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MASS TRANSFER WITH A CHEMICAL REACTION ON UNSTEADY FLOW PAST AN ACCELERATED ISOTHERMAL INFINITE VERTICAL PLATE

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ABSTRACT:
An exact solution to the problem of unsteady flow past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion is presented here, taking in to account the homogeneous chemical reaction of first order. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, chemical reaction parameter and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing chemical reaction parameter.
Keywords: accelerated, isothermal, vertical plate, heat transfer, mass diffusion, chemical reaction.

1. INTRODUCTION

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself (Cussler [3]). Chambre and Young [2] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al[4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [5]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Gupta et al [6] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [7] extended the above problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past a uniformly accelerated vertical plate was studied by Soundalgekar [9]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [8]. Basant Kumar Jha and Ravindra Prasad [1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

Hence, it is proposed to study first order chemical reaction on unsteady flow past an uniformly accelerated isothermal infinite vertical plate in the presence of heat and mass transfer. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study found useful in chemical process industries such as wire drawing, fibre drawing, food processing and polymer production.

2. ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion in the presence of chemical reaction of first order has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is accelerated with a velocity $u = u_0 t'$ in its own plane and the temperature from the plate is raised to T_w and the concentration levels near the plate are also raised to C'_w. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all } y, t' \le 0 \\ t' > 0: & u = u_0 t', & T = T_w, & C' = C'_w & \text{at } \quad y = 0 \\ & u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as } \quad y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{\frac{1}{3}}},\ t = t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}},\ Y = y\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},\ K = K_l\left(\frac{\nu}{u_0^2}\right)^{\frac{1}{3}},\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\beta(T_w - T_\infty)}{u_0},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0} \tag{5}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0 & \text{for all} & Y, t \le 0 \\ t > 0: & U = t, & \theta = 1, & C = 1 & \text{at} & Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{as} & Y \to \infty \end{array} \tag{9}$$

The dimensionless governing equations (6) to (8), subject to the initial and boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \mathrm{erfc}\left(\eta\sqrt{Pr}\right) \tag{10}$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{KtSc})\,\mathrm{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\,\mathrm{erfc}(\eta\sqrt{Sc} - \sqrt{Kt})\right] \tag{11}$$

$$\begin{aligned} U = {} & t(1 - a)\left[(1 + 2\eta^2)\,\mathrm{erfc}\,(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] + 2b\,\mathrm{erfc}\,(\eta) \\ & - b\exp(ct)\left[\exp(2\eta\sqrt{ct})\,\mathrm{erfc}\,(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})\,\mathrm{erfc}\,(\mu - \sqrt{ct})\right] \\ & + at\left[(1 + 2\eta^2 Pr)\,\mathrm{erfc}\,(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 Pr)\right] \\ & - b\left[\exp(2\eta\sqrt{KtSc})\,\mathrm{erfc}\,(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\,\mathrm{erfc}\,(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ & + b\exp(ct)\,[\exp(2\eta\sqrt{Sc\,(K + c)t})\,\mathrm{erfc}\,(\eta\sqrt{Sc} + \sqrt{(K + c)t}) \\ & \qquad + \exp(-2\eta\sqrt{Sc\,(K + c)t})\,\mathrm{erfc}\,(\eta\sqrt{Sc} - \sqrt{(K + c)t})] \end{aligned} \tag{12}$$

where, $a = \frac{Gr}{1-Pr}$, $b = \frac{Gc}{2c(1-Sc)}$, $c = \frac{K\,Sc}{1-Sc}$ and $\eta = Y/2\sqrt{t}$.

4. RESULTS AND DISCUSSION

For physical interpretation of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, K and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air ($Pr = 0.71$). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, chemical reaction parameter, Schmidt number and time are studied graphically.

Figure 1 demonstrates the effect velocity fields for different thermal Grashof number ($Gr = 2,5$), mass Grashof number ($Gc = 5,10$), $K = 0.2$ and $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. Figure 2 represents the effect of velocity profiles for different Schmidt number ($Sc = 0.16, 0.3, 0.6$), $Gr = 5, Gc = 5, K = 0.2$ and $t = 0.2$. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number. The velocity velocity profiles for different values of $(t = 0.2, 0.4, 0,6)$, $K = 0.2$

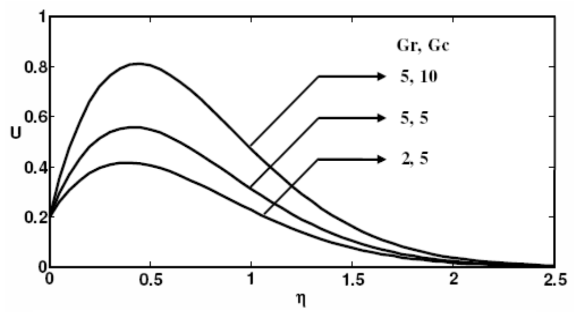

Figure 1: Velocity profile for different values of Gr, Gc

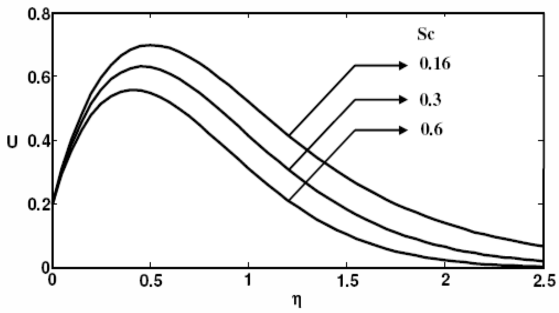

Figure 2: Velocity profile for different values of Sc

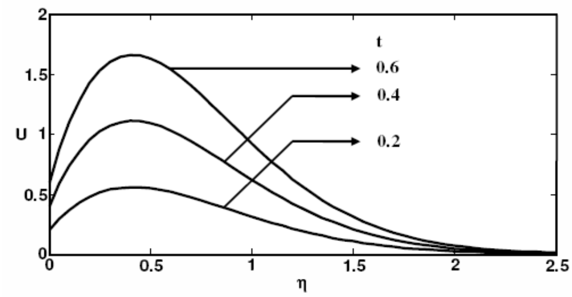

Figure 3: Velocity profile for different values of t

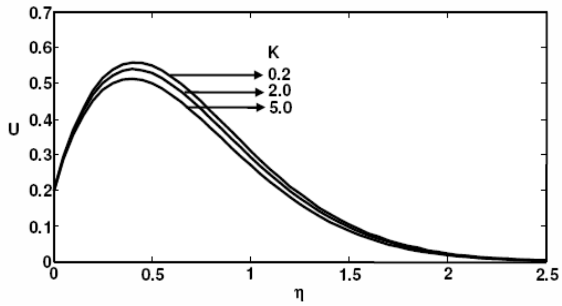

Figure 4: Velocity profile for different values of K

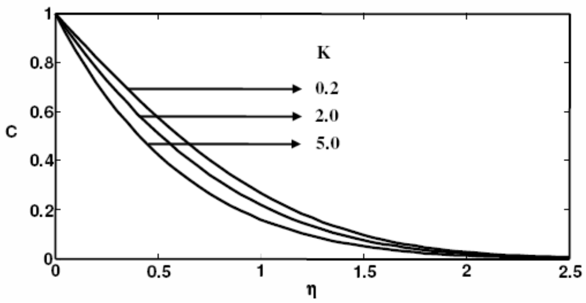

Figure.5: Concentration profile for different values of K

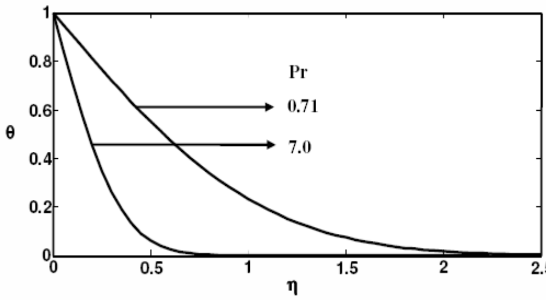

Figure 6: Temperature profile for different values of Pr

$Gr = Gc = 5$ are studied and presented in figure 3. It is observed that the velocity increases with increasing values of the time t. Figure 4 illustrates the effect of velocity for different values of the chemical reaction parameter ($K = 0.2, 2, 5$), $Gr = 5, Gc = 5$ and $t = 0.2$. This shows that the increase in the chemical reaction parameter leads to a fall in the velocity.

The effect of concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5, 10$) and time $t = 0.2$ are presented in figure 5. The effect of the chemical reaction parameter is dominant in concentration field.The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the chemical reaction parameter. It is observed that the concentration increases with decreasing chemical reaction parameter. The temperature profiles for air ($Pr = 0.71$) and water ($Pr = 7.0$) are studied in figure 6. It is observed that the heat transfer is more in air than in water. It is clear that there is a sudden drop in temperature in water compared to that in air.

5. CONCLUSION

The theoretical solution of flow past an uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the chemical reaction parameter.

NOMENCLATURE

C'	concentration $kg.m^{-3}$
C	dimensionless concentration $kg.m^{-3}$
D	mass diffusion coefficient $m^2.s^{-1}$
g	acceleration due to gravity $m^2.s^{-1}$
Gr	thermal Grashof number
Gc	mass Grashof number
Pr	Prandtl number
k	thermal conductivity of the fluid $W.m^{-1}.K^{-1}$
K_l	chemical reaction parameter
K	dimensionless chemical reaction parameter
Sc	Schmidt number
T	temperature K
t'	time S
t	dimensionless time S
u_0	amplitude of the oscillation
u	velocity component in x-direction $m.s^{-1}$
U	dimensionless velocity component in x-direction $m.s^{-1}$
x	spatial coordinate along the plate
y	spatial coordinate normal to the plate
Y	dimensionless spatial coordinate normal to the plate

Greek symbols

α	thermal diffusivity
β	coefficient of volume expansion
β^*	volumetric coefficient of expansion with concentration
η	similarity parameter
μ	coefficient of viscosity $Pa.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid
θ	dimensionless temperature K

Subscripts

w	conditions on the wall
∞	free stream conditions

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