where, $a = \dfrac{R}{Pr}$, $b = \dfrac{R}{1-Pr}$, $d = \dfrac{Gr}{2b^2(1-Pr)}$, $e = \dfrac{Gc}{2c(1-Sc)}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

In order to get the physical insight into the problem, the numerical values of U have been computed from equation (15). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf(a+ib) = erf(a) + \frac{\exp(-a^2)}{2a\pi}[1 - cos(2ab) + isin(2ab)]$$
$$+ \frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}[f_n(a,b) + ig_n a,b)] + \varepsilon(a,b)$$

where, $$f_n = 2a - 2a\,cosh(nb)\,cos(2ab) + n\,sinh(nb)sin(2ab)$$
$$g_n = 2a\,cosh(nb)\,sin(2ab) + n\,sinh(nb)\,cos(2ab)\,;\ |\varepsilon(a,b)| \approx \approx 10^{-16}\,|erf(a+ib)|$$

3. RESULTS AND DISCUSSION

In order to get a physical view of the problem the numerical values of the velocity, temperature and concentration for different values of the phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. The purpose of the calculations given here is to asses the effect of different $\omega t, R, M, Gr, Gc, Sc$ and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of exponential and complementary error function.

The velocity profiles for $t = 0.2$ and different phase angles ($\omega t = 0, \pi/6, \pi/3, \pi/2$), $M = 2, R = 3, Gr = 2, Gc = 2, Sc = 2.01, Pr = 0.71$ are shown in figure 1. It is clear that the velocity increases with decreasing phase angle ωt. The velocity profiles for different magnetic field parameter ($M = 2,5,10$), $\omega t = \pi/6, Gr = Gc = 2, R = 12, Sc = 2.01, Pr = 0.71$ and $t = 0.2$ are presented in Figure 2. It is clear that the velocity increases with decreasing magnetic field parameter. The effect of velocity profiles for different time ($t = 0.2, 0.3, 0.4$), $R = 3, \omega t = \pi/6, M = 2, Gr = 5, Gc = 5, Pr = 0.71$ and $Sc = 2.01$ are shown in figure 3. In this case, the velocity increases gradually with respect to time t. The velocity profiles for different thermal Grashof number ($Gr = 2,10$), mass Grashof number ($Gc = 2,5$), $\omega t = \pi/6, M = 2, R = 3, Sc = 2.01, Pr = 0.71$ and $t = 0.3$ are shown in Figure 4. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number. The effect of velocity for different values of the radiation parameter ($R = 5,8,15$), $\omega t = \pi/6, M = 4, Gr = 10, Gc = 5, Pr = 0.71, Sc = 2.01$ and $t = 0.6$ are shown in figure 5. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

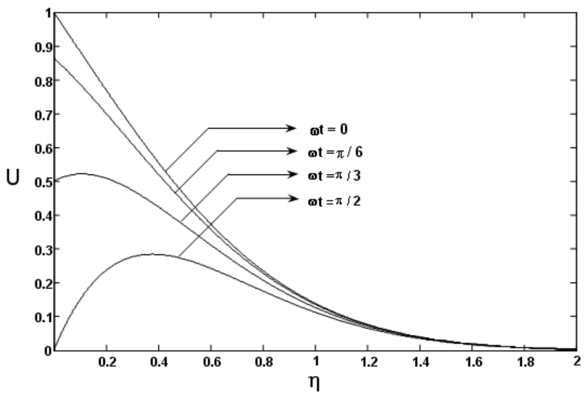

Figure 1.Velocity profiles for different values of ωt

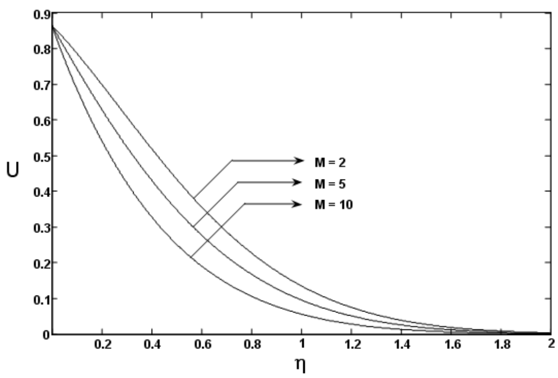

Figure 2. Velocity profiles for different values of M

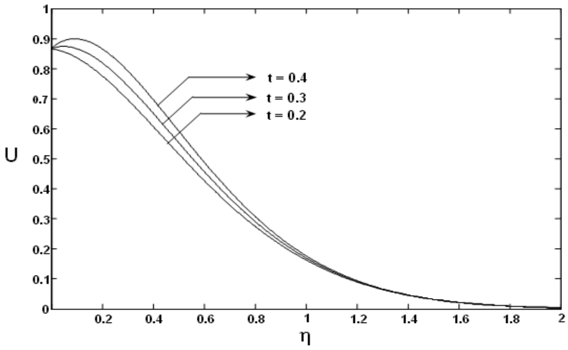

Figure 3. Velocity profiles for different values of t

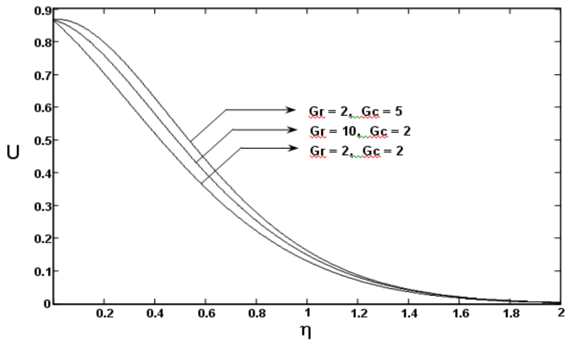

Figure 4. Velocity profiles for different values of Gr, Gc

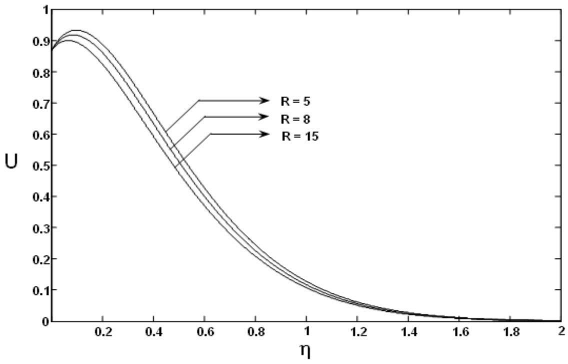

Figure 5. Velocity profiles for different values of R

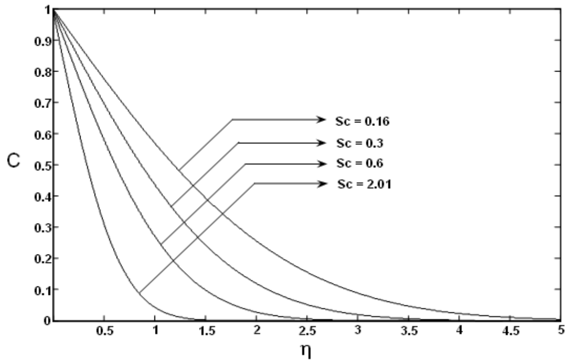

Figure 6. Concentration profiles for different values of Sc

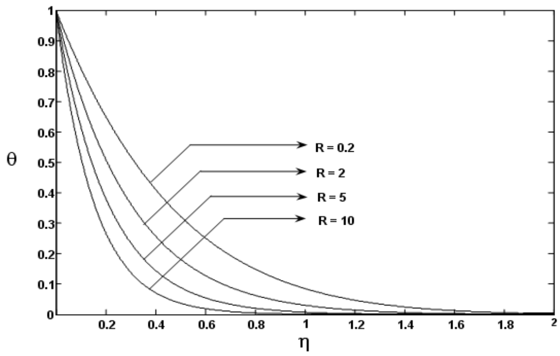

Figure 7. Temperature Profiles for different values of R

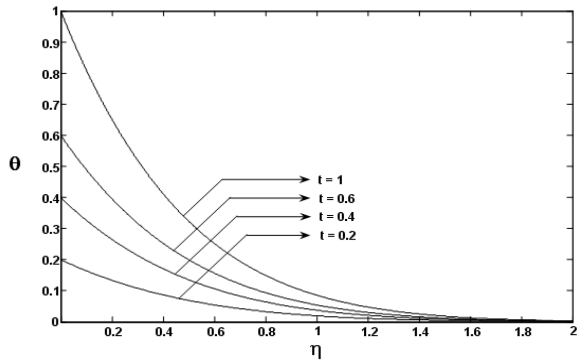

Figure 8. Temperature Profiles for different values of t

Figure 6 represents the effect of concentration profiles at time $t = 1$ for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of thermal radiation parameter ($R = 0.2, 2, 5, 10$) from Equation (13) and these are shown in figure 7. for air (Pr =0.71) at time $t = 1$. The effect of thermal radiation parameter is important in temperature profiles. The trend shows that the temperature increases with decreasing radiation parameter. Figure 8 is a graphical representation which depicts the temperature profiles for different values of the time ($t = 0.2, 0.4, 0.6, 1$) and $Pr = 0.71$ in the presence of thermal radiation $R = 0.2$. It is clear that the temperature increases with increasing values of the time t.

4. CONCLUSIONS

Radiation effects on unsteady flow past an infinite vertical oscillating plate with variable temperature and uniform wall concentration in the presence of transverse magnetic field is studied. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different physical parameters like $\omega t, M, R, Gr, Gc, Sc$ and t are studied graphically. The conclusions of the study are as follows:

(i) The velocity decreases with increasing Radiation parameter or magnetic field parameter.
(ii) The velocity increases with decreasing phase angle ωt.
(iii) The temperature decreases due to high thermal radiation.
(iv) As time increases, it is found that there is a rise in wall temperature.

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Nomenclature

a^+ absorption coefficient
B_o external magnetic field
C' concentration
C dimensionless concentration
D mass diffusion coefficient
erfc complementary error function
g acceleration due to gravity
Gr thermal Grashof number
Gc mass Grashof number
k thermal conductivity of the fluid
M magnetic field parameter
Pr Prandtl number
q_r radiative heat flux in the y-direction
R radiation parameter
Sc Schmidt number
T temperature
t_o time
t dimensionless time
u_o amplitude of the oscillation
u velocity component in x-direction
U dimensionless velocity component in x-direction
x spatial coordinate along the plate
y spatial coordinate normal to the plate
Y dimensionless spatial coordinate normal to the plate

Greek Symbol

α thermal diffusivity
β coefficient of volume expansion
β^* volumetric coefficient of expansion with concentration
η similarity parameter
μ coefficient of viscosity
ν kinematic viscosity
ωt phase angle
σ electric conductivity
θ dimensionless temperature

Subscripts

ω conditions on the wall
∞ free stream conditions

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