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DISCUSSION ON THE VALUE OF EXPONENTIAL WEIGHTS

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ABSTRACT:
In this paper we discuss the possible methodological issues when choosing exponentially derived weights in surveys or when dealing with aggregate indices. Our intention was to make a technical discussion the possibilities when using an appropriate quotient. We have pointed out, that there are at least two opposing conditions.
First, the use of the quotient should minimize its effect on the variability of the underlying data. This was shown on a case study, when comparing results from the statistics needed to perform a paired, two sample test for equal means. Second, the use of the quotient should be helpful from the researcher's perspective.
KEY WORDS:
exponential weights, quotient

1. INTRODUCTION

Aggregation of data using various types of averages is a common and often used technique in research and practice. From the researcher's perspective a representative example may be the calculation of the Human Development Index (HDI), Commitment to Development index (CDI) [1] or ranking of doing business [2]. Another perspective, both attractive from the researcher's and practitioner's view is the use of weighted averages for aggregating responses from surveys. A numerous examples may be found in many papers. As an example, suppose that a questionnaire contains a question, with many options for the respondent, like when asking, which colours you consider to be attractive. The answers are from a nominal scale in form of words: blue, red, yellow, black... etc. The respondent chooses the appropriate colours, and rates them according to his perceived importance. A similar approach may be used when evaluating existing or potential suppliers, where different categories are represented by the desired capabilities required from the supplier. After rating these categories, some form of an aggregation is needed. Probably the simplest way is to use a weighted average (1).

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i w_i \qquad (1)$$

Where X_i represents the value of the observed characteristics, w_i are weights of characteristics, where $i = 1, 2, \dots, n$. When constructing indices like HDI, CDI or rankings like doing business, the X_i often represents a specific values calculated from an independent evaluation. However, in surveys X_i may be a constant, where the formula reduces to a simple average. This is a special case, but even in this case, w_i should be perceived as weights and the importance of assigning weights to various i observations should not be underestimated.

In this paper, we will provide a simple example of how subjective selection of weights based on a geometrical series, can significantly distort even the simplest statistical procedures.

2. SETTING WEIGHTS

The motivation for this paper was a methodological issue from a research study, where in a questionnaire, respondents had to designate preferences between various options, by ranking those options from 12 as the "most important" to 1 as the "least important". Further on, the goal was to verify differences between average weights of selected possibilities. In this particular case, it was clear that using ranking was not an appropriate option, because from the empirical evidence it was deducted, that every more important possibility should have more than linearly increasing weight for the respondent. Thus, the issue was rather methodological regarding the specification of weights (in this case, ex post).

Let's define a set S of respondents preferences, $S := \{1, 2, \ldots, n\}$, where $n \in N$ is the set of natural numbers. However, in reality $S \subset N$ and n rarely reaches "high numbers". Further on $S = C$, where C is the set of respondents choices - options. This condition is not necessary and we will return to this later in this paper. Than the injective function $f(x): S \to R$ maps chosen preferences onto weights. In the case of exponential weights:

$$f(x) := \left(\frac{n(1 \quad q)}{1 - q^n}\right) q^{x-1} \tag{2}$$

The formula (2) has one subjectively drawn parameter q which is actually the quotient, $q > 1$ which sets the pace of increasing weights with regard to the increased level of preferences. How to choose the value of q in the case of an ex post evaluation of the survey? From the statistical point of view, this question reduces to the robustness of statistical methods with regard to the value of q. More specifically, it reduces to some rational trade-off between the:

- robustness of the statistics against the value of q,
- and the need for ensuring some comprehensible values of the exponential weights.

3. METHODOLOGY

Our goal was to observe the trade-off between those two requirements. First, let's observe the behaviour of the mean and standard deviation with regard to the changing q. Because the chosen preferences of the respondents are unknown, they are stochastic in nature; we have simulated them using a discrete uniform probability distribution. The correlations between simulated data were not significant. For this purpose we have simulated a question in the questionnaire with 10 possibilities for which the respondents assigned their preferences from 1 to 10, where 1 was the least preferable option and 10 the most preferable option. The differences between weight were assumed to be exponential as in the (2). The number of "respondents" was set to 30. Next we have simulated their answers using the before mentioned discrete uniform probability distribution.

Further on, we have chosen two options for which the respondents assigned their preferences and calculated the statistics needed for a paired two sample test for the equality of means (only as an example). Thus calculating the mean of the differences $\bar{d}$ and standard deviation of differences s_d. On the next figure 1, we have plotted the corresponding confidence intervals (95% confidence) and the mean with regard to the increasing value of the quotient q.

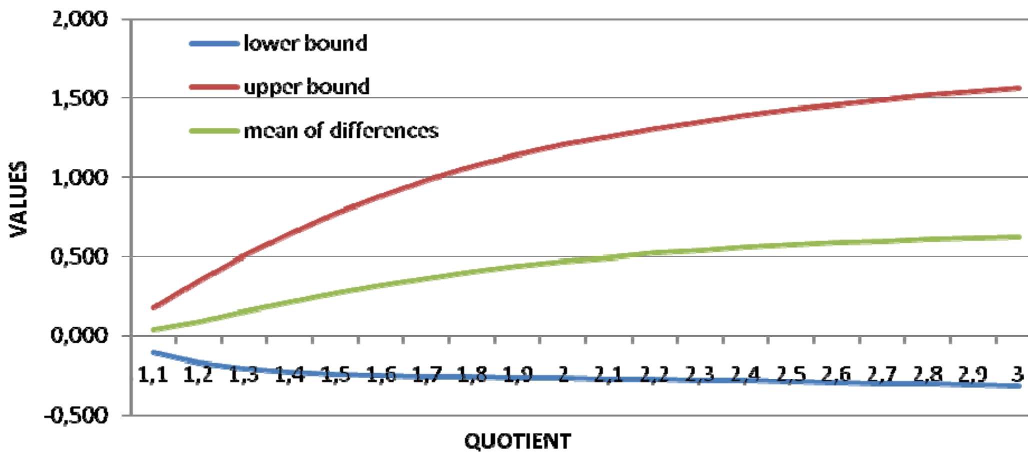

Figure 1. Confidence intervals in the simulation study

Without a more formal approach it can easily be seen that confidence intervals are increasing in range. This effect can be tracked down into the increasing standard deviation of the differences s_d. This is the consequence of increasing value of the quotient q. On the next figure we can observe the effect of increasing quotient on the values of the function (2) i.e. weights.

Intuitively it seems clear that the higher quotient increases the differences between weights of the respondents (More formally, the statement is true, if $Q := \{q : q > 1\}$, than $f(q): Q \to R$). Thus, increasing the s_d. Therefore any statistics related to the variation of the underlying data can be significantly biased by the use of various quotients. However, this level of biasness is not constant and may be the subject of optimization. This can be seen in the first figure, where we have plotted the confidence intervals. For higher quotients the difference between two successive confidence intervals decreases.

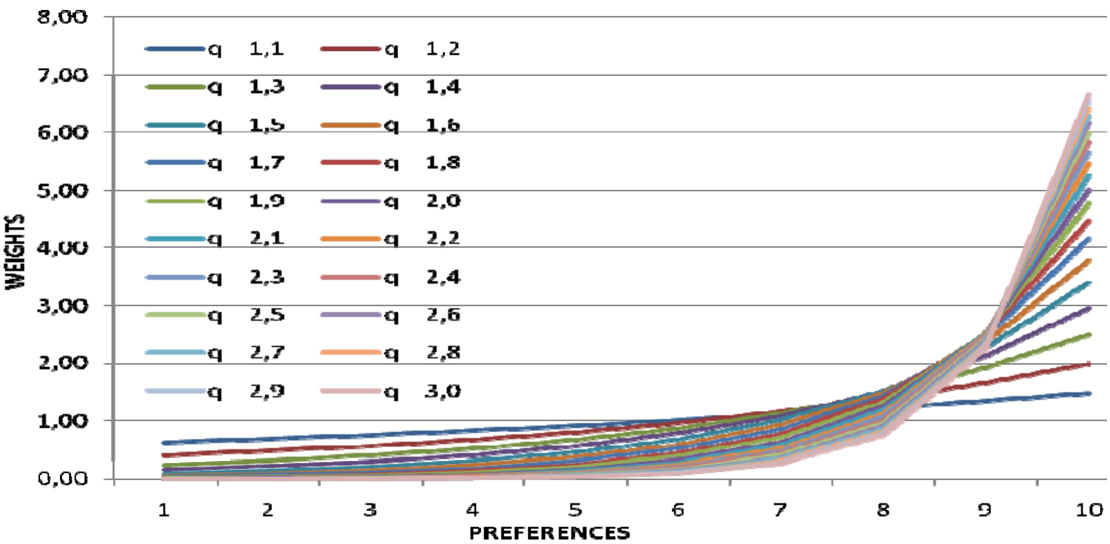

Figure 2. Weights with regard to the changing quotient for N=10

Next it should be defined what is meant by the comprehensible values of the exponential weights. Note that for higher quotient the weight assigned to lower values of the preferences could be viewed as negligible, see table 1. In some way, this may seem like a contradiction to the principle of setting values of preferences (weights) to all the options in the question. The opposite is true for lower quotients. Thus if the previous discussion about the selecting the quotient steered towards choosing higher values, on the other hand it appears as a less convenient approach when regarding the purpose of choosing exponential weights.

Table 1. Values of the weights for choosen quotients for N=10

	PREFERENCES									
QUOTIENTS	1	2	3	4	5	6	7	8	9	10
1,1	0,63	0,69	0,76	0,84	0,92	1,01	1,11	1,22	1,35	1,48
1,2	0,39	0,46	0,55	0,67	0,80	0,96	1,15	1,38	1,66	1,99
1,3	0,23	0,31	0,40	0,52	0,67	0,87	1,13	1,47	1,91	2,49
1,4	0,14	0,20	0,28	0,39	0,55	0,77	1,08	1,51	2,11	2,96
1,5	0,09	0,13	0,20	0,30	0,45	0,67	1,01	1,51	2,26	3,39
2,6	0,00	0,00	0,01	0,02	0,05	0,13	0,35	0,91	2,37	6,16
2,7	0,00	0,00	0,01	0,02	0,04	0,12	0,32	0,86	2,33	6,30
2,8	0,00	0,00	0,00	0,01	0,04	0,10	0,29	0,82	2,30	6,43
2,9	0,00	0,00	0,00	0,01	0,03	0,09	0,27	0,78	2,26	6,55
3	0,00	0,00	0,00	0,01	0,03	0,08	0,25	0,74	2,22	6,67

4. CONCLUSION

The clarification of the “comprehensible values of the exponential weights” is therefore a matter of arbitrary point of view. It seems clear that there is a need of some form of an optimization. The problem could be restated by choosing such a lowest possible weight $min\,[f(x)] := \left(\frac{n(1-q)}{1-q^n}\right)$ for which it is:

1) not negligible by the researcher.
2) robust against the changes of the variability.

Qualitatively, if we consider a respondent which has 10 points and he is allowed to designate these points between 10 various options, than in the case of a constant weight he will designate every option 1 point. When considering exponential weights, the lowest possible weight is less than 1. It is our opinion that if we consider 1 as a benchmark for the undecided respondent, than the lowest possible weight should be at least $1/10$, more generally $min[f(x)] = \frac{1}{n}$, from which $\left(\frac{n(1-q)}{1-q^n}\right) = \frac{1}{n}$ the quotient q can be calculated. In our simulation case, where $n = 10$ this corresponds to the quotient value $q = 1{,}47394$. However, the possible consequences from the statistical point of view should be considered. A practical approach would be to make simulations and see whether the chosen value of the quotient can significantly affect the s_d. Perhaps a more plausible way would be to designate fewer preferences than there are options. In this case $S \subset C$. This would increase the statistical significance, because this allows choosing higher quotients, where those preferences with very low weight are neglected. But also, this alters the possibilities for statistical evaluations. Instead of comparing the importance of various options, it would be more plausible to evaluate the structure of the importance across various options, with regard to different groups of respondents.

This discussion would not be complete if we wouldn’t consider an alternative approach toward selecting weights. Due to these methodological complications, we prefer using analytical hierarchy method [3] for obtaining weights from respondents instead of an arbitrary system of weights. This has one huge drawback, due to the complexity the questioning needs to be performed via face-to-face approach.

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