JOURNAL OF ENGINEERING
annals OF
FACULTY OF ENGINEERING HUNEDOARA

ANALYSIS AND CALCULATION OF RESONANT RESPONSE OF CAVITIES TO USE IN PHOTO ACOUSTIC PHENOMENA

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ABSTRACT
The behaviour of certain resonant acoustic cavities is analyzed. The relative amplitude of the pressure wave on the lateral surface is calculated for a cylindrical cavity of elliptical cross-section. The amplitude of the coherent noise signal resulting from the absorption of the windows that close the cavity is also considered. The amplitude of pressure where the microphone is placed, for spherical and semi-spherical cavities without windows, excited by a led located in the centre, is also calculated. The results are compared with those obtained for a typical cylindrical cavity of circular section with the laser beam centred in its axis.
Keywords:
photo acoustics, coherent noise, quality factor.

1. INTRODUCTION

The cylindrical cavities of circular section have been and are broadly used in photo acoustics investigation [1]. To increase the response of the cell, exciting diverse acoustics vibration mode, some investigators have displaced in parallel form the laser beam from the cylinder axis toward the lateral wall [2], others have proposed a new internal designs for the cylindrical cavity, with the same objective [3]. In all the cases it breaks the symmetry of the cylindrical cavity to excite other resonant modes, instead of the fundamental one, in order to increase the acoustic signal at microphone. The idea of using cavities with other symmetries also pursues the objective of increasing the resonant response at microphone. Therefore, the cylindrical cavity of elliptic cross section, the spherical cavity and the hemispherical cavities resonant responses are analyzed.

The eigenfunctions of Helmholtz equation in coordinate cylindrical elliptic, the Mathieu's functions, especially those of even parity and period π have got a maximum in the centre, at the focuses of the ellipse and in the ends of the greater and minor axes. The Mathieu's functions of even parity and period 2π present maxima at the focuses of the ellipse and in the ends of the greater axis. Therefore, illuminating with a laser the gas inside the cavity in those regions where the Mathieu's functions (that represent to the pressure) have a maximum, the maximum efficiency is achieved in the generation of the photo acoustic phenomenon. This is the foundation to propose the use of a resonant cylindrical cavity with elliptic cross-section.

The semi-spherical cavity is obtained by cutting the spherical cavity with an equatorial plane. By supposing the plane is a perfect acoustic reflector, the self frequencies and self functions of the semi-spherical cavity are equal to those of the spherical cavity.

The spherical and semi-spherical cavities, excited by a led in their centre, do not require windows for their operation: it implies to eliminate the coherent noise. In this cavity, the source is placed at the vertex of the conical excitation volume. Modulating the power source to a self-frequency of the cavity, is achieved the cavities get excited in a self-function.

2. GENERAL EXPRESSION OF THE PRESSURE IN A RESONANT CAVITY

The generation of acoustic and thermal waves in gases is governed [5] by the linear equation wave for the pressure p:

$$\partial_t^2 p(\vec{r},t) - c^2\nabla^2 p(\vec{r},t) = (\sigma - 1)\partial_t H(\vec{r},t) \quad (1)$$

where c, σ and H are, respectively, the speed of the sound, the adiabatic coefficient of the gas and the heat density deposited in the gas by absorption of the light.

Equation (1) has two independent solutions: a weakly acoustic wave dumped with wavelengths in the centimetres range, and a heavily thermal dumped wave, with wavelengths in the submillimeter region [4]. This thermal wave does not propagate beyond the distance of a few wavelengths; then it can be observed only in the vicinity of the exciting light beam.

All the closed cavities used as optoacoustic cells have acoustic resonances and their vibration modes are the solutions of the homogeneous equation of waves:

$$\partial_t^2 p(\vec{r},t) - c^2\nabla^2 p(\vec{r},t) = 0$$

For a temporary variation of harmonic type, it becomes the Helmholtz equation:

$$\nabla^2 p(\vec{r}) + k^2 p(\vec{r}) = 0 \quad (2)$$

The solution of the equation of inhomogeneous wave, equation (1), can be written [1] as:

$$p(\vec{r},t) = A_0(t) + \sum_j A_j(t).p_j(\vec{r}) \quad (3)$$

Since the eigenfunctions of the Helmholtz equation are orthogonals, the coefficients A_j (where j represents a cavity oscillation mode, that is, a ternary r, γ, n) can be calculated by means of the expression [1]:

$$A_0(t) = \frac{(\sigma-1)\int_{V^{(F)}} H(\vec{r})dV}{i\omega V}\exp(i\omega t); \qquad A_j(t) = \frac{i\omega(\sigma-1)\int_{V^{(F)}} H(\vec{r})p_j(\vec{r})dV}{\left(\omega_j^2 - \omega^2 + i\frac{\omega\omega_j}{Q_j}\right)\int_V \left|p_j(\vec{r})\right|^2 dV}\exp(i\omega t) \quad (4)$$

with $Q^{-1}{}_j = \dfrac{c\int_{\sup} p_j^2(\vec{r})\xi ds}{\omega_j \int_V p_j^2(\vec{r})dV}$

The integration region in the integral of the numerator of $A_j(t)$, $V^{(F)}$, is the illuminated volume by the laser; in the integral of the denominator the volume V is the total volume of the cavity. Q_j is the factor of quality of the cavity for the j mode; ω_j is the resonance angular frequency of the cavity for that mode and ω it is the angular frequency that excites the cavity.

Then, the integration region $V^{(F)}$ is the volume of the acoustic emission source as much for the signal as for the noise; in the denominator of A_o, V is the total volume of the cavity. Q_j is the factor of quality (due to the superficial losses) of the cavity for the mode j; ξ is the maximum value of the specific acoustic conductance [3, 4] and informs with regard to the phenomena of energy losses in the interface among the gas (N_2 in our case) and the cavity internal surface. On one hand these losses are due to the transfer of heat of the gas to the wall (by gas viscosity); for other hand, the losses are due to the diffusion of the heat from the sound wave to the wall, both in the very thin layer of adjacent fluid to her. For the nitrogen, the maximum value of ξ is 3.67×10^{-3}.

In the next calculations, the small difference of temporary phase introduced by $A_0(t)$ in the equation (3) was considerated.

3. EIGENFUNCTIONS AND PARAMETERS OF THE ELLIPTIC RESONANT CAVITY

The equation (2) can be solved by separation of variables in coordinated cylindrical elliptic (u, v, z) [5]. The transformation of the Cartesian coordinates to the cylindrical elliptic ones is:

$$x = \frac{d}{2}.\cosh u.\cos v \quad y = \frac{d}{2}.senhu.senv \quad z = z; \quad 0 \le u < \infty \quad 0 \le v \le 2\pi$$

where d is the interfocal distance of the ellipse. Given d, for each value of $u = C$ (with constant C) a particular ellipse is obtained.

The solutions of the equations in u and v are the Mathieu's functions $Sp_{2r+\gamma}(s,v)$ and $Sp_{2r+\gamma}(s,iu)$, [6]. Then, the eigenfunctions of the Helmholtz's equation can be written as:

$$p_j(\vec{r}) = Sp_{2r+\gamma}(s,v)..Sp_{2r+\gamma}(s,iu).\cos(n\pi z / L) \quad (5)$$

where $p_j(\vec{r})$ is the eigenfunction and j symbolize r, γ, n ternary. L is the cavity length. p, in the S expression, indicates the even or odd parity of the function; γ takes 0 or 1 values, when the functions of Mathieu are of period π or 2π respectively. The sub index r enumerates the eigenvalues and the parameter *s* is related with the frequency and the interfocal distance of the ellipse by $\sqrt{s} = \pi f d / c$. The angular functions of Mathieu, those of argument v, are lineal combinations of sines or cosines (according to the parity) with tabulated coefficients [7]. The radial functions of Mathieu, of argument *u*, are lineal combinations of products of Bessel's functions with argument given in function of *u* and their coefficients are also tabulated [7].

The eigenfunctions depends on *z* as $\cos(k_z z)$, with $k_z = n\pi/L$, where *n=0,1,2,...*Since $k^2 = k_t^2 + k_z^2 = (2\pi f/c)^2$, k_z is limited by the values of k. k_t is the number of traverse wave.

The cavity parameters were chosen to be able to excite: 1) the even parity and period π modes (excited when the laser enters to the cavity for the centre of the ellipse); 2) the even parity modes and period π and 2π (when the laser enters for the focus). Therefore, the A_0 and A_j values were calculated for these modes, and the pressure for different points of location of the microphone, equation (3), with $\vec{r} = \vec{r}_m$ ($\vec{r}_m$ is the microphone coordinates). These results were compared with those of an equivalent resonator of circular section. Both cavities (elliptical and circular) have same excitement volume and both oscillate to the same frequency. The laser that excites both cavities is supposed to be the same one.

Once it was chosen a parameter *s,* the solution of the boundary condition for the eigenmode p_j, $\vec{\nabla} p_j(\vec{r}).\hat{n}_{surface} = 0$, gives the *u* values that will determinate the elliptical contours when *d* is fixed, where p_j is maximum. For the calculation of p_j, k_z has been considered null (*n*= 0).

4. LASER CENTERED IN THE AXIS OF THE CAVITY ELLIPTIC SECTION

For this mode of excitement, a 2663 Hz laser modulation frequency was chosen to excite the fundamental mode of a cylindrical cavity of circular section with a diameter inside the range of the dimensions usually used in photo acoustic experiences. The elliptic cavity is excited in the even Mathieu function, of period π with *r=0* and γ *=0*. Associated to that frequency, several *s* and *d* values excite the cavity: each pair of them determines a different ellipse and, therefore, different areas for the section of the cavity.

Table 1. Parameters of the elliptic section cavity for centred laser

a [cm]	b [cm]	e
6.90	5.47	0.61
8.73	7.65	0.48

In table 1, the biggest *a* and the smallest *b* semi axes are specified, and the eccentricity *e* of the resulting ellipse when *s = 4* and *d = 8.4 cm*, determinate for the excitement frequency (*2663 Hz*). With each pair of parameters (*a* and *b),* the corresponding values of A_j were calculated; the results were compared with that of a resonator of equivalent circular section (it resonates to same frequency).

5. LASER ENTERING FOR THE FOCUS OF THE CAVITY OF ELLIPTIC SECTION

In this case, inside the elliptic section cavity the Mathieu functions of even parity and period π and 2π, for *r* and γ different values are excited; they present relative maxima in the focuses of the ellipse and in the ends of the biggest axis. With the same laser modulation frequency (*2663 Hz*), the ellipses obtained have smaller area than those corresponding to the previous case.

Table 2. Parameters of the elliptic section cavity for the laser in the focus

d (cm)	a (cm)	b (cm)	e
4,202	3.89	3.27	0.54
5,042	3.90	2.98	0.65
5,88	3.92	2.59	0.75
6,3025	3.93	2.34	0.8
6,72	3.94	2.04	0.85

In table 2, the interfocal distance *d*, the biggest *a* and smallest *b* semi-axes and the eccentricity values for the resulting ellipses are specified, when the excitement frequency is *2663 Hz*.

6. CALCULATION of A_0 AND A_J FOR ELLIPTIC SECTION CAVITY

6.1 Signal case

The calculation of A_0 and A_j were made (in both forms of entrance of the CO_2 laser to the cavity) for the case in that the laser frequency modulation is equal to the resonance frequency of the cavity in the mode chosen, *j*. It was considered that the gaseous medium fills the cavity is nitrogen to the atmospheric pressure and ambient temperature (*20°C*) with ethylene traces that suppose a very small absorption of the laser radiation, in such way that, to facilitate the calculations, it is possible to consider that the CO_2 laser does not modify its intensity in the itinerary inside the cavity. This means, mathematically, not to include a dependence regarding the axis z. Also it is considered that the intensity of the laser beam is evenly distributed in its section (diameter supposed: 1 cm). With these considerations the function *H(r)* leaves as *H* outside the integral of the numerator in equations (4). The jacobian of the transformation for the integral of A_j is: $|J| = (d/2)^2(\sinh^2 u + \sin^2 v)$. The volume source $V_{(F)}$ is, in this case, the volume reached by the laser radiation: a cylinder of *1* cm of diameter and longitude L equal to the cavity length. *V* is the total volume of the cavity. With these considerations, the time independent expressions of A_0 and A_j are:

$$A_0 = \frac{(\sigma-1)H\int_{V(F)}|J|.dudvdz}{i\omega V}; \qquad A_j = \frac{(\sigma-1)H\int_{V(F)} p_j(\vec{r}).|J|.dudvdz}{\frac{\omega_j}{Q_j}\int_V \left|p_j(\vec{r})\right|^2 |J|dudvdz} \tag{6}$$

The results will be relative to the heat density *H*. This doesn't prevent the comparison among the elliptic and cylindrical cavities responses, because both have same length, both are excited by the same laser and both contain the same gases in equals conditions; then, the laser absorption will be made in them in equal form.

6.2 Noise case

Among the diverse sources of noise, the main one is due to the windows that close the cavity generate coherent noise: the windows absorb part of laser energy and they heat. The heating energy absorbed by the windows is then transmitted to the cavity gaseous medium, generating an acoustic wave in the characteristic modes of the cavity oscillation. The wave of heat, or thermal mode, in the gas by this heating source decays quickly, just as in the case of the wave of heat that arises by laser absorption in the medium and it origin the acoustic signal.

The heating power deposited from the windows in the gaseous medium, N_2, generates a thermal wave whose reach is given by the characteristic propagation longitude in the thermal mode [4], nearly to $\delta = \sqrt{K/(\rho C_p f)}$, in which the heating conductivity of the gas is given by $K=23.86 \times 10\text{-}3\ W(m.°K)^{-1}$; the density for $\rho = 1.16\ kg.m^{-3}$ for atmospheric pressure and ambient temperature; the constant pressure heating capacity $C_p=1.039\ J(g.°K)^{-1}$; and the excitement frequency of the cavity is $f = 2663\ Hz$. Then, it is assumed the heating source volume is given by the product of the cross section of the laser beam by the longitude δ. Given the small value of δ (~ 10-5 m) there will be a noise heat density H_r (different from *H*), constant in the region within δ: it allows extracting H_r outside the integral of the coefficients A_j, just as the signal case, equation (6).

This approach to the problem allows to carry out a similar calculation for the circular section cavity and to compare both.

7-RESULTS FOR THE ELLIPTIC SECTION CAVITY

7.1 Relative signal pressure at the microphone position: laser entering by the cavity axis

The A_0 and A_j values, equation (5), for the resonant frequency (*2663 Hz*) of the elliptic section cavity given by the even period π Mathieu function with $r=0$ and $\gamma=0$, with $s = 4$, were calculated. By means of the equation (3), the relative pressure in term of *H* at the microphone position was obtained. The microphone is located in the extreme of the smaller ellipse axes (of cylindrical elliptic coordinated given by $u=u_C$, $v = \pi/2$ and Cartesian coordinated $x=0$, $y=b$): this is one of the points which the pressure value is maximum. The values are shown in table 3.

Table 3. Relative pressure at the microphone (laser in the axis of the cavity)

e	a [cm]	b [cm]	Relative pressure in x=0, y=b
0.61	6.90	5.47	-1.7×10^{-4}
0.48	8.73	7.65	-1.1×10^{-4}

The values of the relative pressure in the last column of the table 3 for the even function of period π with $r = 0$ were obtained. The contributions of the functions with $r = 1$ and $r = 2$ were calculated, and they were not taken into account because they are 10^{-5} or smaller.

The minus sign in the relative pressure is due to the sign of the maximum of the function $p_j(\mathbf{r})$ on the contour, opposite to the central maximum.

To compare, the coefficients given by the equation (5) for a cell of circular section that resonates to the same frequency (2663 Hz) in the longitudinal mode $k=0$, azimuthal $m=0$ and radial $n=1$, were calculated. The radius of this cell is $R = 8.05$ *cm*. The pressure eigenfunctions $p_j(\mathbf{r})$ for the circular section cavity are given for:

$$p_j(r) = \cos(m\varphi) J_m(\pi\alpha_{m,n} r / R) \cos(k\pi z / L) \tag{7}$$

where L is the longitude of the cell and $\alpha_{m,n}$ is the nth. root of the radial derivate of the Bessel function of m-order (with $r=R$, on the contour, in some point the microphone is located). The value of the $\alpha_{0,1}$ root is *1.2197*. The longitude L of the cell is not necessary to specify because it does not appear in the expression to calculate.

The coefficients, equation (5), were multiplied by $p_{0,1,0}(r_m)$, equation (6), with $r_m = R$ (microphone on the lateral surface). The pressure value, -8.5×10^{-5}, at the microphone position, was obtained. This value is one order of magnitude smaller than those obtained under equivalent conditions (same frequency of resonance; equal constant adiabatic of the gas that fills the interior of both cavities; illuminated by the same laser) for the elliptic section cavity (chart 3). The minus sign has identical explanation to the previous case.

The Q_j value for the cylindrical section cavity is bigger than those corresponding to the modes in that the elliptic section cavity can oscillate because, in the elliptic section cavity, the multiplicity of modes for a single resonance frequency diminishes its cavity selectivity.

7.2 Relative signal pressure at microphone position: laser entering by the cavity focus

The relative pressure to H, equation (3), was calculated in the extreme points of the larger axis of the elliptic contour. In these points, the even period π and 2π Mathieu functions have relative maxima and they are favourable positions to locate the microphone. The relative pressure to H was also calculated at the end of the smaller axis because the even period 2π Mathieu functions have a relative maximum in that point, and it is also a favourable position for the microphone location. The resonance frequency is *2663 Hz* in all the cases. It is assumed that the laser beam is cantered in one of the focuses (what allows to excite the even modes of periods π and 2π). The laser beam was thought to be of *1 cm* of diameter with a cross section circular surface.

With the values given for the frequency and the cavity length ($L = 20$ *cm*), n only can take *0, 1, 2. 3* values (equation (2)). Since the microphone is located in some point $z = L/2$ of the lateral surface cavity, when replacing this value in anyone of the eigenfunctions $p_j(\vec{r})$, equation (3), $\cos(n\pi z / L) = \cos(n\pi/2)$ will be different from zero for $n = 0, 2$. For $n = 2$ the Mathieu eigenfunctions are multiplied by $\cos(2\pi z/L)$. When integrating on z between 0 and L for the calculation of the A_j coefficients, the integral $\int_0^L \cos(2\pi z/L).dz = 0$. This indicates they can only be captured in the microphone location the modes given by the equation (2) with $n = 0$.

The used modes are those even Mathieu functions of period 2π with $\gamma = 1$ and $r = 0, 1$; and the even period π functions with $\gamma = 0$ and $r = 0, 1, 2$.

The relative pressure for different values of the interfocal distance cavity, are shown in table 4.

Table 4. Relative pressure to H (multiplied by 10^4) at microphone location points

d (cm)	e	x=a y=0	x=-a y=0	x=0 y=b
4,202	0.54	8.4	6.12	6.88
5,042	0.65	6.66	3.32	4.44
5,88	0.75	7.08	1.95	3.6
6,3025	0.8	10.6	4.19	6.06
6,72	0.85	13.9	5.22	7.72
a: larger semiaxis; b: smaller semiaxis; d: interfocal distance; e: eccentricity				

In figure 1, the relative pressure versus the interfocal distance, is shown for different microphone positions.

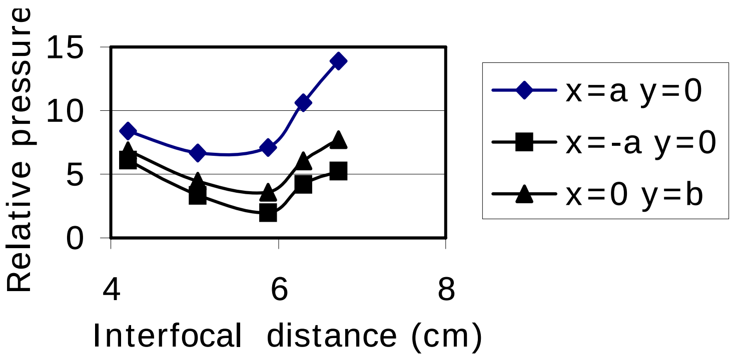

Figure 1. Relative pressure (by 10^4)

7.3 Signal noise: laser beam entering by cavity focus

The coherent noise signal was calculated for both elliptic and cylindrical cavities for the same excitement frequency of *2663 Hz*. In the elliptic case, the noise source volume in the gas is different to the signal source volume; then k_z will be zero for $n=0$ and $31.42\ m^{-1}$ for $n=2$. This implies that in the cavity they can get excited a transverse mode with $k_t=35.76\ m^{-1}$, besides the corresponding to $n=0$ with $k_t=47.60\ m^{-1}$.

For the calculation an elliptic section of interfocal distance $d = 6.72\ cm$ with semi axis $a = 3.93\ cm$ and $b = 2.44\ cm$ was chosen, and it was located the microphone at the end of the smallest semi axis, in $L/2$. In this case, the noise pressure at the microphone, relative to the noise power density H_r, is 3.32×10^{-7}.

It is assumed that the circular section cavity is closed by identical windows and is excited of equal forms as the corresponding elliptic cavity. In this case, the noise pressure at the microphone relative to the noise power density H_r, is 1.27×10^{-7}.

7.4 Quality factor Q

The Q_j values of both cavities, considering the superficial losses mechanisms only, were calculated. The quality factor of circular section cavity is *1200*, as long as the values for the different oscillation modes of the elliptic section cavity (and for different interfocal distances) vary between *100* and *350*. These values for the elliptic section cavity are smaller than those of the circular section cavity in one order of magnitude: it represents a selectivity disadvantage. The higher signal amplitude for the elliptic section cavity could compensate this disadvantage.

8- CIRCULAR AND ELLIPTIC SECTION CAVITIES: CONCLUSIONS

All the calculated relative signal pressure values at the microphone location in the section elliptic cavity when the laser goes in by one of the focuses are an order of magnitude larger than the one obtained for the circular section cavity oscillating in the mode *0,1,0* (to the laser modulation frequency of *2663 Hz*).

For the coherent noise generated by the heating of the windows, the noise signal at microphone has greater relative amplitude than that for the elliptic section cavity in all the analyzed cases.

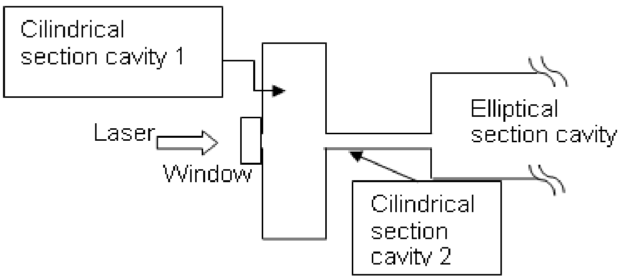

Figura 2. Acoustic filtre

In this paper, it is not possible to calculate the signal-noise relationship for each cavity: the signal amplitude is calculated in relative form and other sources of noise were not considered. They can be related to the signal amplitudes calculated for both cavities on one hand and those corresponding to the noise amplitudes calculated for both cavities, on the other. For an elliptic section cavity of interfocal distance $d=6.72\ cm$ and eccentricity *0.85*, with the microphone located at end of the smallest semi axes, the relationships before mentioned are respectively *6.4* and *2.6*. This result is favourable, in the calculation, to the elliptic section cavity.

Since the group of the elliptical cross section surfaces is between *35* and *40 cm²*, this is smaller than the circular cross section surface (*203 cm2*) and it let us suppose that the volumetric losses would be smaller in the elliptic section cavities than those with circular section.

A possible form to diminish the influence of the noise due to the windows heating that also generates a opt acoustical signal, is to separate each window of the resonant cavity by means of an acoustic filter. This filter could consist of two cylinders like it indicates in figure 2 (the plane of this view is perpendicular to the smaller semi axes than the ellipse; the laser beam enters by the focus of the elliptic section cavity).

Cavity 1 is a cylinder non resonant to the excitement frequency (*2663 Hz*). Cavity 2 is a cylinder of a quarter wavelength and with a section diameter close by the diameter of the laser beam. Then, both cavities, 1 and 2, are not resonant.

For circular section resonant cavity several investigators have proposed diverse alternatives more or less successful to eliminate the noise [2, 3].

These calculations are a strong indication that the cavity of elliptic section excited by the laser beam centred in one of the focuses would have a superior response than an equivalent circular section cavity (it oscillates in the same frequency). The frequency used for the calculation was chosen to make the future experimental determination.

9. SPHERICAL AND SEMISPHERICAL CAVITIES

The proposal light source in the semi spherical cavity is a diode led located in the centre of the equatorial plane that closes the cavity. The advantage is the led can be modulated electronically to frequencies as far as *10 KHz*. These frequencies can not be reached when the source is a mechanically modulated laser (CO_2 or another). This assembly enables to eliminate the presence of the windows (where the laser radiation enters to the cavity) as sources of coherent noise. The led emits in a cone whose maximum opening angle is *30°*; the microphone is located where the pressure is maxima (in the vertex of the cavity).

The spherical resonator, without windows, is fed by a led located in the centre of the sphere; the excitement region is a cone with its vertex at the spherical centre. The microphone is at the point of symmetry axis of the cone on the spherical surface.

In both cavities the microphone is located in the point of maximum pressure.

9.1. Spherical cavity

The pressure eigenfunctions, solutions of the wave equation in the spherical cavity with axial symmetry, are expressed as [8]:

$$p_l(r,\theta) = (k.r)^{-\frac{1}{2}}.J_{l+\frac{1}{2}}(k.r).P_l(\cos\theta) \tag{8}$$

The index $l = 0, 1, 2,...$ and $m = 0$. k is the wave number, $k = \frac{2.\pi}{\lambda}$, where λ is the wavelength correspondent to the resonant frequency. The function $J_{l+\frac{1}{2}}(k.r)$ is the spherical Bessel function [9], and $P_l(\cos\theta)$ is the l-order Legendre polynomials.

The led emits the power inside a cone with maximum opening *π/6*. *R* is the spherical surface radius. The power density absorbed by the gas, *H(r,θ)*, will be expressed as $H(r,\theta) = -\frac{dP}{dV}$, where *P(r,θ)* is the power that spreads in the means and $dP = \frac{\partial P}{\partial r}.dr = -\alpha.P.dr$ is the power lost by the beam, deposited in the gas in an interval radial *dr*, being *α* the absorption coefficient.

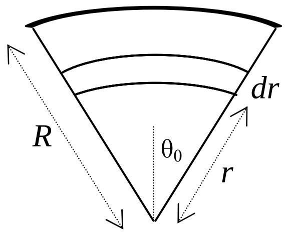

Figure 3. Plane section of the conical active volume (led in the vertex)

The differential of volume is given for $dV = A.dr$; *A* is the area of the spherical cap $A = \Omega.r^2$, being *Ω* the solid angle given by $\Omega = \int_0^{2\pi}\int_0^{\theta_0} sen\theta.d\varphi.d\theta = 2.\pi.(1-\cos\theta_0)$. Therefore, $dV = 2.\pi.(1-\cos\theta_0).r^2.dr$ and, in consequence. $H(r,\theta)=\dfrac{\alpha.P}{2.\pi.(1-\cos\theta_0)}\cdot\dfrac{1}{r^2}$. P is function of r but, since the absorption of the gas is very small ($\alpha.R << 1$), in a first approach it is possible to assume it is constant and to extract it outside of the integral of the coefficients A_n.

Given the symmetry of the source, it is excited, inside the spherical cavity, the mode with $l=2$ and $m=0$. The radius of the spherical surface will be the one that makes zero the value of the pressure normal derived at the surface and that it assures no pressure nodes due to the function of Bessel, in all the volume.

Figure 4 represents the typical graph of the eigenfunction of the spherical cavity like θ function, given by the equation (7) for $l = 2$ with the radius like a parameter.

The main lobules of the eigenfunction are separated by a conical surface that represents the only nodal surface of the pressure inside the cavity, due to a zero of the polynomial of Legendre for $l=2$. This excited mode is not degenerated.

9.2. Semispherical cavity

The excitement modes of the spherical cavity are symmetrical with regard to the equatorial plane. The normal derivative of the pressure, respect to θ in $\theta=\pi/2$, is zero. This implies that the equatorial plane of the sphere is a surface with a maximum of pressure: if it is replaced in their place an acoustical reflective surface (supposed ideal), the semi spherical cavity will have the same eigenfunctions than to the complete spherical cavity (lobules corresponding to the positive part of the vertical axis in figure 4).

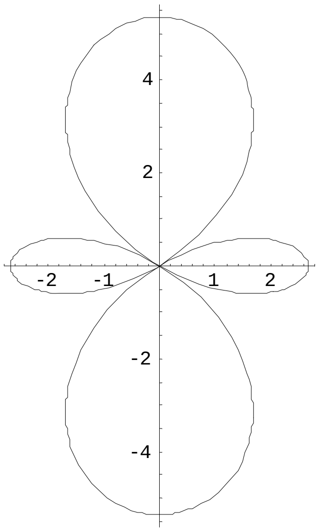

Figure 4. Graphic representation of the eigenfunction inside the spherical cavity

The semi spherical cavity is excited by a led -located at the centre of the equatorial plane that closes the cavity, with $\theta=\pi/2$- that emits symmetrically in the address of the z axis inside the volume closed by a conical surface of $30°$ of maximum opening, it will oscillate in a single mode in agreement with the form of the excitement: the corresponding eigenfunction with $l=2$ and $m=0$, given by the equation (8). In particular, the radius of the cavity is determined by the first root of the derivative of the Bessel function.

With these considerations, the coefficients that enter in the development of the pressure, equation (3), are only A_0 and $A_{l=2,m=0}$ (this last one will be indicated as A_2). Equation (4), will be:

$$A_0=\frac{3.(\gamma-1)}{2.\pi.i.\omega.R^2}.\alpha.P$$

keeping in mind the expressions of the power heating density deposited in the gas, of the differential volume and the factor of quality.

$$A_2=C.\alpha.P.f.\frac{\int_0^{kR}x^{-\frac{1}{2}}.J_{2.5}(x).dx}{\frac{kR}{5}.[J_{2.5}(kR)]^2+\frac{1}{4}\int_0^{kR}[J_{2.5}(x)]^2.dx} \qquad (9)$$
$$C=\frac{(\gamma-1)}{v^2.\xi}\cdot\frac{\cos\theta_a.sen^2\theta_a}{2.(1-\cos\theta_a)}$$

where θ_a is the semi-opening of the source emission cone and equal to $\pi/12$, R is the radius of the hemisphere, f is the led modulation frequency and $k.R=\dfrac{2.\pi.f}{v}.R=\pi.\alpha_{2,0,1}=3.34$; $\alpha_{2,0,1}$ is the first root of the derivative of the spherical function Bessel of order 2.5.

For a frequency of 2663 Hz, and relative to the product αP, $A_0\approx 2.3\times10^{-3}$ and $A_2=4.98$. Since $A_0 << A_2$ for all frequencies, A_2 was not kept in mind for the calculation of the total pressure neither in module nor in phase.

10. CALCULATION OF THE PRESSURE AT THE MICROPHONE

The pressure at the microphone was calculated by equation (3) for spherical and semi spherical cavities. This pressure is relative to the product $\alpha\ P$ since the coefficient A_2, given by equation (4), was calculated relative to this product. Since both cavities have in their interior the same gas (N_2) and the same impurity and they are fed by a led, both will receive the same excitement power. When varying the modulation frequency it should be changed the radius of both cavities, consequently, in order to excite in the mode l=2, m=0, the excitement volume will vary equally in both. The calculated relative pressure, at the microphone position, is a function of the led modulation frequency.

This calculation allows the comparison among the mentioned cavities and with an equivalent cylindrical cavity, of circular cross section [10]. The radio of this cylindrical cavity, resonant in a pure traverse mode, is variable with the modulation frequency of the luminous source and has an excitement volume similar to that of the spherical and semi spherical cavities. This cylindrical cavity is excited by a laser beam directed along its symmetry axis, with circular section. The calculation results are shown in figure 5.

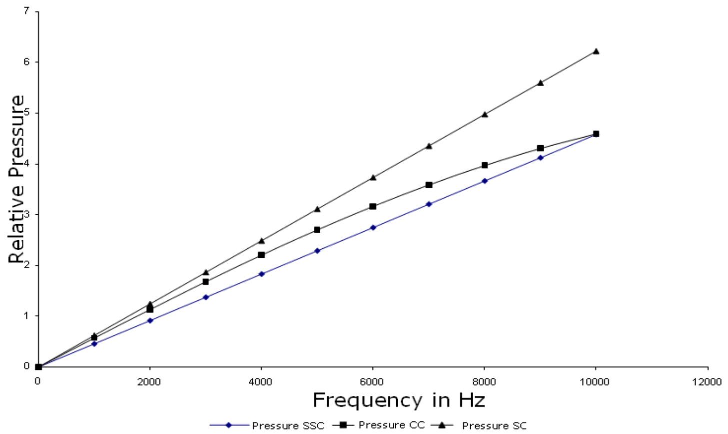

Figure 5. Relative pressure vs. frequency. SSC: semispherical cavity; CC: circular section cylindrical cavity; SC: spherical cavity

11. QUALITY FACTORS *Q*

The factor of quality Q_n depends of the total volume and the internal surface of the cavity, equation (4). It was calculated for the semi spherical, spherical and cylindrical cavities, supposing all excited by same luminous power sources. The values calculated for all the cavities are independent of the modulation frequency, since the cavities were considered resonant for each frequency of the range (between 0 and 10000 Hz). This means that the radius, the volumes (as much the total as the active) and the surfaces of the spherical and semi spherical cavities were modified. In the case of the cylindrical cavity, the radius also varies with the variation of the modulation frequency of the excitatory beam. For this cavity it is kept in mind a corresponding variation in their longitude in order to equal their illuminated volume with that of the spherical and semi spherical cavities.

Under these conditions, the quality factor for all the mentioned cavities is independent of the frequency, and its value is given in the table 5.

Table 5. Quality factor *Q*

Cavity	Value
Semi spherical	*155*
Spherical	*210*
Cylindrical	*551*

12. CONCLUSIONS

Figure 5 shows that, to any frequency, the calculated relative pressure at the microphone position is higher in the spherical cavity than in the semi spherical and in the cylindrical one, to equality of excitement power, and with the position of the luminous source indicated above.

We attribute the smallest performance of the semi spherical cavity in relation to the spherical cavity to is due to the fact that the plane that it closes the semi sphere is not ideal. It is not a perfect acoustic mirror; in the calculation it is related to the specific acoustic conductivity ξ.

An advantage of the luminous source assembly in the geometric centre of the spherical cavity or the centre of the plane surface in the semi spherical cavity is that the entrance of the luminous beam to the cavity doesn't cross windows as it does in the case of the cylindrical cavity.

Another advantage of the mentioned assembly is that it allows to use led like luminous sources easily electronically modulated until *10 kHz*.

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