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AN APPROXIMATE TECHNIQUE FOR DAMAGE IDENTIFICATION IN BEAMS USING NONLINEAR REGRESSION ANALYSIS OF BENDING FREQUENCY CHANGES

Elma EKINOVIC[1], Vlatko DOLECEK[2], Slavko DOLINSEK[3], Nedim HODZIC[1]

[1] University of Zenica, [2] University of Sarajevo, BOSNIA & HERZEGOVINA
[3] University of Primorska, Koper, SLOVENIA

ABSTRACT:
This paper addresses the problem of damage identification in beam-like structures on the base of bending frequency changes. The identification of damage location and its depth is performed by use of regression relations between changes in natural frequencies and damage parameters. Input data for establishing the regression relations are collected using numerical analysis (FEA) of the beam structure with and without damage. The damage is simulated as a narrow open notch perpendicular to the beam axis. The efficiency and limitations of the proposed technique are assessed through a series of damage scenarios.
KEYWORDS:
damage identification, finite element analysis, nonlinear regression

1. INTRODUCTION

Identification of damages in structures and components is an important aspect of their proper operation. Failure to detect damages has various consequences, and they vary based on the application and importance of the structures and components. A number of non-destructive techniques are available now to detect faults and defects in a structure. Nevertheless, great efforts are still put towards developing new, more reliable, efficient, and less tedious detection techniques. The most common of these methods is the vibration based damage detection and identification.

The basic idea behind the vibration based identification technique is that any change in the physical properties of a structure caused by the presence of a defect will directly cause some changes in the modal parameters, such as natural frequencies, damping factors, and mode shapes. These changes can be measured using the available modal testing methods. On the other hand, having sufficient data from numerical analysis enables an engineer to derive the regression relations that correlate specific shifts in one or more of the modal parameters as a function of the size and location of defects, [6].

Numerous methods have been proposed to identify damage parameters in structures. The method of using experimental modal analysis for detecting cracks appeared in the 1940s. A comprehensive survey of the available literature was given by Dimarogonas [4] and Doebling et al. [5]. These reports reviewed various technical literatures on detection and identification of structural damage using vibration based testing. Salawu [12] presented identification techniques that use only frequency information to identify damage in structures.

This paper proposes a technique to identify the location and magnitude of a damage in a beam-like structure using numerical analysis and nonlinear regression. The changes that occur in the first four of its lowest bending frequencies are used for the identification of the crack parameters.

2. PROPOSED IDENTIFICATION TECHNIQUE

2.1. Formulation of the problem

The fundamental idea underlying the process of crack identification is based on the fact that a change in physical parameters of the structure causes a change in its modal parameters. This fact is represented by the following matrix equation for undamped natural vibrations, Eq.(1):

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \qquad (1)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, and $\ddot{\mathbf{q}}$ and $\mathbf{q}$ are the acceleration and displacement vectors, respectively. The eigenvalues of Eq. (1) correspond to the undamped natural frequencies of the structure.

A damage that occurs in a structure causes a change in mass and stiffness matrices and, consequently, in the natural frequencies. Some theoretical models are established to determine these changes in structural matrices, for instance in [1, 2, 3]. These procedures lead to differential equations that usually require a great effort to be solved, whereas a numerical analysis of a damaged structure can provide the values of these changes much faster and easier. Using the frequency shifts of several natural frequencies, it is possible to determine the magnitude and location in many cases of a single damage, [12].

2.2. Numerical analysis of a free-free beam

The technique proposed in this paper will be shown considering a simple case of a free-free beam, Fig.1. Let the numerical model of the beam has the following properties: length L_b=400 mm, height H=8,16 mm, width B=8.12 mm, modulus of elasticity E=2.068x10^{11} Pa, mass density =7820 kg/m^3, and Poisson's coefficient =0.29. The beam is modeled using solid elements in software I-DEAS Master Modeler 9, Fig.2. The damage is simulated as a narrow open notch perpendicular to the beam axis. The location of the damage is L_d , its depth is a, and the width of the notch is 1mm.

Relative location L = L_d / L_b and relative depth of the damage D = a / H were varied and the first four natural frequencies corresponding to the bending modes of the undamaged and the damaged beam were calculated using the numerical model. Due to structural symmetry, the location of the notch L_d measured from the left end of the beam was varied from 10 mm to 200 mm in 10 mm increments. The depth a of the notch was varied from 1 mm to 4 mm in 1 mm increments.

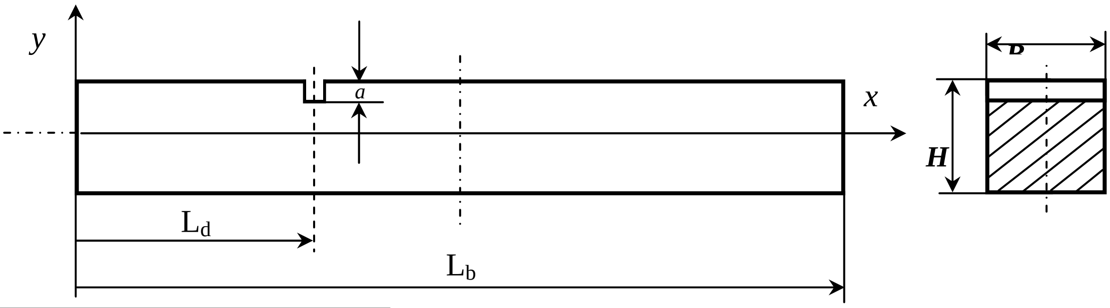


Figure 1. Geometry of the free-free beam with a notch

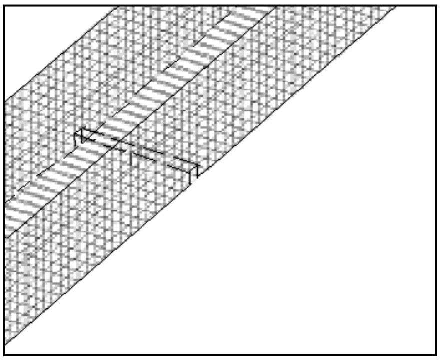


Figure 2. Numerical model of the beam with the notch

Then, the relative frequency parameters $F_I = f_{I(d)} / f_{I(u)}$, I=1,2,3,4, were calculated. Here, f_I represents the Ith natural frequency, subscript (d) denotes the damaged beam, and subscript (u) refers to the undamaged beam structure.

The calculated changes of relative frequency parameters F_I in relation to the relative depth D and relative location L of the notch are presented in Fig.3 and Fig.4.

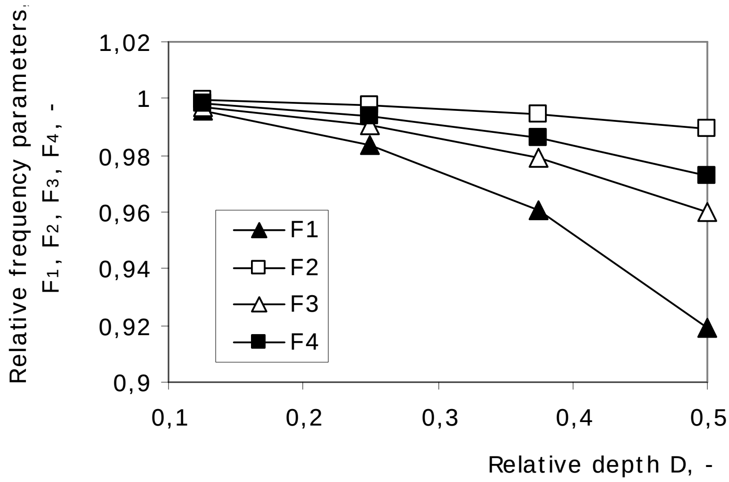


Figure 3. Relative frequency parameters f_1- f_4 versus relative depth d (plotted for the case of relative damage location l=0.45)

One can see that these changes are lower for smaller depth of damage, and that the damage located at the characteristic node of a mode does not affect the frequency of that mode. Since the notch in numerical model eliminates a certain part of the structural mass, there are some places where the influence of the mass change prevails over that of stiffness, so in these points the relative frequency change F_I is higher than unity. In case of a real crack, all relative frequency changes F_I would be always smaller than unity (or equal to unity in nodal points) since the crack decreases the stiffness of the structure and does not influence the structural mass.

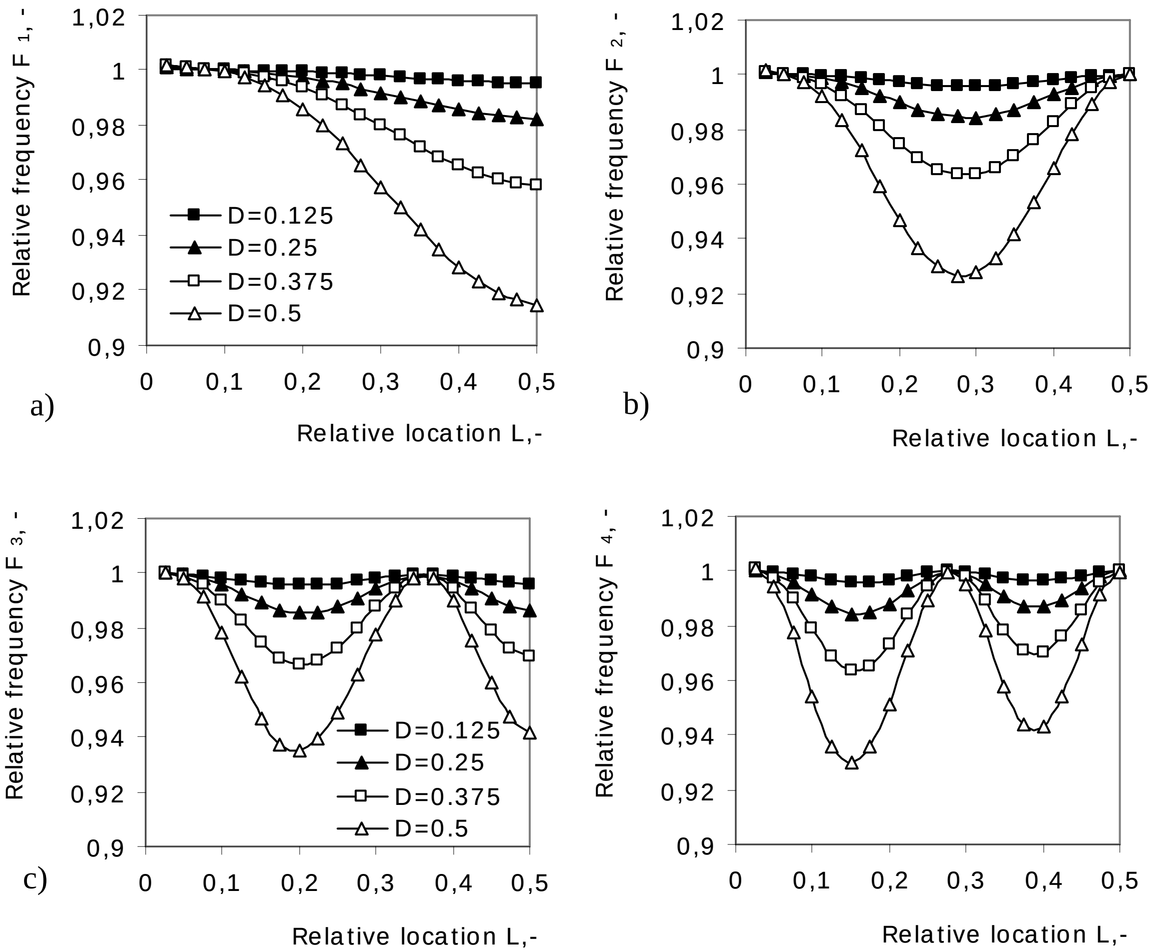


Figure 4. Numerically obtained relative frequency parameters f_1- f_4
In relation to the relative location l and relative depth d of the notch: F_1, (b) F_2 , (c) F_3, (d) F_4

2.3. Regression analysis of the frequency changes

Numerical values of the relative frequency parameters F_I, I=1,2,3,4, were taken as the input data for establishing the regression curves that describe the relations between the relative frequency changes and damage parameters (relative location and relative depth). The software STATISTICA 6.0 (Nonlinear Estimation option) was used for statistical estimation of these regression relations. Nonlinear Estimation computes the relationship between a set of independent variables and a dependent variable. It should be emphasized here that Nonlinear Estimation leaves it up to you to specify the nature of the relationship; for example, you may specify the dependent variable to be a logarithmic function of the independent variable(s), an exponential function, a function of some complex ratio of independent measures, etc., [14].

The graphical presentation of frequency changes in relation to the relative depth D and relative location L, Fig.3 and Fig.4, shows nonlinear behavior. As mentioned above, the choice of appropriate nonlinear functions is wide and can be suggested by these graphical presentations. After several attempts the best fit can be chosen, taking into account the coefficient of correlation and shape of the nonlinear curve.

Generally, in case of uniform beam-like structures, the best results are obtained when the relative frequency changes are expressed by regression relation that include parabolic influence of the relative depth D (in this case quadratic) and polynomial influence of the relative location L, Eq. (2):

$$F_I = 1 - B_{1(I)} D^2 \left(B_{2(I)} + B_{3(I)} L + B_{4(I)} L^2 + \ldots + B_{N(I)} L^{N-2} \right), \quad (2)$$

where $B_{1(I)}$, $B_{2(I)}$, $B_{3(I)}$, ... , $B_{N(I)}$ are regression coefficients for the relative frequency parameter F_I.

The following regression relations for the first four relative frequency parameters F_I, I=1,2,3,4, are obtained for the beam under consideration, Eqs.(3), (4), (5) and (6):

$$F_1 = 1 - 0.177566\,D^2\,(-0.01948 - 0.85975\,L + 6.32585\,L^2 + 47.5372\,L^3 - 83.912\,L^4) \tag{3}$$

$$F_2 = 1 - 0.42922\,D^2\,(0.065133 - 3.8651\,L + 45.7407\,L^2 - 44.275\,L^3 - 267.41\,L^4 + 406.144\,L^5) \tag{4}$$

$$F_3 = 1 - 11.0353\,D^2\,(0.006469 - 0.37663\,L + 5.74127\,L^2 - 16.533\,L^3 - 32.81\,L^4 + 180.968\,L^5 - 177.36\,L^6) \tag{5}$$

$$F_4 = 1 - 69.938\,D^2\,(0.00252 - 0.17049\,L + 3.30183\,L^2 \;\; 18.862\,L^3 + 21.3852\,L^4 + 121.863\,L^5 - 375.07\,L^6 + 298.339\,L^7) \tag{6}$$

The corresponding coefficients of correlation are very high in these cases (0.996 for F_1, F_2, F_3, and 0.998 for F_4). These regression surfaces are plotted in 3D in Fig.5.

The largest differences between the values obtained numerically and those calculated by regression relations given by Eqs.(3)-(6) are at those locations of the beam where nodes and maximal amplitudes of the particular mode shapes occur. This results from the nature of regression relations to smooth the data at extreme points.

The appropriate regression relations can be found by trial-and-error or intuitively for any type of the real beam taking into account its dimensions, material, boundary conditions, etc. Besides the numerical simulation for collecting the input data for regression analysis, one can use the values of frequency changes for different locations and depths of the damage calculated by expressions that are given by some authors, for instance [7, 13].

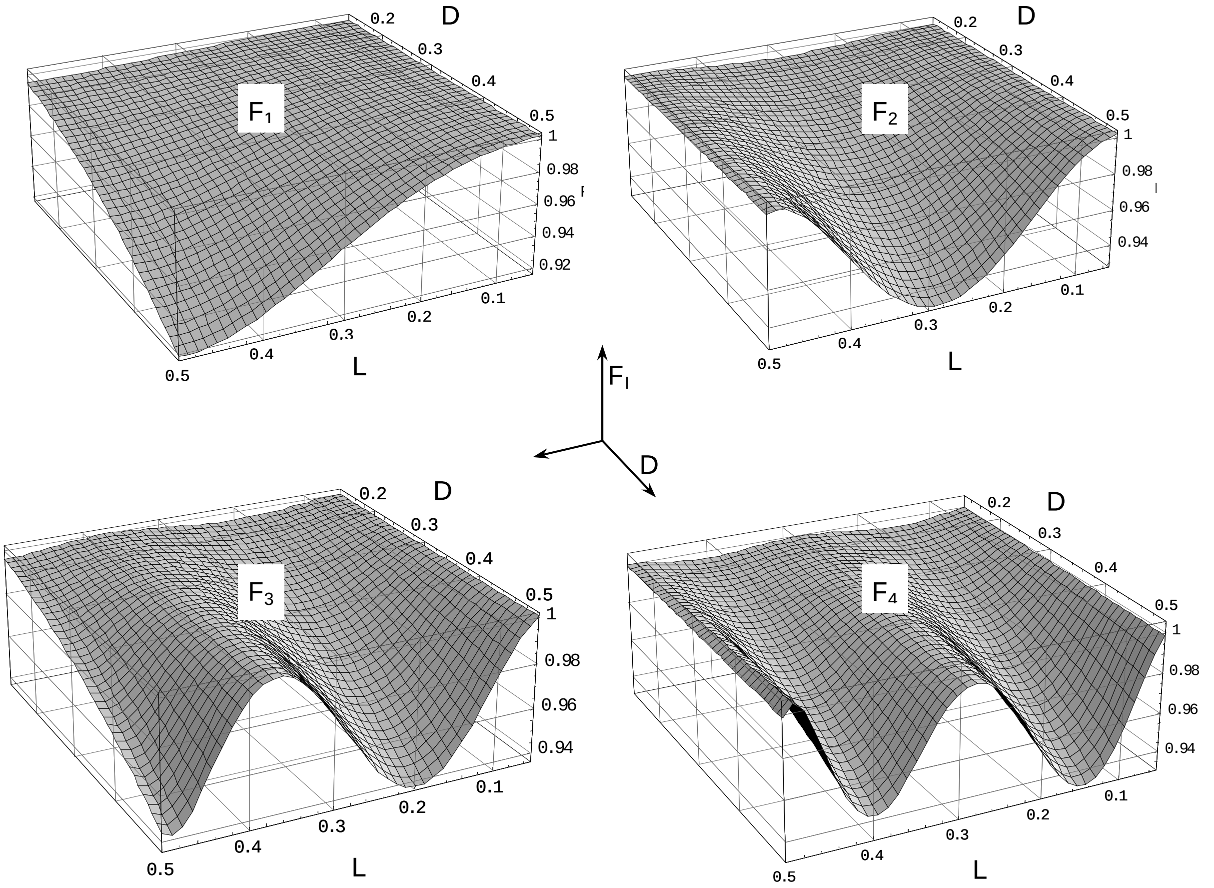


Figure 5. 3D graphical presentation of regression relations Eqs. (3)-(6) for the frequency change parameters F_1- F_4

2.4. Identification of damage parameters

Each of the regression surfaces shown in Fig.5 can be cut by an appropriate horizontal plane that corresponds to a specific value of the frequency change. The intersection curve projected on D-L plane shows all possible values of D and L that correspond to the same frequency change, Fig.6. The procedure is based on the fact that all of the intersection curves should theoretically give a common intersection point in D-L plane, which shows the values of damage parameters D_{est} and L_{est}, see [10, 11, 15]. But, the procedure proposed here allows the identification of damage parameters without plotting, i.e. visualization of the intersection curves as in [11, 15], which simplifies the identification procedure a lot.

The basic idea for finding the intersection points (i.e. sets of two unknown coordinates D and L) in this example (for the regression relations of the form Eq.2) will be shown here. The regression relations given by Eqs. (3) and (4) for the first two frequency changes can be written more compactly as:

$$F_1 = 1 - k_1 D^2 g_1(L)\,, \tag{7}$$

$$F_2 = 1 - k_2 D^2 g_2(L)\,, \tag{8}$$

by substituting

$$k_1 = B_{1(1)}\,, \quad g_1(L) = B_{2(1)} + B_{3(1)} L + B_{4(1)} L^2 + B_{5(1)} L^3 + B_{6(1)} L^4\,, \tag{9}$$

$$k_2 = B_{1(2)}\,, \quad g_2(L) = B_{2(2)} + B_{3(2)} L + B_{4(2)} L^2 + B_{5(2)} L^3 + B_{6(2)} L^4 + B_{7(2)} L^5\,. \tag{10}$$

From equations (7) and (8), it follows:

$$D^2 = (1 - F_1) / k_1 g_1(L) = (1 - F_2) / k_2 g_2(L) \tag{11}$$

which results in the relationship that involves only the unknown L:

$$(1 - F_1) / k_1 g_1(L) = (1 - F_2) / k_2 g_2(L)\,. \tag{12}$$

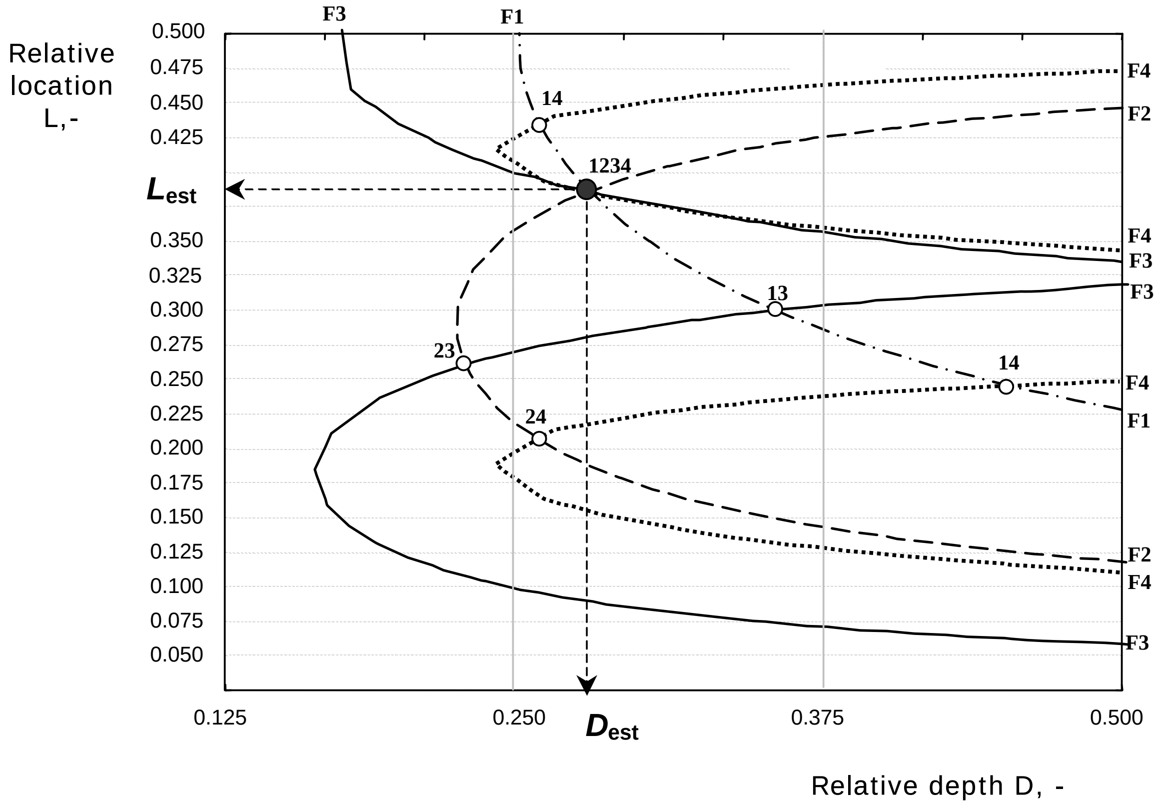


Figure 6. Graphical presentation of finding the intersection point
Of intersection curves in d-l plane

For the measured frequency changes F_1 and F_2, one can find all values of the relative location L that satisfy Eq.(12), taking into account that complex values of the relative location L are not acceptable. The corresponding values of the relative depths D (positive values) can be found directly from Eq.(11). Similar relations can be established for other pairs of frequency changes, and an appropriate set of intersection points with real values of L and D can be found. Of course, the proposed method of finding the intersection points is valid for regression relations of the form (2), but for other types of the best-fit regression relations a feasible way to solve a set of generally nonlinear equations should be found.

Theoretically, changes in two frequencies (basically in the F_1 and F_2) should be sufficient to find the intersection point, i.e. the pair (D,L) that corresponds to the actual damage parameters. However, due to inevitable differences between the numerical model and the real structure (modeling errors), one should not rely solely on only two measured frequency changes to find the intersection point (D,L). It is much safer and reliable to locate this point using more frequency changes, which also contribute to the robustness of the technique. Basically, the first four frequencies are shown to be sufficient to identify a crack in the beam, [15]. However, in practice, it would be very hard to obtain the same intersection point for all four frequency curves as the numerical values of intersection point coordinates can differ slightly, Fig.7.

Also, as shown in Figure 6, there are another intersection points in the D-L plane between the curves obtained by a pair of frequency changes, but they are isolated and not confirmed by the other frequency changes.

For that reason, this technique proposes finding three closest intersection points of frequency curves, i.e. those that give minimal sum of their distances from their mean value. The coordinates of their mean value can be adopted to represent the damage parameters. The only prerequisite here is that this mean value should not estimate the damage much beyond the numerically observed ranges of L and D (in this example for L=0.025 to 0.5 and D=0.125 to 0.5) that are covered by regression relations. In such a case, the next combination of three intersection points giving minimal sum of distances from their mean value should be appropriate.

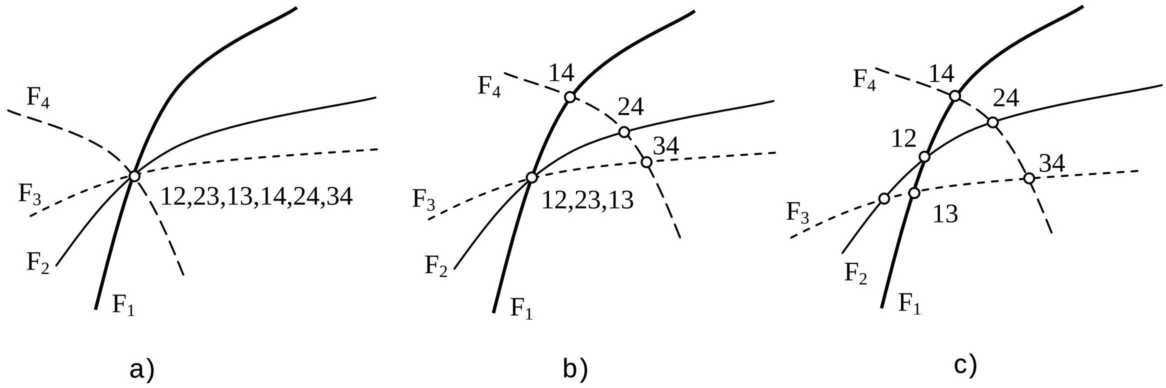


Figure 7. Characteristic cases of intersections of frequency curves:
a) Ideal case; b) special case; c) general case
(note: the intersection point of f_1 and f_2 is marked as 12, etc.)

3. NUMERICAL VALIDATION OF THE PROPOSED TECHNIQUE

The proposed procedure will be checked by numerically obtained values of frequency changes. These frequency changes could represent the data from a simulated experiment in which the real structure and the numerical model match ideally. All errors that appear here are only the result of a smoothing property of regression representation and the proposed procedure of finding damage parameters by the mean value of three intersection points.

Numerical example. Let find the parameters D and L of a damage if the frequency changes are: F_1=0.972339, F_2=0.97026, F_3=0.998684, F_4=0.97827.

Using the relations given in Eqs.(3) and (4) for the first two frequency changes and similar relations for other frequency pairs, the set of all possible locations L can be found from appropriate equations like Eq.(12). The corresponding values of D are then calculated using Eq.(11) for the first two frequency changes and similar relations for other frequency pairs. The calculated real values of L and positive values of D are given in Table 1.

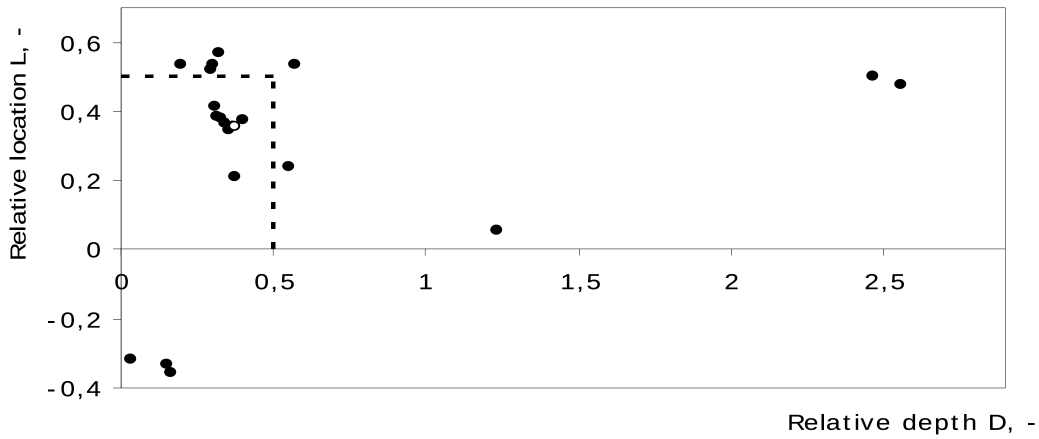
The set of 23 calculated intersection points are shown on Fig.8a in the D-L plane (remember, plotting of these points is not necessary). The procedure of finding the closest three points gave the following results: D_{est}= 0,367870 for the relative depth and L_{est} =0,355564 for the relative location, Fig.8b. This estimation is determined by three intersection points of F_2, F_3 and F_4. (No.7, No.16, No. 21), i.e. by three frequency changes. Additionally, in this case, the intersection points that belong to the F_1 are very close to the estimated mean value, which also confirms the obtained parameters.

Table 1. The values of relative location L and relative depth D calculated for all combinations of the first four frequencies (for numerical example)

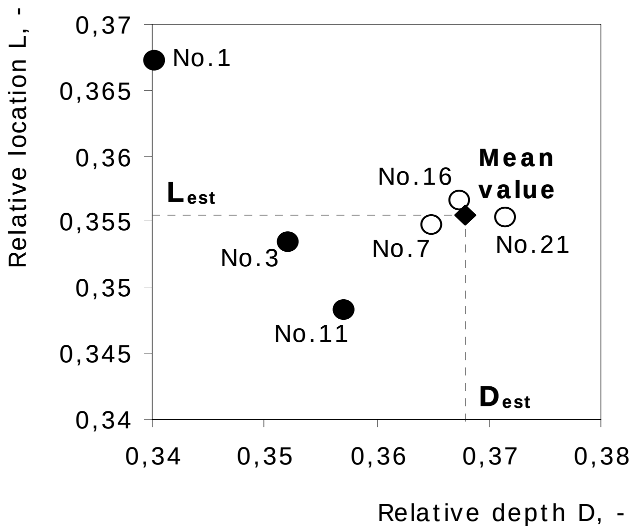
Frequency curves	Point No.	Relative depth D	Relative location L	Frequency curves	Point No.	Relative depth D	Relative location L
F_1& F_2			-0.442196	F_1& F_4			-0.356333
F_1& F_2			0.036980	F_1& F_4			0.028417
F_1& F_2			0.056247	F_1& F_4			0.044768
F_1& F_2	No.1	0.357032	0.348285	F_1& F_4	No.10	0.548767	0.239997
F_1& F_2	No.2	0.320403	0.567199	F_1& F_4	No.11	0.340148	0.367285
				F_1& F_4	No.12	0.311512	0.411703
				F_1& F_4	No.13	0.295111	0.521355
F_1 & F_3			-0.332444	F_2 & F_4	No.14	0.161899	-0.355078
F_1 & F_3			0.028788	F_2 & F_4			0.026855
F_1 & F_3			0.052778	F_2 & F_4			0.044184
F_1 & F_3	No.3	0.352057	0.353525	F_2 & F_4	No.15	0.371219	0.208984
F_1 & F_3	No.4	0.328735	0.382536	F_2 & F_4	No.16	0.367375	0.356630
F_1 & F_3	No.5	0.299612	0.535157	F_2 & F_4	No.17	2.558701	0.476092
				F_2 & F_4	No.18	2.466915	0.499526
F_2 & F_3	No.6	0.148874	-0.331798	F_3 & F_4			-1.328760
F_2 & F_3			0.028644	F_3 & F_4	No.19	0.030632	-0.318502
F_2 & F_3			0.052741	F_3 & F_4			0.028834
F_2 & F_3	No.7	0.364841	0.354727	F_3 & F_4	No.20	1.231357	0.054114
F_2 & F_3	No.8	0.400340	0.376023	F_3 & F_4	No.21	0.371393	0.355336
F_2 & F_3	No.9	0.569456	0.536062	F_3 & F_4	No.22	0.314828	0.383896
				F_3 & F_4	No.23	0.195270	0.533383

Note: Empty spaces in the column D refer to complex values

As the data for frequency changes F_1, F_2, F_3 and F_4 used in this example are previously calculated values of relative frequency changes for the numerical case of damage with D_{act}=0.375 and L_{act} =0.35, it follows that the error of estimation is -0,713% for the relative depth and 0.5564% for relative location. In absolute values, the estimated location of the damage is 142.2256 mm measured from the left end of the beam and the value of damage depth is 2.94396 mm (the actual location is 140 mm and the depth 3mm). Due to the free-free beam symmetry, the estimated location also assumes the possibility that the damage is located at the same distance from the right end of the beam, but this ambiguity (inherent to symmetrical beams) can be eliminated by some other procedures, for instance by adding a mass to disturb the beam symmetry.



a)



b)

Figure 8. The intersection points for the numerical example:
a) all intersection points; b) magnified view on three characteristic intersection points and their mean value

The procedure described above was repeated for 28 cases of damage parameters (7 damage locations with 4 depths) using the calculated frequency changes. The results of this training examination are very satisfactory except for the estimation of depth for damages near to the free end, where nodes of all four bending modes appear (i.e. for small frequency changes), Fig.9.

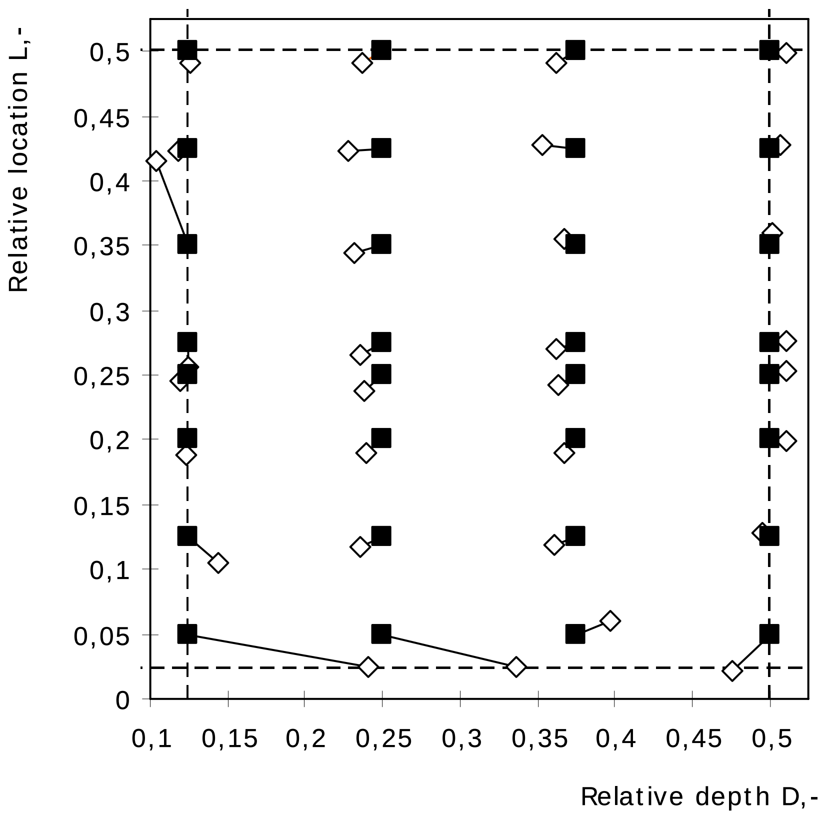


FIGURE 9. RESULTS OF DAMAGE IDENTIFICATION USING NUMERICALLY OBTAINED FREQUENCY CHANGES
(■ ACTUAL DAMAGE PARAMETERS, ◇ESTIMATED DAMAGE PARAMETERS)

4. CONCLUSION

The paper shows how damage parameters can be estimated analytically (without plotting intersection curves and points) by establishing the appropriate nonlinear regression relation between bending frequency changes and damage parameters.

The accuracy of the technique depends on:

- the quality of numerical model representing the real structure,
- the number of numerical simulations used for the estimation of regression relationships,
- the quality of frequency measurements in real practice.

Although the use of regression analysis makes this method approximate, the results are quite satisfactory and at least as good as the results obtained by some other methods, for instance in [1, 8, 9]. The identification of minor cracks would require better mesh refinement and higher number of numerical calculations to provide a sufficient number of points for establishing the regression relations. Also, measurement of small frequency changes due to minor cracks could impose an additional problem in practice.

A future research would include the extension of the proposed technique to different cross-section geometries of the beam, the use of frequency data of other classes of vibration modes (axial, torsional etc.), and the application of the proposed technique to fatigue cracks. Also, it would be interesting to investigate if the proposed technique could be used for multiple-damage scenarios.

In conclusion, we hope that the presented investigation will help spread out the perception that regression analysis can be successfully used in the field of non-destructive damage identification, [6].

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