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METHOD OF GEOMETRIC CALCULATION OF INVOLUTE HELICAL GEAR DRIVES, MODIFIED IN TWO DIRECTIONS

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ABSTRACT:
The peculiarities of involute helical gear drives, originating from the two-direction modification of the pinion and the gear in radial and tangential direction are explained. For the most general case of the external involute meshing, where the gear teeth are helical and modified in two directions, an engineering method for geometric calculation is developed. Two different problems (right and reverse) of the geometric design are defined. The calculating formulas for the determination of the geometric dimensions of gears, the location of the characteristic points of the tooth profiles, the dimensions of the control of the relative position of profiles and quality indices of gears and gear drive are specified. The limiting conditions, ensuring the efficiency of involute meshing regarding the geometric parameters, are determined. A classification of involute gear drives, showing the connection of the two-direction modification of gears and the centre distances is proposed.
KEYWORDS:
Gear, Involute meshing, Helical gear drive, Modification, Geometric design, Contact ratio

1. INTRODUCTION

Involute gears, used in the up-to-date technique, in the majority of cases are *radial modified*. Unmodified gears find application in limited cases, mainly in kinematic gear drives, where the loadings and transmitted powers are negligible. The questions related to the geometric essence of the radial modification and its influence on the carrying capacity of involute gear drives, in details are studied by many authors. Using the radial modification, to a certain extent is broadened the area of existence of the designed gear drive and simultaneously there are created suitable conditions for the improvement of its quality indices. From a practical point of view, related to the radial modification of gears, significant is the contribution of the team of I. A. Bolotovskii, who composed an album comprising a great number of blocking contours [4] for the choice of coefficients of radial shift of the rack, at specified teeth number of pinion and gear. Despite the indisputable advantages of the involute helical gear drives, resulting from the radial modification of gears, the area of their existence by the traditional method of design remains limited regarding the relative position of the opposite involute profiles.

For broadening the area of existence of the involute meshing the Russian scientist E. B. Vulgakov [5] and his students [7-8] have designed a new method, known as "*direct gear design*" (DGD). The basic theses in this method are obtained as a result of the elimination of the rack-cutter when defining the geometry of gears and the introduction of new independent parameters, connected directly with the involute teeth profiles. With DGD are designed such gear drives, for which it was considered that it would not be possible to be realized by involute meshing. The positive results even so do not result in the large-scale application of DGD in practice mainly because of the use of unpopular principles and the circumstance that the proposed method is obtained as a denial of the traditional method, but not through its improvement and development.

For overcoming the shortcomings of the traditional method and DGD the author of the present paper proposes a *generalized method* for geometric calculation of gear drives [1-3], where the conventional principles of the traditional method are preserved. In the generalized method of design the location of the opposite profiles is determined by the proposed *tangential modification* of gears, which, in combination with their radial modification, additionally broadens the area of existence of the gear drive. The proposed method is based on the development and improvement of the traditional method, but not by its denial. Due to this the generalized method allows with traditional principles to obtain untraditional results. A generalized presentation of the geometry of the gear drive using the proposed method is a result from defining the *generalized model of involute meshing*, from which under certain conditions the respective variant is obtained. This approach is set also in the base of the designed generalized method for geometric design of two-direction modified involute gear drives of helical teeth. As a result of the conventional method for geometric design of involute helical gear drives [4] is obtained as a private case of the proposed method.

2. GEOMETRIC CALCULATIONS

2.1 Peculiarities in meshing of helical gear drives with a rack-cutter

The cut of *helical gear* drives with a rack-cutter is done in a similar way as of spur gear drives. The basic difference between both cases is related to the shape of the teeth of the rack-cutter. In one case a rack-cutter of helical teeth is imitated, and in the other case - a rack-cutter of spur teeth.

It is assumed that the geometric shape of the helical rack-cutter is determined by its profile in its *normal section* (Fig. 1). It means that the *basic rack profile* (BRP), that is standardized, is specified in a normal section. In *transverse section*, perpendicular to the rotation axis of the gear, the rack-cutter profile has a changed geometry. Despite, the depth dimensions of the rack-cutter in normal and transverse section are kept one and the same, and the longitudinal dimensions change depending on the value of the helix angle β. The pressure angle BRP also changes. When designating the values of the transverse section it is accepted to add the index t to the respective symbol of the normal section.

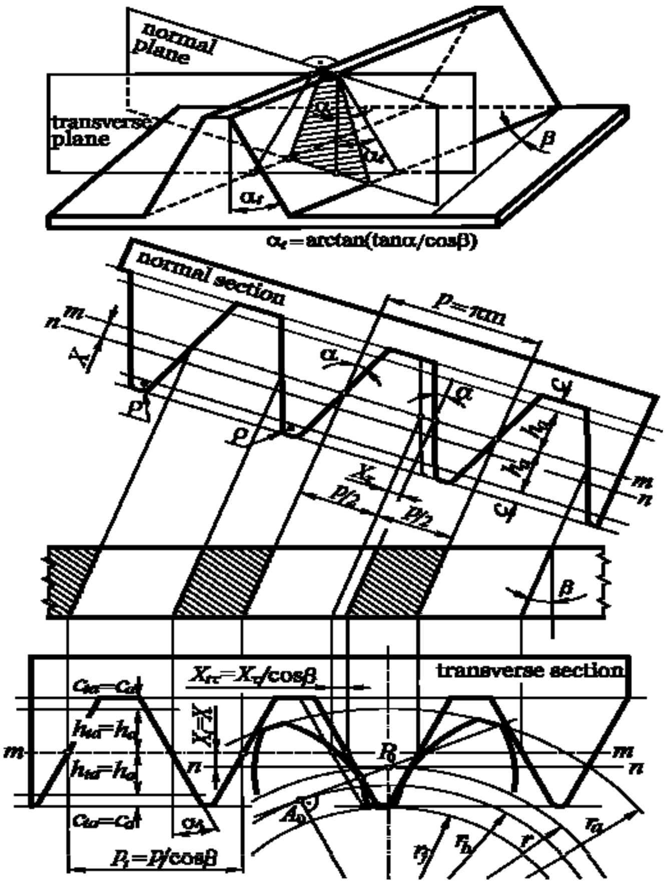

Figure 1. Normal and transverse section of the rack-cutter

If the independent parameters of BRP are assigned in normal section (the normal module m, profile angle α, depth coefficient of the addendum part of the tooth h_a^*, coefficient of bottom clearance c^*), for the longitudinal dimensions in transverse section are valid the equations

$$m_t = m/\cos\beta ,$$

$$p_t = p/\cos\beta = \pi m/\cos\beta ,$$

$$\tan\alpha_t = \tan\alpha/\cos\beta , \qquad (1)$$

and the depth dimensions remain unchanged, i.e.

$$h_{ta} = h_a = h_a^* m , \quad c_t = c = c^* m \qquad (2)$$

The geometry of helical gears is determined with the help of the meshing of rack-cutter with the gear in transverse section, shown on Fig. 1. In this case the tooth profile of the gear is involute only in transverse section (in normal section the profile is not an involute curve). In the two-direction modification of the gear it is assumed that its relative location to the rack-cutter is determined by the radial shift X of the rack-cutter and the shift X_{τ} of its side profiles, where

$$X = xm , \qquad X_{\tau} = x_{\tau} m , \qquad (3)$$

In equations (3) with x and x_{τ} are specified the coefficients of the relative shift in a normal section. It is seen from Fig. 1 that for shifts X_t and $X_{t\tau}$ in transverse section are valid the equations

$$X_t = X = xm , \qquad X_{t\tau} = X_{\tau}/\cos\beta = x_{\tau} m/\cos\beta . \qquad (4)$$

By equations (4) it is determined that the radial shift in normal and transverse section is one and the same, while the tangential shift X_{τ} is different in both sections.

2.2 Right problem

As initial values for the geometric calculation of two-direction modified cylindrical gear drives, formed by involute helical gears, are specified the independent parameters m, α, h_a^*, c^*, ρ^* of BRP (with ρ^* is specified the coefficient of the fillet radius of the rack-cutter tooth) and the teeth number z_1

and z_2 of gears. Besides, in accordance with the solved problem are specified five more from the following six mutually related parameters: $a_w, x_1, x_{\tau 1}, x_2, x_{\tau 2}$ and β (with a_w is designated the centre distance of gear drive). By the geometric design of gear drives most often two basic problems are solved. In the first problem, defined as a *right problem*, are assigned x_1, $x_{\tau 1}$, x_2, $x_{\tau 2}$ and β, and the centre distance a_w is unknown. On the contrary, in the second problem, known as a *reverse problem*, the centre distance is assigned. A combined variant is also possible, where after solving the right problem the centre distance is rounded and afterwards the reverse problem is solved.

The right problem begins with clarifying the initial data, shown in Table 1. Then the calculation process is done using the formulas in Table 2 and Table 3, where in the last columns are given the results of the calculated numerical example. In the right problem initially is specified the centre distance according to the procedure, given in Table 2. Then, with formulas from rows 4 to 15 of Table 3 are calculated the diametrical dimensions of gears and teeth thicknesses on the reference circles and addendum circles. The location of the characteristic points of involute profiles and the related values are determined by the formulas from rows 16 to 23. The rows from 24 to 30 are used for the determination of the respective quality indices of gears and involute meshing. The solving of the right problem ends with the determination of the necessary geometric dimensions, related to the control of gears (rows from 31 to 40).

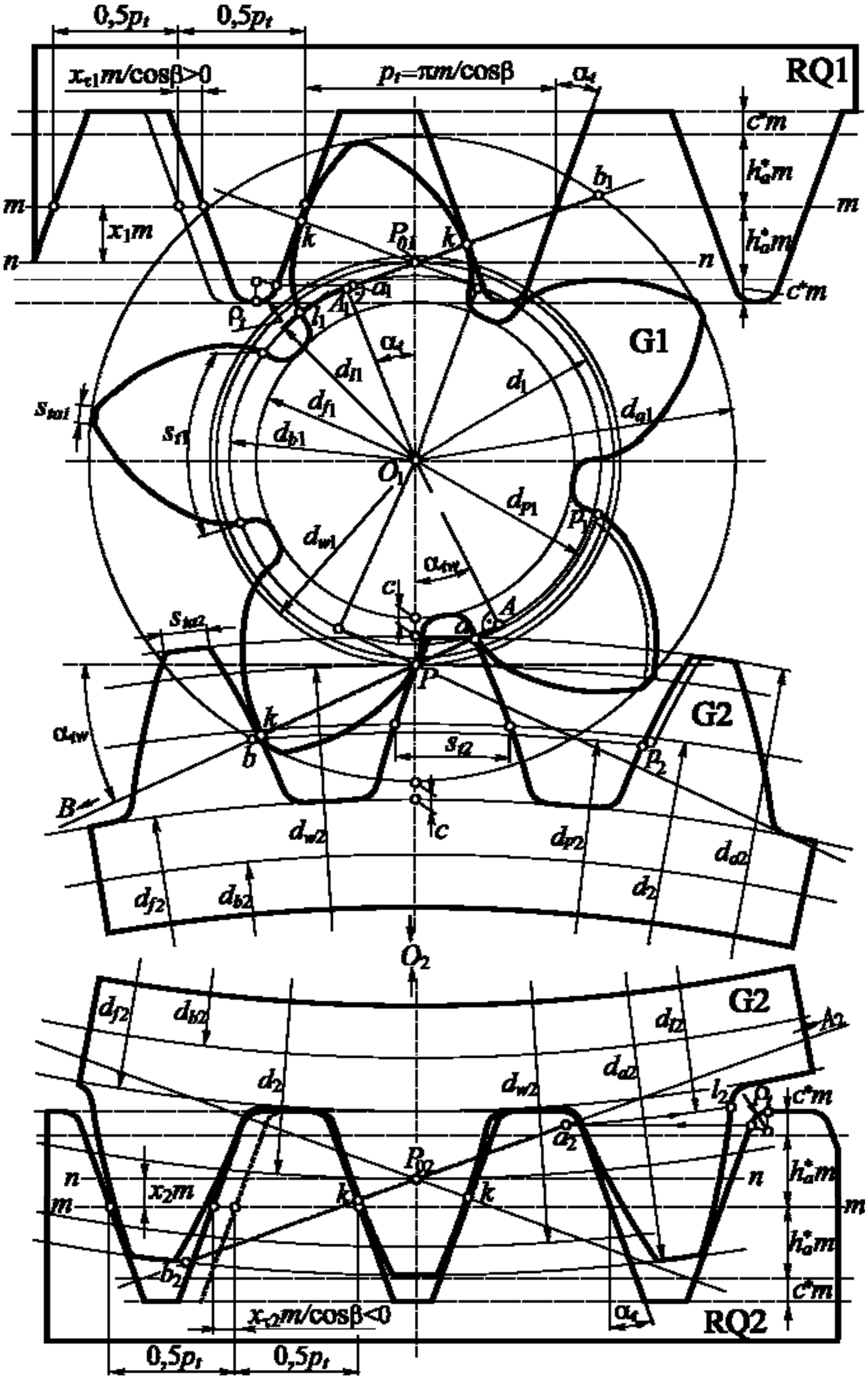

Figure 2. Meshing of two-direction modified gears

With the got geometric dimensions on Fig. 2, corresponding to the calculated results and mutually related are drawn: meshing of pinion G1 with the rack-cutter RQ1; meshing of gear G2 with the rack-cutter RQ2; involute meshing between the pinion and the gear.

Table 1: Initial data for geometric calculations

Independent parameters		Symbol	Dimension	Example
Module		m	mm	10
Helix angle teeth on the reference cylinder		β	deg	20°
Number of teeth	Of the pinion	z_1	–	5
	Of the gear	z_2	–	50
Basic rack profile (BRP)	Normal profile angle	α	deg	20°
	Addendum coefficient	h_a^*	–	1
	Coefficient of bottom clearance	c^*	–	0,25
	Coefficient of fillet radius	ρ^*	–	0,30
Coefficient of radial modification	Of pinion	x_1	–	0,71
	Of gear	x_2	–	0,40098
Coefficient of tangential modification	Of pinion	$x_{\tau1}$	–	0,24
	Of gear	$x_{\tau2}$	–	– 0,19

Table 2: Determination of the centre distance a_w

№	Parameters	Symbol	Dimension	Equation	Example
1	Reference centre distance	a	mm	$a = m(z_1 + z_2)/2\cos\beta$	292,6489
2	Transverse profile angle of rack-cuter	α_t	deg	$\alpha_t = \arctan(\tan\alpha/\cos\beta)$	21,1728°
3	Total coefficient of radial modification	x_Σ	–	$x_\Sigma = x_1 + x_2$	1,11098
4	Total coefficient of tangential modification	$x_{\tau\Sigma}$	–	$x_{\tau\Sigma} = x_{\tau1} + x_{\tau2}$	0,05
5	Working transverse pressure angle	α_{tw}	deg	$\mathrm{inv}\alpha_{tw} = \frac{x_{\tau\Sigma} + 4x_\Sigma \tan\alpha}{2(z_1 + z_2)} + \mathrm{inv}\alpha_t$	25,7578°
6	Centre distance	a_w	mm	$a_w = a\cos\alpha_t/\cos\alpha_{tw}$	303

Table 3: Geometric dimensions and parameters of gears and gear drives

№	Parameters	Symbol	Equation	Example
1	Minimum coefficient of radial modification	x_{min}	$x_{min} = h_a^* + c^* - \rho^*(1 - \sin\alpha) - z\sin^2\alpha_t/2\cos\beta$	0,7055 −2,4180
2	Minimum coefficient of tangential modification	$x_{\tau min}$	$x_{\tau,min} = 2\tan\alpha(h_a^* + c^*) - \pi/2$	−0,6609
3	Maximum coefficient of tangential modification	$x_{\tau max}$	$x_{\tau,max} = \pi/2 - 2\tan\alpha(h_a^* + c^* - \rho^* + \rho^*/\sin\alpha)$	0,2407
4	Reference circle diameter	d	$d = mz/\cos\beta$	53,209 532,089
5	Base circle diameter	d_b	$d_b = d\cos\alpha_t$	49,617 496,170
6	Root circle diameter	d_f	$d_f = d - 2m(h_a^* + c^* - x)$	42,409 515,109
7	Pitch circle diameters	d_{w1}	$d_{w1} = 2a_w/(z_2/z_1 + 1)$	55,091
		d_{w2}	$d_{w2} = 2a_w - d_{w1}$	550,909
8	Centre distance modification coefficient	Δy	$\Delta y = x_\Sigma - (a_w - a)/m$	0,07587
9	Addendum circle diameters	d_{a1}	$d_{a1} = 2a_w - d_{f2} - 2c^*m$, or $d_{a1} = d_1 + 2m(h_a^* + x_1 - \Delta y)$	85,891
		d_{a2}	$d_{a2} = 2a_w - d_{f1} - 2c^*m$, or $d_{a2} = d_2 + 2m(h_a^* + x_2 - \Delta y)$	558,591
10	Tooth thickness over the reference circle in transverse section	s_t	$s_t = \frac{m}{\cos\beta}\left(\frac{\pi}{2} + x_\tau + 2x\tan\alpha\right)$	24,770 17,800

11	Tooth thickness over the reference cylinder in normal section	s	$s = m(\frac{\pi}{2} + x_{\tau} + 2x\tan\alpha)$	23,276 16,727
12	Pressure angle at the addendum diameter	α_a	$\alpha_a = \arccos(d_b/d_a)$	54,7131° 27,3454°
13	Helix angle of the tooth over the addendum cylinder	β_a	$\beta_a = \arctan(d_a \tan\beta/d)$	30,4356° 20,9118°
14	Tooth thickness over the addendum circle in transverse section	s_{ta}	$s_{ta} = d_a[\pi/2z + (2x\tan\alpha + x_{\tau})/z +$ $+ \mathrm{inv}\,\alpha_t - \mathrm{inv}\,\alpha_a]$	2,165 6,352
15	Tooth thickness over the addendum cylinder in normal section	s_a	$s_a = s_{ta}\cos\beta_a$	1,867 5,934
16	Pressure angle at the external cross point of the involute profiles	θ	$\mathrm{inv}\,\theta = (\pi/2 + 2x\tan\alpha + x_{\tau})/z + \mathrm{inv}\,\alpha_t$	55,4177° 29,5619°
17	Tip circle diameter of the cross point of involute profiles	d_{Δ}	$d_{\Delta} = d\cos\alpha_t/\cos\theta$	87,4171 570,4264
18	Profile curvature at its boundary point	ρ_l	$\rho_1 = 0{,}5\,d\sin\alpha_t -$ $- m[h_a^* + c^* - \rho^*(1-\sin\alpha) - x_i]/\sin\alpha_t$	0,124 78,049
19	Profile curvatures at its internal contact point	ρ_{p1}	$\rho_{p1} = a_w\sin\alpha_{tw} - 0{,}5d_{b2}\tan\alpha_{a2}$	3,378
		ρ_{p2}	$\rho_{p2} = a_w\sin\alpha_{tw} - 0{,}5d_{b1}\tan\alpha_{a1}$	96,619
20	Pressure angle at the boundary point	α_l	$\tan\alpha_1 = 2\rho_1/d_b$	0,2853° 17,4639°
21	Pressure angles at the internal contact point	α_{p1}	$\tan\alpha_{p1} = \tan\alpha_{tw} - \frac{z_2}{z_1}(\tan\alpha_{a2} - \tan\alpha_{tw})$	7,7548°
		α_{p2}	$\tan\alpha_{p2} = \tan\alpha_{tw} - \frac{z_1}{z_2}(\tan\alpha_{a1} - \tan\alpha_{tw})$	21,2788°
22	Diameter of location of the boundary profile point	d_l	$d_1 = d_b/\cos\alpha_1$, or $d_1 = \sqrt{d_b^2 + 4\rho_1^2}$	49,618 520,146
23	Diameter of location of the internal contact point	d_p	$d_p = d_b/\cos\alpha_p$, or $d_{pi} = \sqrt{d_b^2 + 4\rho_p^2}$	50,075 532,471
24	Theoretical line of action	l_{AB}	$l_{AB} = a_w\sin\alpha_{tw}$, or $l_{AB} = 0{,}5(d_{b1} + d_{b2})\tan\alpha_{tw}$	131,6740
25	Real line of action	l_{ab}	$l_{ab} = 0{,}5(d_{b1}\tan\alpha_{a1} + d_{b2}\tan\alpha_{a2}) - a_w\sin\alpha_{tw}$	31,6769
26	Gear potential	ε_p	$\varepsilon_p = z(\tan\alpha_a - \tan\alpha_1)/2\pi$	1,120 1,612
27	Gear drive potential	ε_{pw}	$\varepsilon_{pw} = (z_1 + z_2)\tan\alpha_{tw}/2\pi$	4,223
28	Transverse contact ratio	ε_α	$\varepsilon_\alpha = \frac{z_1(\tan\alpha_{a1} - \tan\alpha_{tw}) + z_2(\tan\alpha_{a2} - \tan\alpha_{tw})}{2\pi}$	1,016
29	Axial contact ratio	ε_β	$\varepsilon_\beta = (b_w\sin\beta)/\pi m$, | b_w - width of meshing (b_w=200)	2,177
30	Total contact ratio	ε	$\varepsilon = \varepsilon_\alpha + \varepsilon_\beta$	3,194
31	Constant chord in normal section	s_c	$s_c = m[(\pi/2 + x_{\tau})\cos^2\alpha + x\sin 2\alpha)$	20,554 14,770
32	Depth to the constant chord	h_c	$h_c = 0{,}5(d_a - d - s_c\tan\alpha)$	12,601 10,563
33	Tooth thickness over a specified circle in normal section	s_y	$s_y = d_y\cos\beta_y[\pi/2z + (2x\tan\alpha + x_{\tau})/z +$ $+ \mathrm{inv}\,\alpha_t - \mathrm{inv}\,\alpha_y]$; $\alpha_y = \arccos(d_b/d_y)$; $\beta_y = \arctan(d_y\tan\beta/d)$ $d_b < d_y < d_a$, in example: d_{y1}=50; d_{y2}=535	22,8355 15,7238
34	Tooth thickness over a chord of a specified circle in normal section	s_{hy}	$s_{hy} = d_y\sin\gamma_y$, where $\gamma_y = \frac{s_y}{d_y}\frac{180}{\pi}$	22,0499 15,7216

35	Depth to the measured chord of the specified circle	h_{hy}	$h_{hy}=0{,}5[d_a-d_y+d_y(1-\cos\gamma_y)]$		20,5080 11,9111
36	Minimum number of engaged teeth when measuring the base tangent length	$z_{w,\min}$	$z_{w,\min}=z[\tan\alpha_1-(x_\tau+2x\tan\alpha)/z-\mathrm{inv}\alpha_t]/\pi+0{,}5$		1 6
37	Maximum number of engaged teeth when measuring the base tangent length	$z_{w,\max}$	$z_{w,\max}=z[\tan\alpha_a-(x_\tau+2x\tan\alpha)/z-\mathrm{inv}\alpha_t]/\pi+0{,}5$		2 8
38	Base tangent length	W	$W=m\cos\alpha\,[\pi(z_w-0{,}5)+$ $+x_\tau+2x\tan\alpha+z\,\mathrm{inv}\,\alpha_t]$	pinion	$W_1=22{,}709$ $W_2=52{,}230$
				gear	$W_6=171{,}685$ $W_7=201{,}206$ $W_8=230{,}727$
39	Helix angle of the tooth over the basic cylinder	β_b	$\cos\beta_b=\cos\beta\cos\alpha/\cos\alpha_t$		18,7472°
40	Minimum width of the gear when measuring the base tangent length	$b_{\min}$	$b_{\min}=W_{\min}\sin\beta_b$, where $W_{\min}=W$ at $z_w=z_{w,\min}$		7,298 55,178

2.3 Reverse problem

The solving of the reverse problem begins with preliminary determination of the pressure angle α_{tw} of the gear drive in the transverse section from the formula

$$\alpha_{tw}=\arccos[m(z_1+z_2)\cos\alpha_t/2a_w\cos\beta]\,. \qquad (5)$$

Then, from the *basic equation of meshing* of the type

$$x_{\tau\Sigma}+4x_\Sigma\tan\alpha=2(z_1+z_2)(\mathrm{inv}\alpha_{tw}-\mathrm{inv}\alpha_t)\,, \qquad (6)$$

is obtained the connection between the summarized coefficients of shift x_Σ and $x_{\tau\Sigma}$, where

$$x_\Sigma=x_1+x_2\,,\qquad x_{\tau\Sigma}=x_{\tau1}+x_{\tau2} \qquad (7)$$

Further, after clarifying the coefficients x_1, x_2, $x_{\tau1}$ and $x_{\tau2}$, the solving of the reverse problem is done in the same way and sequence, as the right problem. The determination of coefficients x_1, x_2, $x_{\tau1}$, $x_{\tau2}$ is one of the existing stages of the reverse problem, because from their choice to a great extent the quality indices of meshing and the load carried by the gear drive depend.

3. RESTRICTING CONDITIONS

In the process of calculations obligatorily are checked *five restricting conditions*, relating to: 1) ensuring of a real existence of the rack-cutters; 2) avoiding the undercut of involute profiles (cutter interference); 3) avoiding the sharpening of teeth; 4) avoiding the gear interference between the meshed profiles; 5) ensuring a constant meshing by the value of the contact ratio.

3.1 Restrictions in the rack-cutter

The tangential modification directly influences the location of the side profiles of the rack-cutter. At specified values of $x_{\tau1}$ and $x_{\tau2}$ it may turn out that the rack-cutter does not exist because of the self-cross of its profiles. In this connection on Fig. 3a is shown the tooth shape of the rack-cutter, when the tangential shift is maximum, and on Fig. 3b – when the same shift is minimum. In the general case the fillet radius $\rho=\rho^* m$ at the tip of the rack-cutter does not depend directly on the bottom clearance $c=c^* m$.

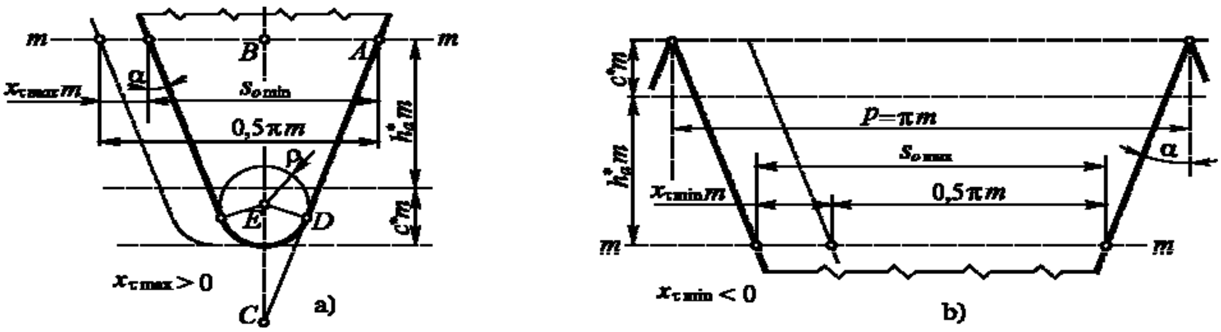

Figure 3. Boundaries of tangential shift of profiles: a) $x_\tau=x_{\tau\max}$; b) $x_\tau=x_{\tau\min}$

In cases of tangential modification it is expedient, the coefficient of the fillet radius ρ^* to be chosen within the scope $0 < \rho^* \leq c^*/(1-\sin\alpha)$. From a strength point of view recommendable are the larger values of this coefficient, even though by teeth grinding most often $\rho^* = 0$. In correspondence with the designated dimensions on Fig. 3, for the minimum coefficient of tangential shift $x_{\tau\min}$ is obtained the formula, given in row 2, and for the maximum coefficient $x_{\tau\max}$ – the formula from row 3 of Table 3. With these formulas, at specified values of the independent parameters of BRP, the possible range of shift of the coefficient of tangential shift is obtained.

The condition, defining the real existence of the rack-cutters s determined from the inequalities

$$x_{\tau 1} \geq x_{\tau\min}, \qquad x_{\tau 1} \leq x_{\tau\max}, \qquad x_{\tau 2} \geq x_{\tau\min}, \qquad x_{\tau 2} \leq x_{\tau\max}, \tag{8}$$

where the tolerance values of $x_{\tau\min}$ and $x_{\tau\max}$ are determined by the shown rows of Table 3.

3.2 Undercutting of involute profiles

The restrictive conditions, with which the undercut is avoided, are expressed by inequations

$$x_1 \geq x_{1\min}, \qquad x_2 \geq x_{2\min}, \tag{9}$$

whose right sides are defined by the formula from row 1 of Table 3.

3.3 Sharpening of teeth

After the calculation of teeth thickness over their addendum circles (row 15) the conditions for non-sharpening of teeth are checked

$$s_{a1} \geq s_a^* m, \qquad s_{a2} \geq s_a^* m, \tag{10}$$

where $s_a^* = 0{,}15...0{,}3$ is a coefficient, depending on the respective heat treatment of the wheel rim.

3.4 Gear interference the meshed profiles

In the general case the check for avoiding the gear interference, at absence of undercut of involute profiles, requires the satisfaction of conditions

$$\rho_{p1} \geq \rho_{l1}, \qquad \rho_{p2} \geq \rho_{l2}, \tag{11}$$

in which the corresponding values are determined from rows 18 and 19 in Table 3.

3.5 Contact ratio

In order to determine the contact ratio of the helical gear drive preliminary from rows 28 and 29 in Table 3 are obtained the transverse and axial contact ratio. The continuous transmission of movement in the gear drive is guaranteed by condition

$$\varepsilon \geq \varepsilon_{\min}, \tag{12}$$

in which the total contact ration ε is determined from row 30, and the minimum values of this coefficient is $\varepsilon_{\min} = 1{,}1...1{,}15$.

4. CLASSIFICATION OF INVOLUTE MESHING

The formation of involute gear meshing, composed from two two-direction modified gears, is explained with the help of Fig. 2. The shown gears have helical teeth, by reason of which the picture of involute meshing is displayed in transverse section.

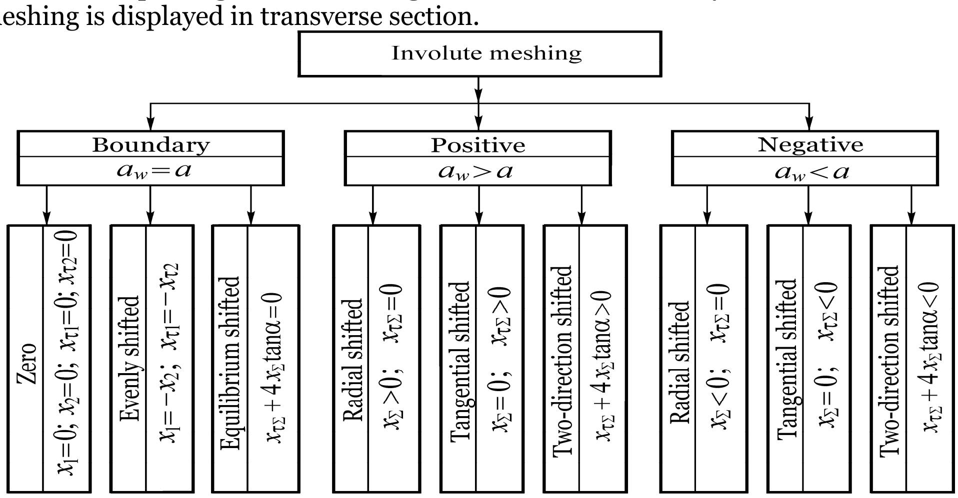

Figure 4. Classification of involute gear drives

Each gear, engaged in tooth meshing, can be modified in a different way in radial and tangential direction. As a result a large variety of the formed involute meshings is obtained. The most general indication, in which are differentiated the got meshings, is directly related to the comparison of the *reference* centre distance a (row 1 from Table 2) with the *real* centre distance a_w (row 6 in Table 2). According to this indication in the proposed on Fig. 4 classification, the involute meshing is divided in three types: *boundary* ($a_w = a$); *positive* ($a_w > a$); *negative* ($a_w < a$).

It is well known that in the case, in which $a_w = a$, the pressure angle α_{tw} of the helical gear drive is equal to the profile angle α_t of the rack-cutter in transverse section. Then, after nullification of the right side of the basic equation of meshing (6) it is established that in this case the condition is satisfied

$$x_{\tau\Sigma} + 4x_{\Sigma}\tan\alpha = 0 \tag{13}$$

The got gear meshing, for which is valid the equation (13), is called a *boundary meshing*. It, on its own behalf is *zero* at $x_1 = x_2 = x_{\tau1} = x_{\tau2} = 0$, *evenly shifted* at $x_1 = -x_2$ and $x_{\tau1} = -x_{\tau2}$ *and equilibrium shifted,* if for the coefficients x_1, x_2, $x_{\tau1}$ and $x_{\tau2}$ is valid the equation

$$(x_{\tau1} + x_{\tau2})/(x_1 + x_2) = 4\tan\alpha, \quad \text{where} \quad x_{\tau1} + x_{\tau2} \neq 0 \text{ и } x_1 + x_2 \neq 0. \tag{14}$$

The positive and negative meshing on its behalf is divided in *radial shifted, tangential shifted* and *two-direction shifted*. At radial meshing $x_{\tau\Sigma} = 0$, and at tangential shifted meshing $x_{\Sigma} = 0$. The two-side shifted meshing is positive, if the right side of equation (13) is positive ($x_{\tau\Sigma} + 4x_{\Sigma}\tan\alpha > 0$), and negative – in the opposite case ($x_{\tau\Sigma} + 4x_{\Sigma}\tan\alpha < 0$).

5. CONCLUSION

The proposed method for geometric calculation of involute gear drives can be used directly by engineers and designers for the design of different types of drives and transmissions. The shown formulas for geometric calculation refer to the general case, in which the gear teeth are helical and modified in radial and tangential direction. Because of this the methodology is applicable for each private variety of involute meshing, without being necessary to draw up new calculation formulas. In this sense the methodology can be used also for the design of spur gear drives of two-direction modified gears. From a practical point of view the proposed methodology insures an analytical possibility for the realization of one common software for a computer design of all types of cylindrical gear drives of external involute meshing.

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