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CHEMICAL REACTION EFFECTS ON FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE

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ABSTRACT
Exact solution to the problem of unsteady flow past an infinite vertical exponentially accelerated plate in the presence of uniform heat flux and variable mass diffusion is presented here, taking into account the homogeneous chemical reaction of first order. The governing equations are solved in closed form by the Laplace Transform technique. The influence of various parameters, entering in the problem, on the velocity field, temperature field and concentration field is discussed with the help of graphs.
Keywords
Chemical reaction, Heat and mass transfer, Exponential, Accelerated, Vertical plate.

1. INTRODUCTION

Diffusion rates can be altered tremendously by chemical reactions. The effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production.

Chambre and Young [6] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al. [8] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [9]. The dimensionless governing equations were solved by the usual Laplace Transform technique.

Gupta [2] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [5] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen kumar [1]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [4]. Basant kumar Jha [3] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Recently Muthucumaraswamy et al. [7] studied mass transfer effects on exponentially accelerated isothermal vertical plate.

The objective of the present investigation is to study unsteady flow past an exponentially accelerated infinite vertical plate with uniform heat flux and variable mass diffusion in the presence of a homogeneous chemical reaction of first order. The dimensionless governing equations are solved using the Laplace Transform technique. The solutions for the temperature, concentration and velocity are in terms of exponential and complementary error function

2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform heat flux and mass diffusion is considered. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. Initially the plate and the fluid are at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all points. At time $t'>0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature of the plate is raised at an uniform rate and the level of concentration near the plate is raised linearly with time. All the physical properties of the fluid are considered to be constant except the influence of the body-force term, in accordance with Boussinesq's approximation. The unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r\left(C' - C'_\infty\right) \tag{3}$$

with the following initial and boundary conditions

$$t' \le 0,\ u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \quad \text{for all } y'$$

$$t' > 0,\ u' = u_0 \exp(a't'),\ \frac{\partial T'}{\partial y'} = \frac{-q}{\kappa},\ C' = C'_\infty + \left(C'_w - C'_\infty\right)At' \ \text{ at } y' = 0$$

$$u' = 0,\ T' \to T'_\infty,\ C' \to C'_\infty,\ \text{as } y' \to \infty. \tag{4}$$

where $A = \dfrac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0}, t = \frac{t'u_0^2}{\nu}, y = \frac{y'u_0}{\nu}, \theta = \frac{T' - T'_\infty}{\left(\dfrac{q\nu}{\kappa u_0}\right)}, G_r = \frac{g\beta q\nu^2}{\kappa u_0^4}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$G_m = \frac{g\beta^*\nu\left(C'_w - C'_\infty\right)}{u_0^3}, P_r = \frac{\mu C_p}{\kappa}, S_c = \frac{\nu}{D}, K = \frac{K_r\nu}{u_0^2}, a = \frac{a'\nu}{u_0^2}. \tag{5}$$

In equations (1) to (4), leads to

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - KC \tag{8}$$

The initial and boundary conditions in non-dimensional form are

$$t \le 0: \quad u = 0,\ \theta = 0,\ C = 0 \quad \text{for all } y$$

$$t > 0: \quad u = \exp(at), \frac{\partial \theta}{\partial y} = -1,\ C = t \quad \text{at } y = 0$$

$$u = 0, \theta \to 0, C \to 0 \quad \text{as } y \to \infty. \tag{9}$$

All the physical parameters are defined in the nomenclature. The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace Transform technique and the solutions are derived as follows.

$$\theta = 2\sqrt{t}\left[\frac{\exp\left(-\eta^2 P_r\right)}{\sqrt{\pi P_r}} - \eta\, erfc\left(\eta P_r\right)\right] \tag{10}$$

$$C = \frac{t}{2}\left[\exp\left(2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}+\sqrt{Kt}\right)+\exp\left(-2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}-\sqrt{Kt}\right)\right]$$
$$-\frac{\eta\sqrt{tS_c}}{2\sqrt{K}}\left[\exp\left(-2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}-\sqrt{Kt}\right)-\exp\left(2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}+\sqrt{Kt}\right)\right] \tag{11}$$

$$u = \frac{\exp(at)}{2}\left[\exp\left(-2\eta\sqrt{at}\right)erfc\left(\eta-\sqrt{at}\right)+\exp\left(2\eta\sqrt{at}\right)erfc\left(\eta+\sqrt{at}\right)\right]$$
$$+bt\sqrt{t}\left[\frac{4\left(1+\eta^2\right)}{\sqrt{\pi}}\exp\left(-\eta^2\right)-\frac{4\left(1+\eta^2 P_r\right)}{\sqrt{\pi}}\exp\left(-\eta^2 P_r\right)\right.$$
$$\left.+\eta\sqrt{P_r}\left(6+4\eta^2 P_r\right)erfc\left(\eta\sqrt{P_r}\right)-\eta\left(6+4\eta^2\right)erfc(\eta)\right]$$
$$+2\,det\left[\left(1+2\eta^2\right)erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp\left(-\eta^2\right)\right]-2e\,erfc(\eta)$$
$$+e\exp(-dt)\left[\exp\left(-2\eta\sqrt{-dt}\right)erfc\left(\eta-\sqrt{-dt}\right)+\exp\left(2\eta\sqrt{-dt}\right)erfc\left(\eta+\sqrt{-dt}\right)\right]$$
$$+e(1-dt)\left[\exp\left(-2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}-\sqrt{Kt}\right)+\exp\left(2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}+\sqrt{Kt}\right)\right]$$
$$+\frac{ed\eta\sqrt{tS_c}}{\sqrt{K}}\left[\exp\left(-2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}-\sqrt{Kt}\right)-\exp\left(2\eta\sqrt{KtS_c}\right)erfc\left(\eta\sqrt{S_c}+\sqrt{Kt}\right)\right]$$
$$-e\exp(-dt)\left[\exp\left(-2\eta\sqrt{(K-d)tS_c}\right)erfc\left(\eta\sqrt{S_c}-\sqrt{(K-d)t}\right)\right.$$
$$\left.+\exp\left(2\eta\sqrt{(K-d)tS_c}\right)erfc\left(\eta\sqrt{S_c}+\sqrt{(K-d)t}\right)\right] \tag{12}$$

where $\eta = \frac{y}{2\sqrt{t}}$, $b = \frac{G_r}{3\left(P_r-1\right)\sqrt{P_r}}$, $d = \frac{KS_c}{S_c-1}$, $e = \frac{G_m}{2d^2\left(S_c-1\right)}$

3. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameter a, G_r, G_m, S_c, P_r, K and t upon the nature of the flow and transport. The value of the Schmidt number S_c is taken to be 0.6 which corresponds to water-vapour. Also, the value of prandtl number P_r is chosen as 0.71, which represents air.

Figure (1) represents the velocity profiles due to the variations in a in cases of cooling and heating of the plate. It is evident from figure that the velocity increases with an increase in a in cases of both cooling and heating of the plate.

Figure (2) reveals the velocity variations with G_r and G_m in cases of cooling and heating of the plate. It is observed that greater cooling of surface (an increase in G_r) and increase in G_m results in an increase in the velocity. It is due to the fact increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. The reverse effect is observed in case of heating of the plate.

Figure (3) displays the effects of S_c on the velocity field for the cases of cooling and heating of the plate. In the case of cooling of the plate, the velocity increases with decreasing Schmidt number. The opposite phenomenon is observed for heating of the plate.

Figure (4) illustrates the effects of P_r on the velocity field for the cases of cooling and heating of the plate. In the case of cooling of the plate, the velocity is observed to decrease with increasing Prandtl number. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. The reverse effect is observed in case of heating of the plate.

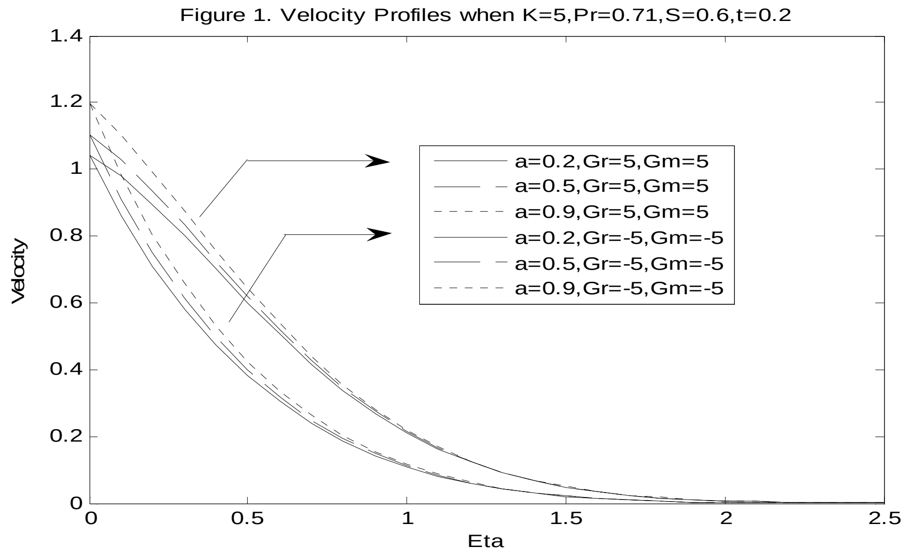
Figure 1. Velocity Profiles when K=5,Pr=0.71,S=0.6,t=0.2
Velocity
Eta
a=0.2,Gr=5,Gm=5
a=0.5,Gr=5,Gm=5
a=0.9,Gr=5,Gm=5
a=0.2,Gr=-5,Gm=-5
a=0.5,Gr=-5,Gm=-5
a=0.9,Gr=-5,Gm=-5

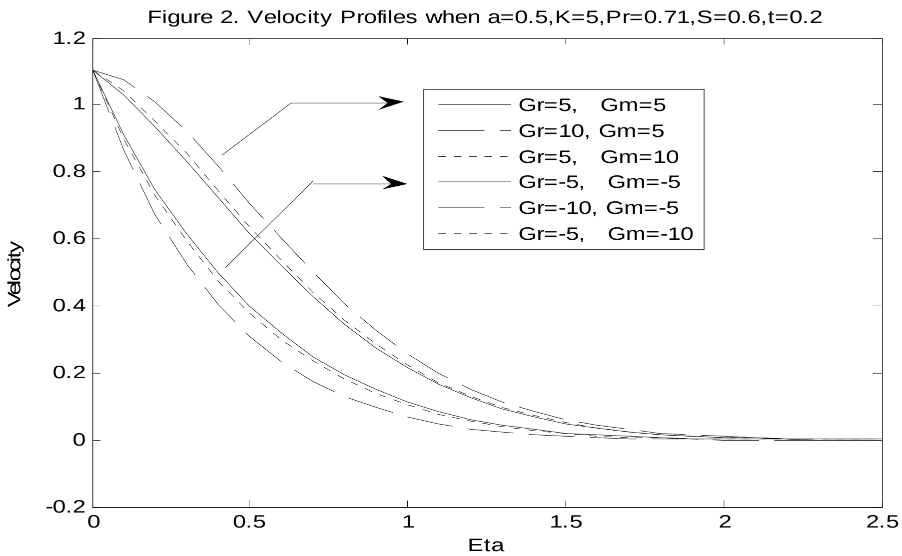
Figure 2. Velocity Profiles when a=0.5,K=5,Pr=0.71,S=0.6,t=0.2
Velocity
Eta
Gr=5, Gm=5
Gr=10, Gm=5
Gr=5, Gm=10
Gr=-5, Gm=-5
Gr=-10, Gm=-5
Gr=-5, Gm=-10

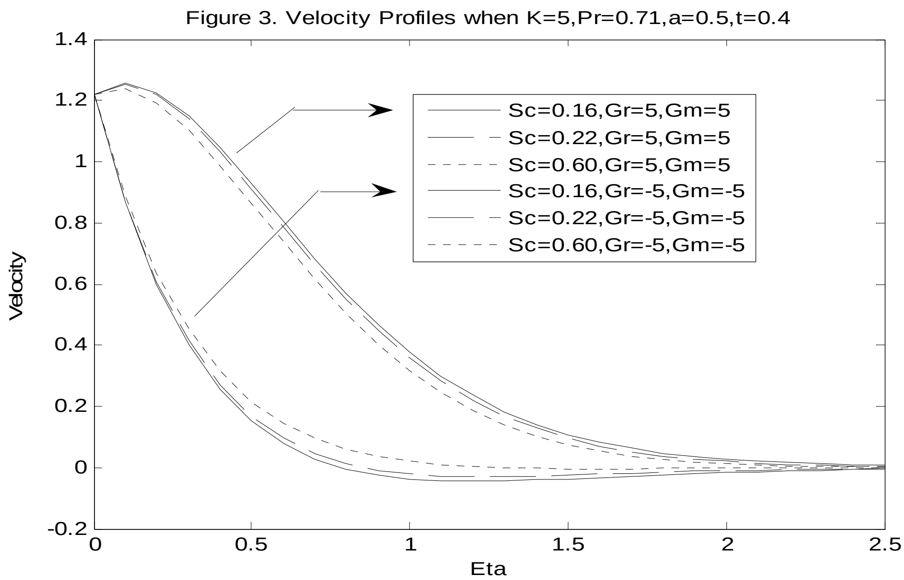
Figure 3. Velocity Profiles when K=5,Pr=0.71,a=0.5,t=0.4
Velocity
Eta
Sc=0.16,Gr=5,Gm=5
Sc=0.22,Gr=5,Gm=5
Sc=0.60,Gr=5,Gm=5
Sc=0.16,Gr=-5,Gm=-5
Sc=0.22,Gr=-5,Gm=-5
Sc=0.60,Gr=-5,Gm=-5

Figure 4. Velocity Profiles when K=5,Sc=0.60,a=0.5,t=0.2

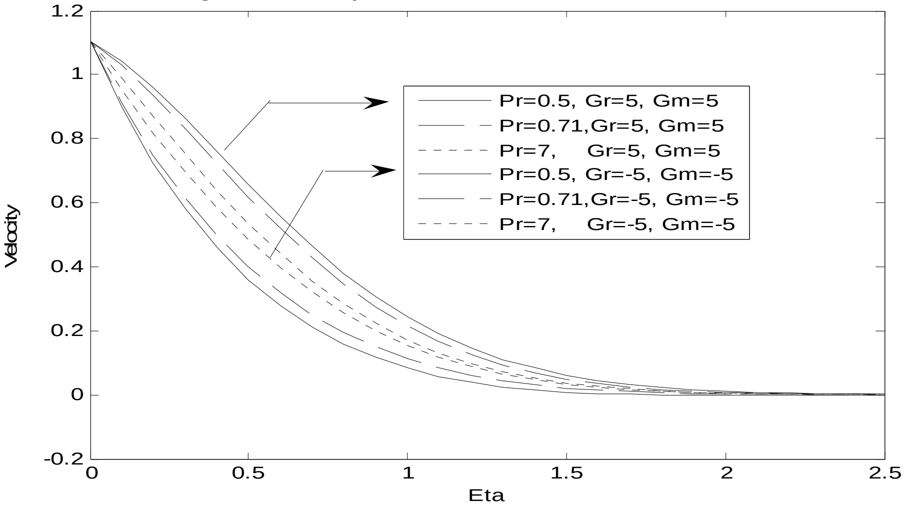

Figure 5. Velocity Profiles when Pr=0.71,Sc=0.60,a=0.5,t=0.4

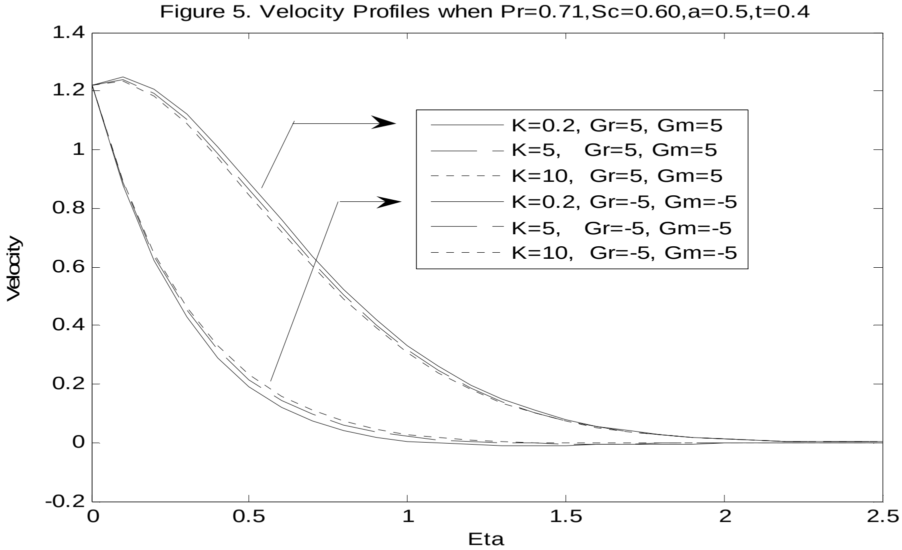

Figure 6. Velocity Profiles when Pr=0.71,Sc=0.60,a=0.5,K=5

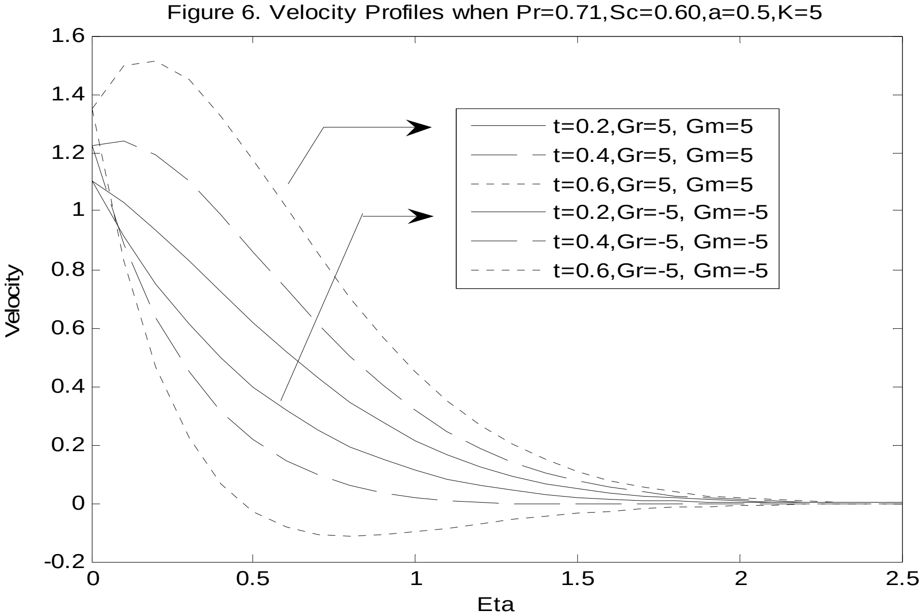

Figure 7. Temperature profiles

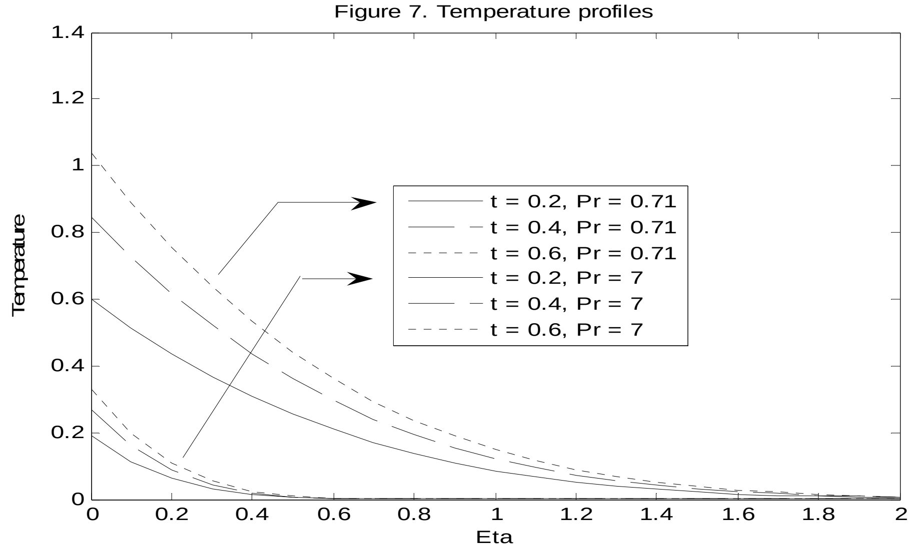

Figure 8. Concentration profiles when Sc=0.60

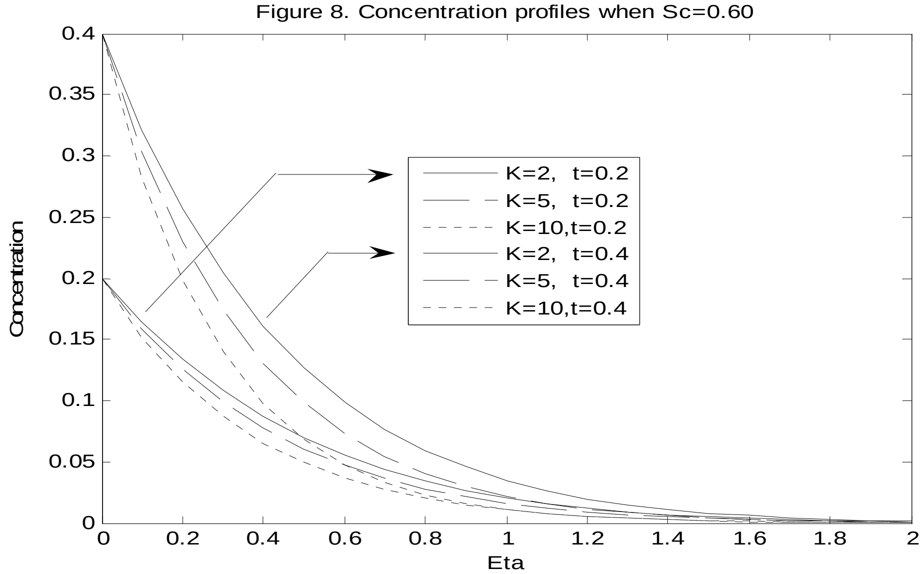

Figure 9. Concentration profiles when K=5

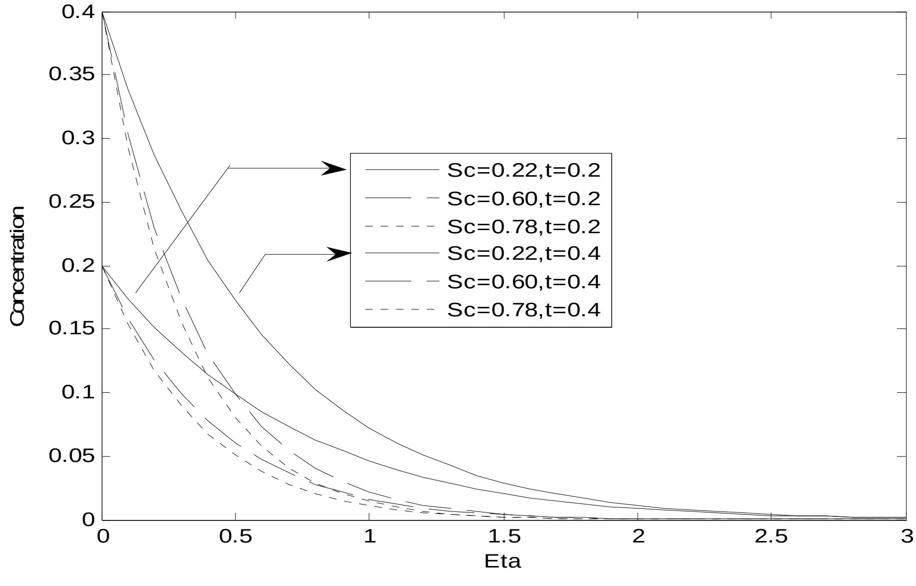

Figure (5) represents the velocity profiles for different values of the chemical reaction parameter in cases of cooling and heating of the plate. It is observed that in the case of cooling the velocity increases with decreasing chemical reaction parameter and the reverse effect is observed in the case of heating of the plate.

The velocity profiles for different values of time t are presented in Figure (6) for the cases of cooling and heating of the plate. It is observed that in the case of cooling the velocity increases with an increase in t, but in the case of heating u increases with increase in t near the plate and the reverse effect is observed far away from the plate.

Figure (7) depicts the temperature profiles against η (distance from plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically. The temperature for air is greater than that of water, this is due to the fact that thermal conductivity of fluid decreases with increasing P_r, resulting a decrease in thermal boundary layer thickness. It is also observed that the temperature increases with an increase in t for both P_r =7(water) and P_r =0.71(air).

Figure (8) demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter and time t. It is observed that the concentration increases with decreasing chemical reaction parameter. It is also observed that the concentration increases with an increase in t.

Figure (9) concerns with the effect of S_c on the concentration. Like temperature, the concentration is maximum at the surface and falls exponentially. The concentration decreases with an increase in S_c. Further, it is noted that concentration falls slowly and steadily for S_c =0.22 (hydrogen) in comparison to other gases S_c =0.60 (water vapour) and S_c =0.78 (ammonia).

Nomenclature

A - *Constant*
C' - *Species concentration in the fluid*
C'_w - *Concentration of the plate*
C'_∞ - *Concentration in the fluid far away from the plate*
C - *Dimensionless concentration*
C_p - *Specific heat at constant pressure*
D - *Chemical Molecular diffusivity*
g - *Acceleration due to gravity*
G_r - *Thermal Grashof number*
G_m - *Mass Grashof number*
κ - *Thermal conductivity of the fluid*
K - *Chemical reaction parameter*
P_r - *Prandtl number*
q - *Heat flux*
S_c - *Schmidt number*
T' - *Temperature of the fluid near the plate*
T'_w - *Temperature of the plate*
T'_∞ - *Temperature of the fluid far away from the plate*
t' - *Time*
t - *Dimensionless time*
u' - *Velocity of the fluid in the x'-direction*
u_0 - *Velocity of the plate*
u - *Dimensionless velocity*
y' - *Coordinate axis normal to the plate*
y - *Dimensionless coordinate axis normal to the plate*

Greek symbols

α - *Thermal diffusivity*
β - *Volumetric coefficient of thermal expansion*
β^* - *Volumetric coefficient of expansion with concentration*
μ - *Coefficient of viscosity*
ν - *Kinematic viscosity*
ρ - *Density of the fluid*
θ - *Dimensionless temperature*
erf - *Error function*
$erfc$ - *Complementary error function*

Subscripts

w - *Conditions on the wall*
∞ - *Free stream conditions*

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