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# RADIATIVE HEAT TRANSFER EFFECTS ON LINEARLY ACCELERATED ISOTHERMAL VERTICAL PLATE WITH MASS DIFFUSION

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**ABSTRACT:**
Theoretical solution of unsteady flow past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion in the presence of thermal radiation is presented here. The plate temperature is raised to $T_w$ and the concentration level near the plate is also raised to $C_w'$. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, radiation parameter and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. But the trend is just reversed with respect to the thermal radiation parameter.
**Key words:**
accelerated, isothermal, vertical plate, heat transfer, mass diffusion, radiation.

## 1. INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along an isothermal vertical plate was studied by Hossain and Takhar (1996). Raptis and Perdikis(1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al* (1996) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Gupta *et al* [6] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis[7] extended the above problem to include mass transfer effects subjected to variable suction or injection. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar[9]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [8]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Basant Kumar Jha and Ravindra Prasad [1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

Hence, it is proposed to study heat and mass transfer effects on unsteady flow past a uniformly accelerated infinite isothermal vertical plate in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a steady found useful process industries such as wire drawing, fibre drawing, food processing and polymer production.

## 2. ANALYSIS

The unsteady flow of a viscous incompressible fluid past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion in the presence of thermal radiation has been considered. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_\infty$. At time $t' > 0$, the plate is accelerated with a velocity $u = u_0 t'$ in its own plane and the temperature from the plate is raised to $T_w$ and the concentration level near the plate are also raised to $C_w'$. It is assumed that the effect of viscous dissipation is negligible in the energy equation. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_P \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{aligned} &u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad for \ \ all \ \ y, t' \leq 0 \\ &t' > 0: \ u = u_0 t', \quad T = T_w, \quad C' = C'_w \quad at \quad y = 0 \\ &u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad as \quad y \to \infty \end{aligned} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty{}^4 - T^4) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^4$ in a Taylor series about $T_\infty$ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_P \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty{}^3(T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{\frac{1}{3}}}, \quad t = t'\left(\frac{u_0{}^2}{\nu}\right)^{\frac{1}{3}}, \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \ Gr = \frac{g\beta(T_w - T_\infty)}{u_0}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0} \tag{8}$$

$$R = \frac{16a^*\sigma T_\infty{}^3}{k}\left(\frac{\nu}{u_0{}^2}\right)^{\frac{1}{3}}, \quad \Pr = \frac{\mu C_P}{k}, \ Sc = \frac{\nu}{D}$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\,\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} &U = 0,\quad \theta = 0,\quad C = 0 \quad for\ \ all\ \ Y, t \le 0 \\ &t > 0:\ U = t,\quad \theta = 1,\quad C = 1 \quad at\ \ Y = 0 \\ &U \to 0,\ \theta \to 0,\ C \to 0 \ \ as\ \ Y \to \infty \end{aligned} \tag{12}$$

The dimensionless governing equations (9) to (11), subject to the initial and boundary conditions (12) are solved by the usual Laplace-transform technique and the solution are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\ erfc\left(\eta\sqrt{\Pr} + \sqrt{bt}\right) + \exp(-2\eta\sqrt{Rt})\ erfc\left(\eta\sqrt{\Pr} - \sqrt{bt}\right)\right] \tag{13}$$

$$C = erfc\left(\eta\sqrt{Sc}\right) \tag{14}$$

$$\begin{aligned} U = {} & t(1-d)\left[(1+2\eta^2)\,erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] + 2a\ erfc(\eta) \\ & - a\exp(ct)\left[\exp(2\eta\sqrt{ct})\ erfc\left(\eta + \sqrt{ct}\right) + \exp(-2\eta\sqrt{ct})\ erfc\left(\eta - \sqrt{ct}\right)\right] \\ & + dt\left[(1+2\eta^2 Sc)\ erfc\left(\eta\sqrt{Sc}\right) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc)\right] \\ & - a\left[\exp(2\eta\sqrt{Rt})\ erfc\left(\eta\sqrt{\Pr} + \sqrt{bt}\right)\right. \\ & \left. + \exp(-2\eta\sqrt{Rt})\ erfc\left(\eta\sqrt{\Pr} - \sqrt{bt}\right)\right] \\ & + a\exp(ct)\left[\exp(2\eta\sqrt{\Pr(b+c)t})\ erfc\left(\eta\sqrt{\Pr} + \sqrt{(b+c)t}\right)\right. \\ & \left. + \exp(-2\eta\sqrt{\Pr(b+c)t})\ erfc\left(\eta\sqrt{\Pr} - \sqrt{(b+c)t}\right)\right] \end{aligned}$$

where $a = \frac{Gr}{2c(1-\Pr)}$, $b = \frac{R}{\Pr}$, $c = \frac{R}{1-\Pr}$, $d = \frac{Gc}{1-Sc}$ and $\eta = Y/2\sqrt{t}$.

## 4. RESULTS AND DISCUSSION

For physical interpretation of the problem, numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc*, *Pr*, *R* and *t* upon the nature of the flow and transport. The value of the Schmidt number *Sc* is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number *Pr* are chosen such that they represent air (*Pr* = 0.71). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, thermal radiation parameter, Schmidt number and time are studied graphically.

The effect of velocity for different values of the radiation parameter (*R* = 0.2, 5, 20), *Gr* = 2 = *Gc* and *t* = 0.2 are shown in figure 1. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation. The velocity profiles for different time (*t* = 0.2, 0.3, 0.4), *Gr* = *Gc* = 2 and *R* = 10 are studied and presented in figure 2. It is observed that the velocity increases with increasing values of the time *t*.

Figure 3 demonstrates the effect velocity fields for different thermal Grashof number (*Gr* = 2, 5), mass Grashof number (*Gc* = 2, 5), *R* = 20 and *t* = 0.2. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

Figure 4 represents the effect of concentration profiles for different Schmidt number (*Sc* = 0.16, 0.3, 0.6, 2.01) and *t* = 1. The trend shows that the wall concentration increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

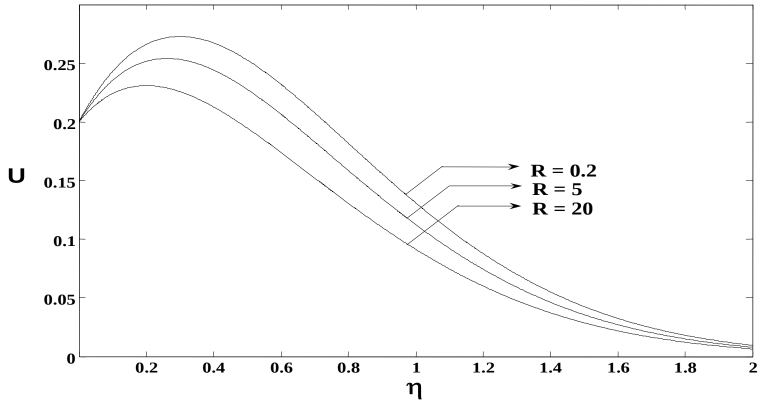


Figure 1.Velocity profiles for different values of R

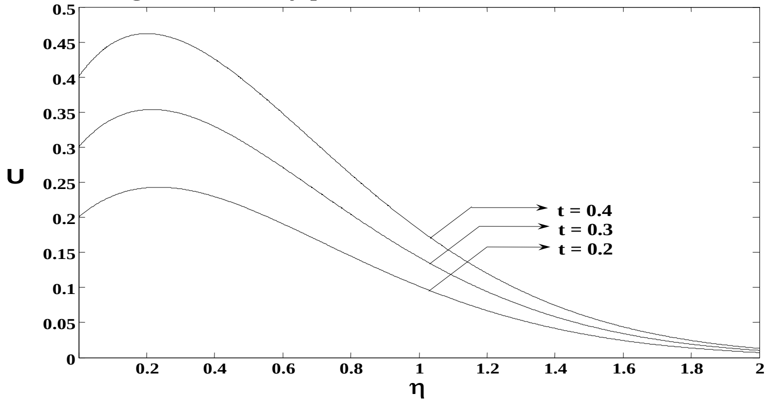


Figure 2. Velocity profiles for different values of t

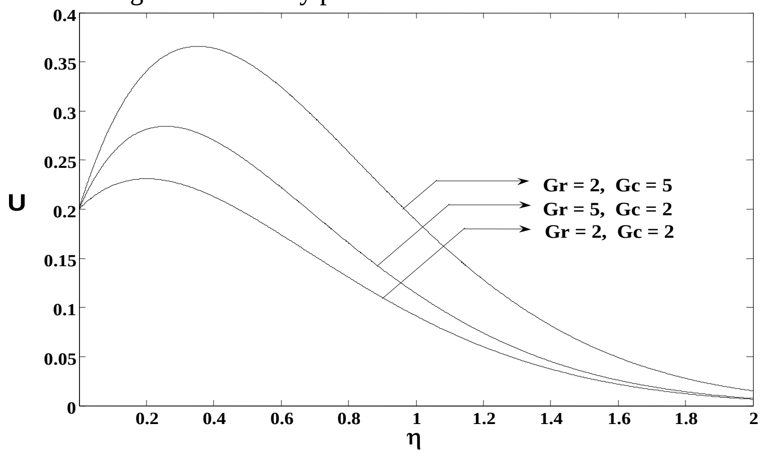


Figure 3.Velocity profiles for different values of Gr, Gc

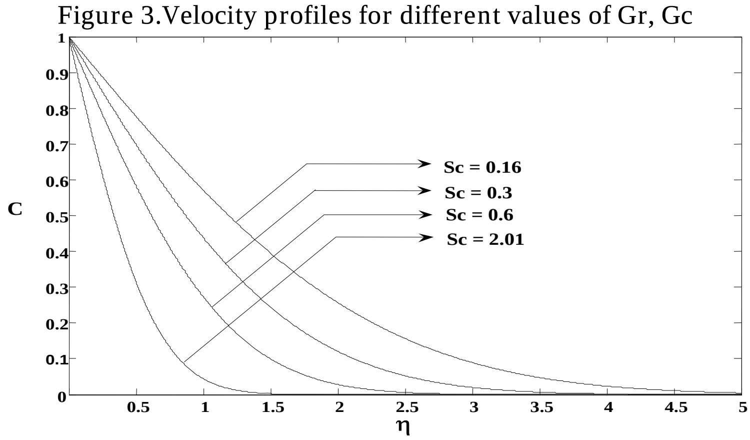


Figure 4. Concentration profiles for different values of Sc

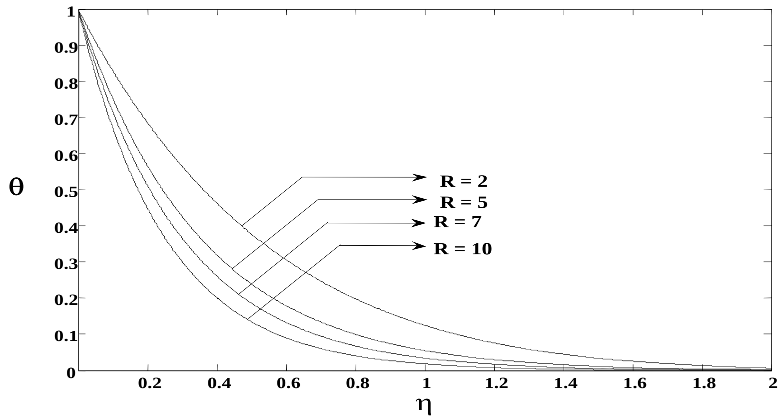


Figure 5. Temperature profiles for different values of R

The temperature profiles are calculated for different values of thermal radiation parameter ($R$ = 2, 5, 7, 10) and time ($t$ = 0.4) from Equation (13) and these are shown in Figure 5. For air ($Pr$ = 0.71). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

## 5. CONCLUSION

The theoretical solution of flow past a uniformly accelerated isothermal infinite vertical plate with uniform mass diffusion, in the presence of thermal radiation has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different physical parameters like thermal Grashof number, mass Grashof number, thermal radiation parameter and $t$ are studied graphically. It is observed that the velocity increases with increasing values of $Gr$, $Gc$ and $t$. But the trend is just reversed with respect to the thermal radiation parameter.

## NOMENCLATURE

*C' - concentration*
*C - dimensionless concentration*
*D - mass diffusion coefficient*
*G - acceleration due to gravity*
*Gr - thermal Grashof number*
*Gc - mass Grashof number*
*Pr - Prandtl number*
*Qr - radiative heat flux in the y- direction*
*R - radiation parameter*
*K - thermal conductivity of the fluid*
*Sc - Schmidt number*
*T - temperature*
*t' - time*
*t - dimensionless time*
*$u_0$ - amplitude of the oscillation*
*u - velocity component in x- direction*
*U - dimensionless velocity component in x- direction*
*x - spatial coordinate along the plate*
*y - spatial coordinate normal to the plate*
*Y - dimensionless spatial coordinate normal to the plate*

***Greek symbols***

$\alpha$ *- thermal diffusivity*
$\beta$ *- coefficient of volume expansion*
$\beta^*$ *- volumetric coefficient of expansion with concentration*
$\eta$ *- similarity parameter*
$\mu$ *- coefficient of viscosity*
$\nu$ *- kinematic viscosity*
$\rho$ *- density of the fluid*
$\theta$ *- dimensionless temperature*

***Subscripts***

*w - conditions on the wall*
$\infty$ *- free stream conditions*

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