

MAGNETIC FLUID BASED SQUEEZE FILM BETWEEN ANNULAR PLATES AND TRANSVERSE SURFACE ROUGHNESS EFFECT

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Abstract:

An endeavor has been made to analyze the behavior of a magnetic fluid based squeeze film between transversely rough annular plates. The bearing surfaces are assumed to be transversely rough. The stochastic film thickness characterizing the random roughness is considered to be asymmetric with non-zero mean and variance. A magnetic fluid is used as a lubricant and the external magnetic field is oblique to the lower plate. The associated Reynolds' equation is stochastically averaged with respect to the random roughness parameter. This equation is then solved with suitable boundary conditions to obtain the bearing performance characteristics, such as load carrying capacity and response time. The results are presented graphically. It is clearly seen that the transverse surface roughness adversely affects the bearing system. The results show that magnetic fluid lubricant increases the load carrying capacity significantly. In addition, the load carrying capacity increases due to the negatively skewed roughness. Same is the case of negative variance. It is observed that the negative effect induced by the standard deviation and variance(+ve) can be compensated up to a considerable extent by the positive effect of magnetization in the case of negatively skewed roughness. Results of the present investigation tend to suggest that the roughness must be accorded due consideration while designing such bearing system.

Keywords:

Squeeze film, magnetic fluid lubricant, transverse roughness, Reynolds' equation and load carrying capacity.

1. THE INTRODUCTION

The performance of a squeeze film between various geometrical configurations of flat surfaces was analyzed by Archibald [3]. Wu [27] dealt with the squeeze film behavior between porous annular irrotational disks. This analysis was modified by Gupta & Sinha [12] by considering the effect of axial current. Ting [23] improved the analysis of Gupta & Sinha [12] considerably, taking only the lower disk to be rotating. Murti [16] discussed the squeeze film between curved circular plates describing the film thickness by an exponential expression. Following the analysis of Murti [16], Gupta & Vora [13] studied the squeeze film behavior between curved annular plates. However, in the above analysis the lower plate was considered to be flat. Ajwalia [2] extended the investigation of Gupta & Vora [13] by taking lower plate along a surface generated by an exponential form. Patel & Deheri [20] investigated the configuration of Gupta & Vora [13] by considering the lower plate as well as the upper plate along the surfaces generated by hyperbolic form. Subsequently, Patel & Deheri [17] modified the approach of Ajwalia [2] to consider both plates along the surfaces determined by secant forms. They concluded that the performance improved in comparison with that of Ajwalia [2]. Lin et. al [15] analyzed magneto-hydrodynamic squeeze film characteristics between curved annular plates wherein, the magnetic field effect characterized by the Hartmann number provided an enhancement to the load carrying capacity. All the above studies considered conventional lubricant.

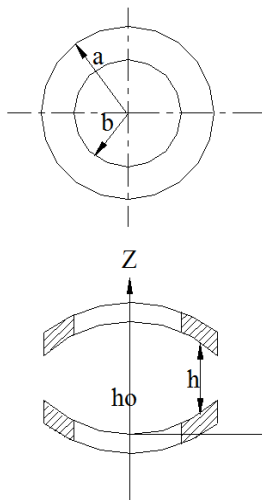
Use of magnetic fluid as a lubricant modifying the performance of the bearing has been very well recognized. Verma [26] investigated the application of magnetic fluid as a lubricant taking into account the tangential slip velocity at the porous matrix lubricated interface, while Agrawal [1] discussed the effect of magnetic fluid by considering no slip condition. Bhat & Deheri [4, 5] analyzed the performance of a magnetic fluid based squeeze film behavior between curved annular disks and curved circular plates and found that its performance with the magnetic fluid as lubricant was relatively better than with a conventional lubricant. Patel & Deheri [19] studied the magnetic fluid based squeeze film between curved annular plates. But we know that bearing surfaces particularly,

after having some run in and wear develop roughness. Various methods have been proposed to study and analyze the effect of surface roughness of the bearing surfaces on the performance of squeeze film bearings. Several investigators have adopted a stochastic approach to mathematically model the randomness of the roughness (Tzeng & Seibel [24], Christensen & Tonder [7, 8, 9]). A comprehensive general analysis was presented by Christensen & Tonder [7, 8, 9] for surface roughness (both transverse as well as longitudinal) based on a general probability density function by modifying the approach of Tzeng and Seibel [24]. Later on this approach of Christensen & Tonder [7, 8, 9] laid down the basis of the analysis to discuss the effect of surface roughness on the performance of the bearing system in a number of investigations (Ting [23], Prakash & Tiwari [22], Prajapati [21], Guha[11] and Gupta & Deheri [14]).

Although, the effect of the transverse roughness is adverse in general, the investigation by Deheri, Patel & Patel [10] reported that the negatively skewed roughness induced a better performance. Hence, it was deemed appropriate to launch an investigation into the performance of a magnetic fluid based squeeze film between transversely rough annular plates, by incorporating some special boundary conditions commensurations with physical aspects.

2. THE ANALYSIS

The configuration of the bearing system presented below consists of the annular disks.



The upper face moves normal towards the lower disk with uniform velocity $\dot{h} = \frac{dh}{dt}$. Both the disks are considered to have transversely rough surfaces. Assuming an axially symmetric flow of the magnetic fluid lubricant between the disks under an oblique magnetic field $\vec{H}$ whose magnitude H is a function of r vanishing at $r = a$ (outer radius) and $r = b$ (inner radius), the modified Reynolds' equation governing the film pressure (Agrawal [1], Patel and Deheri [18] and Vadher et.al [25]) is obtained as

$$\frac{1}{r} \frac{d}{dr} \left[Ar \frac{d}{dr} (p - 0.5 \mu_0 \bar{\mu} H^2) \right] = 12 \mu \dot{h} \quad \dots \dots \dots (1)$$

$$H^2 = (a - r)(r - b); \quad b < r < a,$$

$$A = h^3 + 3h^2\alpha + 3h(\sigma^2 + \varepsilon) + 3\sigma^2\alpha + \alpha^3,$$

μ_0 is permeability of the free space, $\bar{\mu}$ is the magnetic permeability, μ is the viscosity of the fluid and the film thickness has been taken as $h(r,t) + h_s(r,\xi)$, where in h denotes the smooth part of the film thickness and h_s is the part due to surface roughness measured from the mean level and its random character is described by the variable ξ . σ , α and ε are the standard deviation, variance and measure of symmetry respectively of the stochastic film thickness distribution defined by the relationships

$$\alpha = E(h_s)$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by $E(R) = \int_{-c}^c R f(h_s) dh_s$

$$\text{while } f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^{3/2}; & -c \leq h_s \leq c \\ 0, & \text{elsewhere} \end{cases}$$

The boundary conditions $p(a) = 0$

$$\text{and} \quad \left(\frac{dp}{dr} \right)_{r=b} = \frac{\pi}{2b} (b^2 - a^2) \mu_0 \bar{\mu} \quad \dots \dots \dots (2)$$

are taken from the discussion carried out in [6] from bearings longevity point of view. Integrating the modified Reynolds' equation (1) under the boundary conditions (2) one finds that

$$p = 0.5 \mu_0 \bar{\mu} (a-r)(r-b) + \frac{3\mu \dot{h} (r^2 - a^2)}{A} + \left\{ -\frac{\pi \mu_0 \bar{\mu} (a^2 - b^2)}{2} - 0.5 \mu_0 \bar{\mu} (a-b)b - \frac{6\mu \dot{h} b^2}{A} \right\} \log \left(\frac{r}{a} \right) \quad \dots \dots (3)$$

Introducing the non-dimensional quantities

$$R = \frac{r}{b}, \quad \sigma^* = \frac{\sigma}{h_0}, \quad \varepsilon^* = \frac{\varepsilon}{h_0^3},$$

$$\mu^* = -\frac{\mu_0 \bar{\mu} h^3}{\mu \dot{h}}, \quad \alpha^* = \frac{\alpha}{h_0}, \quad k = \frac{a}{b}$$

$$P = -\frac{h^3 p}{\pi \mu \dot{h} (a^2 - b^2)}$$

the pressure distribution in dimensionless form is calculated from equation (3) as

$$P = \frac{\mu^* (k-R)(R-1)}{2\pi (k^2-1)} - \frac{3}{\pi B} \frac{(R^2 - k^2)}{(k^2-1)} + \log \left(\frac{R}{k} \right) \left\{ \frac{\mu^*}{2} - \frac{\mu^*}{2\pi(k+1)} + \frac{6}{\pi B(k^2-1)} \right\} \quad \dots \dots (4)$$

where

$$B = 1 + 3\alpha^* + 3(\alpha^{*2} + \sigma^{*2}) + \varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3}$$

The load carrying capacity of the bearing is given by $w = 2\pi \int_b^a r p(r) dr$ whose non-dimensional

$$\text{form is obtained as } W = -\frac{h^3 w}{\pi \mu \dot{h} (a^2 - b^2)^2} = \frac{\mu^*}{12} (k-1)^2 - \frac{3}{2B} (1-k^2) - \left[\frac{1}{2} \ln k - \frac{1}{4} (k^2-1) \right] \left[\pi \mu^* + \frac{12}{B(k^2-1)} - \frac{\mu^*}{k+1} \right] \quad \dots \dots (5)$$

Lastly, the response time Δt taken by the upper plate to reach a film thickness h_2 at t_2 starting from h_1 at t_1 is given by $\Delta t = -\frac{\mu W (a^2 - b^2)^2}{w} \int_{h_1}^{h_2} \frac{1}{g(h)} dh$,

where $g(h) = h^3 + 3h^2\alpha + 3h(\alpha^2 + \sigma^2) + \varepsilon + 3\sigma^2\alpha + \alpha^3$,

This leads to the dimensionless form of the response time given by $\Delta T = -\frac{w h_0^2 \Delta t}{\pi \mu (a^2 - b^2)^2}$

$$= -W \int_{\bar{h}_1}^{\bar{h}_2} \frac{1}{g(\bar{h})} d\bar{h} \quad \dots \dots (6)$$

where $g(\bar{h}) = \bar{h}^3 + 3\sigma^{*2} \bar{h} + 3\alpha^* \bar{h}^2 + 3\alpha^{*2} \bar{h} + 3\sigma^{*2} \alpha^* + \alpha^{*3} + \varepsilon^*$

and $\bar{h}_1 = \frac{h_1}{h_0}$, $\bar{h}_2 = \frac{h_2}{h_0}$, $\bar{h} = \frac{h}{h_0}$

3. THE RESULTS AND DISCUSSION

It is clear that the dimensionless film pressure, load carrying capacity and response time are determined from equations (4), (5) and (6) respectively. Further, it is easily seen that these performance characteristics depend on various parameters such as $\mu^*, \sigma^*, \alpha^*, \varepsilon^*$ and k . These parameters characterize the effect of magnetic fluid lubricant, surface roughness and aspect ratio respectively. It appears that for a bearing with smooth surfaces this study reduces to the investigation [5] of Bhat & Deheri's magnetic fluid based squeeze film between annular plates in the limiting case μ^* tending to zero in the absence of porosity.

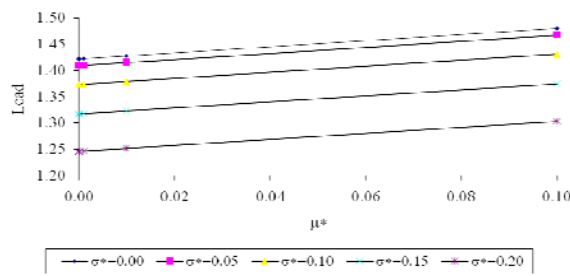


Figure: 1 Variation of load carrying capacity with respect to μ^* and σ^*

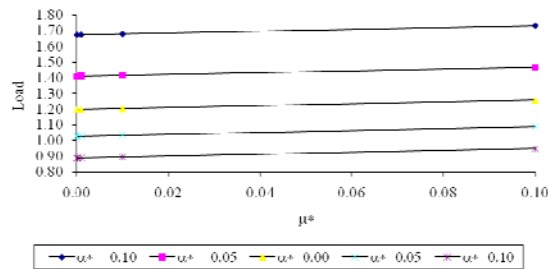


Figure: 2 Variation of load carrying capacity with respect to μ^* and α^*

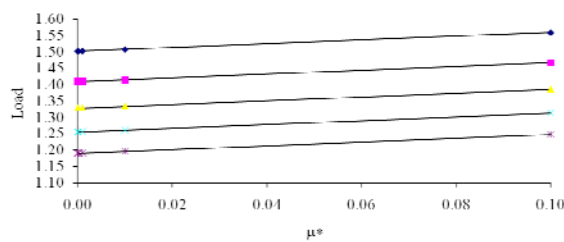


Figure: 3 Variation of load carrying capacity with respect to μ^* and ε^*

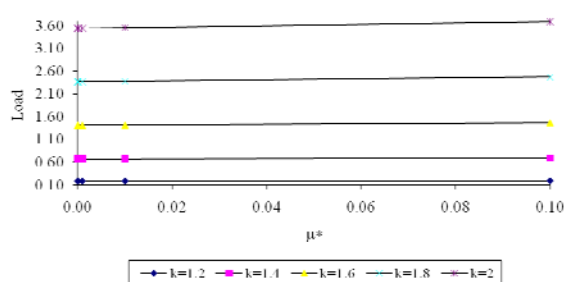


Figure: 4 Variation of load carrying capacity with respect to μ^* and k

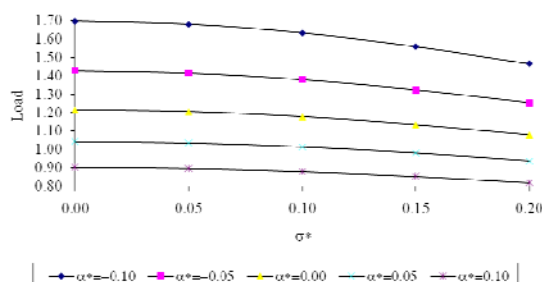


Figure: 5 Variation of load carrying capacity with respect to σ^* and α^*

Fig. (1) - (4) present the variation of load carrying capacity with respect to the magnetization parameter for different values of roughness parameters and the aspect ratio. It is observed that the load carrying capacity registers an increase due to the magnetization. The standard deviation, (+ve) variance and (+ve) skewness decrease the load carrying capacity while negatively skewed roughness and negative variance tend to increase the load carrying capacity. Besides, the load carrying capacity increases owing to the aspect ratio. Furthermore, a comparison of Fig. (1) - (4) indicates that the effect of magnetization is almost negligible in the case of aspect ratio.

We have the distribution of load carrying capacity with respect to the standard deviation for various values of the other two roughness parameters and the aspect ratio in Fig.(5) - (7). These figures make it clear that the effect of standard deviation is considerably adverse in nature. This negative effect of standard deviation gets further increased due to skewness (+ve) and (+ve) variance. In addition, it is seen that upto certain values of the aspect ratio the effect of standard deviation is almost negligible.

The effect of variance with respect to the skewness and aspect ratio is shown in Fig. (8) - (9) respectively. Fig. (8) makes it clear that the combined effect of ε^* (+ve) and α^* (+ve) is considerably negative while the combined effect of variance (-ve) and negatively skewed roughness is strongly positive. Fig. (9) suggests that even for a moderate value of α^* the aspect ratio increases the load carrying capacity sharply. Also, increase due to α^* (-ve) is relatively more as compared to the positive effect of the aspect ratio. Finally, the combined effect of the aspect ratio and negatively skewed roughness is almost identical with the combined effect of aspect ratio and variance which is the message from Fig. (10) in conjunction with Fig. (9).

As indicated by equation (8), the response time more or less trades the path of the load carrying capacity.

A comparison of this study with the investigation carried out by Bhat & Deheri [5] indicates that the performance of the bearing system can be improved further by suitably choosing the boundary conditions. This article also underlines that in spite of the fact that the surface roughness effect is adverse, the bearing system can be made to perform better by involving the magnetization parameter in the boundary conditions. Even, this arrangement may lead to a longer life period of the bearing system.

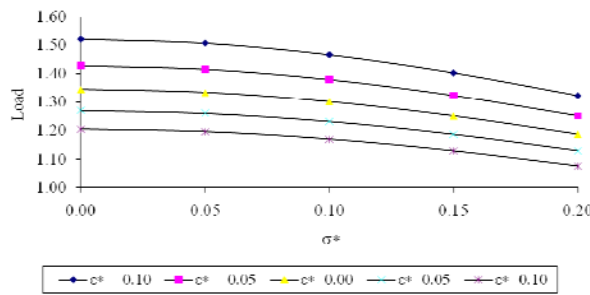


Figure: 6 Variation of load carrying capacity with respect to σ^* and ϵ^*

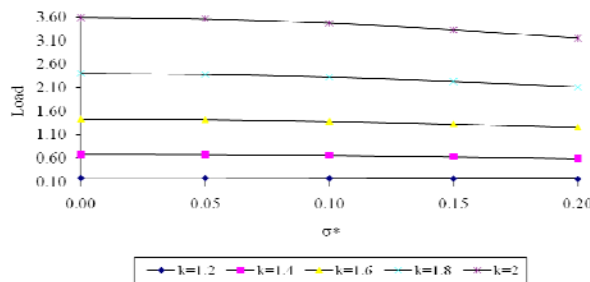


Figure: 7 Variation of load carrying capacity with respect to σ^* and k

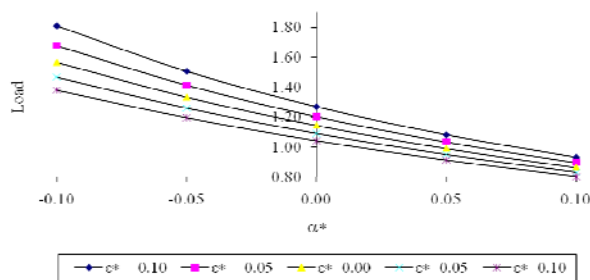


Figure: 8 Variation of load carrying capacity with respect to α^* and ϵ^*

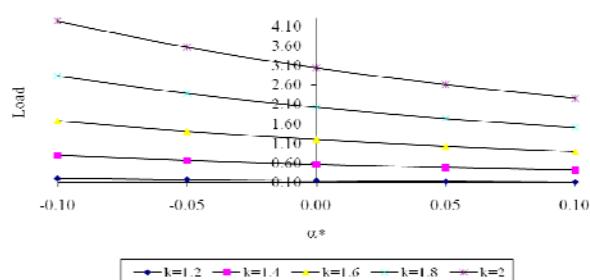


Figure: 9 Variation of load carrying capacity with respect to α^* and k

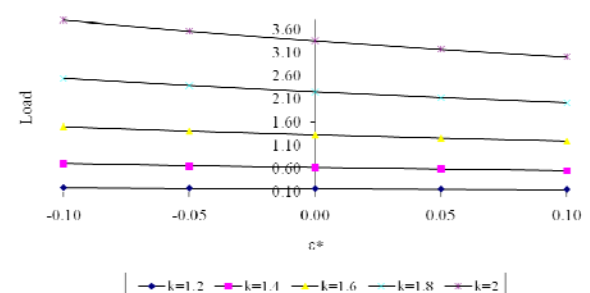


Figure: 10 Variation of load carrying capacity with respect to ϵ^* and k

4. CONCLUSIONS

The negative effect of the surface roughness can be neutralized by taking a suitable combination of the magnetization parameter and aspect ratio in the case of negatively skewed roughness by choosing proper boundary conditions. This study makes it mandatory that due consideration must be given to roughness while designing the bearing system although suitable boundary conditions involving the magnetization parameter are brought in.

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