The paper presents reliability evaluation models of the pumping system for slag and ashes discharge, from thermo-electric power plants (TPP). The paper it is structured in four parts. The first two parts present the system reliability modelling using the Markov model, respectively binomial model. The time and outflow availability modelling of analyzed system are represented in the third part. The last part presents the conclusions. In order to facilitate the understanding of the models it has been concretized with reference to the slag and ashes exhausting system from CET I Oradea, equipped with Bagger pumps.

**Keywords:** hydro mechanical systems, reliability modeling, Markov model, binomial model, time and outflow availability.

1. INTRODUCTION

Usually, the slag and ashes which result from coal burning, are evacuated using the Bagger pumps. For thermo-electric power plants it has been established that the slag and ashes continuous evacuation directly conditioning the cauldron working. Therefore, the number of Bagger pumps establishment and their drive back connections, represent the subject for the technical and economic reliability and optimization calculus.

The reliability modeling, it has been made for the functioning configuration in which the Bagger pumps stations (BgPS) are "n+k" systems (n in work, k in reserve). For forecasting reliability analyzing the most used methods are [1, 2, 5, 6]:

- the binomial method, where the elements are characterized by states probabilities (p,q);
- the Markov method with continuous parameter, where the elements are characterized by fundamental reliability indicators ($\lambda_i$, $\mu_i$).

2. THE MARKOV METHOD USED TO RELIABILITY MODELLING OF SLAG AND ASHES PUMPING SYSTEM FROM CET I ORADEA

There are three Bagger pumps stations provided foe slag and ashes exhausting in CET I Oradea:

- the Bagger station 1: attends the 1, 2 and 3 cauldrons and it is equipped with 5 Bagger pumps;
- the Bagger station 2: attends the 4 and 5 cauldrons and it is equipped with 4 Bagger pumps;
- the Bagger station 3: attends the cauldron 6 and it is equipped with 3 Bagger pumps.

The continuous and safety functioning of Bagger pumps it is very important for continuous and nominal output functioning cauldrons.

For Bagger pumps dimensioning like "n+k" systems, the forecasting reliability indicators calculus are following presented.

**a). The Bagger station 1** has 5 Bagger pumps, SIGMA 250-NBA-580 type and an outflow of Q = 800 m$^3$/h. Functioning configurations is "3+2" (3 in work and 2 in reserve). Because the groups are identical it has been admitted the same values for reliability indicators. The total number states of a system with 5 elements are 2$^5$=32. In this case...
(identical elements) states are merged and RED (reliability equivalent diagram) is represented in figure [2, 3]:

![Rediagram](image)

**Figure 1.** The RED of “3+2” system

The states graph is presented in figure 2:

![Diagram](image)

**Figure 2.** The states graph of “3+2” system

The transition intensities matrix \([q_{ij}]\) has the 6th rank and it is \([1, 2, 3, 5, 6]\):

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & -5\lambda & \mu & - & - & - \\
2 & 5\lambda & -4\lambda & 2\mu & - & - \\
3 & -4\lambda & -2\mu-3\lambda & 3\mu & - & - \\
4 & - & 3\lambda & -3\mu-2\lambda & 4\mu & - \\
5 & - & - & 2\lambda & -4\mu-\lambda & 5\mu \\
6 & - & - & - & \lambda & -5\mu \\
\end{array}
\]

The equations system:

\[
\begin{align*}
-5\lambda p_1 + \mu p_2 &= 0 \\
5\lambda p_1 - (\mu + 4\lambda)p_2 + 2\mu p_3 &= 0 \\
4\lambda p_2 - (2\mu + 3\lambda)p_3 + 3\mu p_4 &= 0 \\
3\lambda p_3 - (3\mu + 2\lambda)p_4 + 4\mu p_5 &= 0 \\
2\lambda p_4 - (4\mu + \lambda)p_5 + 5\mu p_6 &= 0 \\
\lambda p_5 - 5\mu p_6 &= 0 \\
\sum_{i=1}^{6} p_i &= 1
\end{align*}
\]

The system solution leads to probability vector determination \([p_i], i=1\div6\) with which the reliability indicators are calculated.

Similarly the other Bagger pumps stations from slag and ashes exhausting system will be analyzed and the numerical data will be tabular represented.

**b) The Bagger station 2** has 4 Bagger pumps, SIGMA 250-NBA-580 type and an outflow of \(Q = 800 \text{ m}^3/\text{h}\). Functioning configuration is ”2+2” (2 in work, 2 in reserve).

The states graph is presented in figure 3:

![Diagram](image)

**Figure 3.** The states graph of ”2+2” system
The equations system:

\[
\begin{align*}
-4\lambda p_1 + \mu p_2 &= 0 \\
4\lambda p_1 - (\mu + 3\lambda)p_2 + 2\mu p_3 &= 0 \\
3\lambda p_2 - (2\mu + 2\lambda)p_3 + 3\mu p_4 &= 0 \\
2\lambda p_3 - (3\mu + \lambda)p_4 + 4\mu p_5 &= 0 \\
\lambda p_4 - 4\mu p_5 &= 0 \\
\sum_{i=1}^{5} p_i &= 1
\end{align*}
\]

The system solution leads to probability vector determination \([p_i], i=1\div5\) with which the reliability indicators are calculated.

c) The Bagger station 3 has 3 Bagger pumps, SIGMA 250-NBA-580 type and an outflow of \(Q = 800 \text{ m}^3/\text{h}\). Functioning configuration is ”1+2” (1 in work, 2 in reserve).

The states graph is presented in figure 4:

![Figure 4](image_url)

**Figure 4.** The states graph of ”1+2” system

The equations system:

\[
\begin{align*}
-3\lambda p_1 + \mu p_2 &= 0 \\
3\lambda p_1 - (\mu + 2\lambda)p_2 + 2\mu p_3 &= 0 \\
2\lambda p_2 - (2\mu + \lambda)p_3 + 3\mu p_4 &= 0 \\
\lambda p_3 - 3\mu p_4 &= 0 \\
\sum_{i=1}^{4} p_i &= 1
\end{align*}
\]

The system solution leads to probability vector determination \([p_i], i=1\div4\) with which the reliability indicators are calculated.

The states grouping for each one of the pumping stations is done in the following way:

- **BgPS 1**: \([S = [S_1, S_2, S_3], R = [S_4, S_5, S_6]]\)
- **BgPS 2**: \([S = [S_1, S_2, S_3], R = [S_4, S_5]]\)
- **BgPS 3**: \([S = [S_1, S_2, S_3], R = [S_4]]\)

The calculus expressions of reliability indicators for the Bagger pumps stations are represented in table 1.

<table>
<thead>
<tr>
<th>The reliability indicators</th>
<th>Bagger pumps stations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BgPS 1</strong></td>
<td><strong>BgPS 2</strong></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(P_S)</td>
<td>(3 \sum_{i=1}^{3} p_i)</td>
</tr>
<tr>
<td>(P_R)</td>
<td>(6 \sum_{i=4}^{5} p_i)</td>
</tr>
</tbody>
</table>

Table 1. The calculus of states probabilities and reliability indicators for the Bagger pumps stations.
Table 1 (continuation)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha(T_A) )</td>
<td>( \sum_{i=1}^{3} p_i \cdot T_A )</td>
<td>( \sum_{i=1}^{3} p_i \cdot T_A )</td>
<td>( \sum_{i=1}^{3} p_i \cdot T_A )</td>
<td></td>
</tr>
<tr>
<td>( \beta(T_A) )</td>
<td>( \frac{6}{p_4} \cdot T_A )</td>
<td>( \frac{5}{p_4} \cdot T_A )</td>
<td>( \frac{3}{p_4} \cdot T_A )</td>
<td></td>
</tr>
<tr>
<td>( \nu(T_A) )</td>
<td>( 3p_3 \lambda T_A )</td>
<td>( 2p_3 \lambda T_A )</td>
<td>( p_3 \lambda T_A )</td>
<td></td>
</tr>
<tr>
<td>MTBF</td>
<td>( \frac{3}{p_4} \cdot \sum_{i=1}^{3} p_i )</td>
<td>( \frac{2}{p_4} \cdot \sum_{i=1}^{3} p_i )</td>
<td>( \frac{3}{p_4} \cdot \sum_{i=1}^{3} p_i )</td>
<td></td>
</tr>
<tr>
<td>MTM</td>
<td>( \frac{6}{p_4} \cdot \sum_{i=1}^{3} p_i )</td>
<td>( \frac{5}{p_4} \cdot \sum_{i=1}^{3} p_i )</td>
<td>( \sum_{i=1}^{3} p_i )</td>
<td></td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>( \frac{3}{p_4} \lambda T_A )</td>
<td>( \frac{2}{p_4} \lambda T_A )</td>
<td>( \frac{3}{p_4} \lambda T_A )</td>
<td></td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>( \frac{6}{p_4} \lambda T_A )</td>
<td>( \frac{6}{p_4} \lambda T_A )</td>
<td>( \frac{6}{p_4} \lambda T_A )</td>
<td></td>
</tr>
</tbody>
</table>

Admitting the values of Bagger pumps fault mean rate, respectively recovery mean rate from [7]: \( \lambda_{BGP} = 40 \cdot 10^{-4} \text{ h}^{-1} \); \( \mu_{BGP} = 119 \cdot 10^{-4} \text{ h}^{-1} \) the following results in table 2 had been obtained.

Table 2. Numerical values of reliability indicators for Bagger pumps stations

<table>
<thead>
<tr>
<th>The reliability indicators</th>
<th>Bagger pumps stations</th>
<th>BgPS 1</th>
<th>BgPS 2</th>
<th>BgPS 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( P_S )</td>
<td></td>
<td>0.8934395</td>
<td>0.9485405</td>
<td>0.9840781</td>
</tr>
<tr>
<td>( P_R )</td>
<td></td>
<td>0.1065605</td>
<td>0.0514595</td>
<td>0.0159216</td>
</tr>
<tr>
<td>( \alpha(T_A) ) [h]</td>
<td></td>
<td>7826.53</td>
<td>8309.2148</td>
<td>8620.5242</td>
</tr>
<tr>
<td>( \beta(T_A) ) [h]</td>
<td></td>
<td>933.46998</td>
<td>450.78522</td>
<td>139.47322</td>
</tr>
<tr>
<td>[faults per year]</td>
<td></td>
<td>27,8477</td>
<td>9,958424</td>
<td>4,979212</td>
</tr>
<tr>
<td>MTBF [h]</td>
<td></td>
<td>281,04692</td>
<td>834.39054</td>
<td>1731.3029</td>
</tr>
<tr>
<td>MTM [h]</td>
<td></td>
<td>33,520457</td>
<td>45,266723</td>
<td>28,01103</td>
</tr>
<tr>
<td>( \lambda_s ) [h⁻¹]</td>
<td></td>
<td>3.5581158 \cdot 10^{-3}</td>
<td>1.1984796 \cdot 10^{-3}</td>
<td>5.7759968 \cdot 10^{-4}</td>
</tr>
<tr>
<td>( \mu_s ) [h⁻¹]</td>
<td></td>
<td>0.0298325</td>
<td>0.0220912</td>
<td>0.0357001</td>
</tr>
</tbody>
</table>

3. FORECASTING RELIABILITY EVALUATION OF BgPS USING BINOMIAL METHOD

The binomial method appeals to an easier mathematical model than the Markov method.

In this case for reliability indicators evaluation, we must start from the binomial theorem expression. For "n+k" BgPS type is:

\[
(p + q)^{n+k}
\]  \( (5) \)

The reliability indicators evaluation has been made by the following relations: The time safety of system with "n" groups in work (successfully probability) is:

\[
P_S = \sum_{i=n}^{n+k} C_{n+k}^i \cdot p^i \cdot (1-p)^{n+k-i}
\]  \( (6) \)
The time safety of BgPS with \(n+k-j\) groups in work is:

\[
P_{n+k-j} = \sum_{i=n}^{k-j} C_{n+k-j}^i \cdot p^i (1-p)^{n+k-j-i} \quad \text{cu} j \leq k \tag{7}
\]

The feasible states of BgPS I are presented in tables 3 and 4. The functioning probability, respectively the failure probability for Bagger pumps, including the electrical equipment are \([7], p_{BgP}=0.748; q_{BgP}=0.252\).

Table 3. The feasible states of BgPS 1

<table>
<thead>
<tr>
<th>State nr.</th>
<th>BgP state</th>
<th>State probability</th>
<th>Annual mean time [h/\text{an}]</th>
<th>Achieved mean outflow [m³/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(s)</td>
<td>3 2 -</td>
<td>(p^5)</td>
<td>(p^5 \cdot T_A)</td>
<td>(3Q_{BgP})</td>
</tr>
<tr>
<td>2(s)</td>
<td>3 1 1 3</td>
<td>(5p^4 q)</td>
<td>(5p^4 q \cdot T_A)</td>
<td>(3Q_{BgP})</td>
</tr>
<tr>
<td>3(s)</td>
<td>3 - 2</td>
<td>(10p^3 q^2)</td>
<td>(10p^3 q^2 \cdot T_A)</td>
<td>(3Q_{BgP})</td>
</tr>
<tr>
<td>4(sp)</td>
<td>2 - 3</td>
<td>(10p^2 q^3)</td>
<td>(10p^2 q^3 \cdot T_A)</td>
<td>(2Q_{BgP})</td>
</tr>
<tr>
<td>5(sr)</td>
<td>1 - 4</td>
<td>(5pq^4)</td>
<td>(5pq^4 \cdot T_A)</td>
<td>(1Q_{BgP})</td>
</tr>
<tr>
<td>6(r)</td>
<td>- - 5</td>
<td>(q^5)</td>
<td>(q^5 \cdot T_A)</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4. Numerical values of the BgPS 1 feasible states

<table>
<thead>
<tr>
<th>State nr.</th>
<th>BgP state</th>
<th>State probability</th>
<th>Annual mean time [h/\text{an}]</th>
<th>Achieved mean outflow [m³/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(s)</td>
<td>3 2 -</td>
<td>0.2341574</td>
<td>2226</td>
<td>2400</td>
</tr>
<tr>
<td>2(s)</td>
<td>3 1 1 3</td>
<td>0.3944363</td>
<td>3455</td>
<td>2400</td>
</tr>
<tr>
<td>3(s)</td>
<td>3 - 2</td>
<td>0.2657999</td>
<td>2328</td>
<td>2400</td>
</tr>
<tr>
<td>4(sp)</td>
<td>2 - 3</td>
<td>0.0895374</td>
<td>784</td>
<td>1600</td>
</tr>
<tr>
<td>5(sr)</td>
<td>1 - 4</td>
<td>0.0150825</td>
<td>132</td>
<td>800</td>
</tr>
<tr>
<td>6(r)</td>
<td>- - 5</td>
<td>1,01625·10^{-3}</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

rs – the reserve state; sp – the partial success (66.6 %); sr – the reduced success (33.3 %); \(Q_{BgP}\) – the pump outflow

The successfully probabilities expressions for the other pumping systems are given in table 5. The reliability indicators calculus are made according to the previous models.

Table 5. Numerical values of successfully probability for pumping systems

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Bagger station</th>
<th>Configuration</th>
<th>The indicator (P_S) relation</th>
<th>Numerical results for (P_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>SPBg 2</td>
<td>4xSIGMA</td>
<td>(p^4 + 4p^3 q + 6p^2 q^2)</td>
<td>0.9480862</td>
</tr>
<tr>
<td>2.</td>
<td>SPBg 3</td>
<td>3xSIGMA</td>
<td>(p^3 + 3p^2 q + 3pq^2)</td>
<td>0.9839969</td>
</tr>
</tbody>
</table>

Corroborating with groups outflow the availability indicators of BgSP can be calculated:

- The successfully probability is:
  \[
P_S = p^5 + 5p^4 q + 10p^3 q^2 = 0.8943636 \tag{8}
\]

- The failure probability is:
  \[
P_R = 20p^2 q^3 + 5pq^4 + q^5 = 0.1056364 \tag{9}
\]

- The medium number of functioning groups, respectively the failures groups:
  \[
m_f = 5p \approx 4; m_q = 5q \approx 1 \tag{10}
\]

- The standard deviation in comparison with the mean value (\(m_i\)):
  \[
  \sigma = \sqrt{5pq} = 0.94248 \tag{11}
\]

- The pumping volume during the analysis interval:
\[ V_p = \sum_{i=1}^{5} Q_i \cdot T_i = 20 \cdot 10^6 \text{ m}^3 / \text{an} \] (13)

- The unavailable volume during the analysis interval:
  \[ \Delta V_I = V_N - V_p = 4 \cdot Q_{PBg} \cdot T_A - V_p = 8 \cdot 10^6 \text{ m}^3 / \text{an} \] (14)
- The availability and unavailability indicators:
  \[ D_Q = \frac{V_p}{V_N} = 0,7134703 \]
  \[ I_Q = 1 - D_Q = 0,2865298 \] (15)

4. CONCLUSIONS

1. In reference material one cannot find a specific treating (dedicated, adequate, distinct and profound) of BgPS forecasting reliability;
2. For Bagger pumps system reliability evaluation the following models are recommended:
   - the Markov model for "n+k" system ("3+2", "2+2", "1+2"),
   - the binomial model "n+k",
   - the outflow availability and unavailability evaluation, using the binomial;
3. With reference to the numerical results obtained for CET I Oradea BgPS it has been ascertained a better behavior of Bagger station 3, from the reliability point of view.

REFERENCES