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ASYMPTOTIC BEHAVIORS OF STOCHASTIC EVOLUTION COCYCLE

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Abstract:
The paper presents the properties of exponential stability for stochastic evolution semiflow and stochastic evolution cocycles. Some characterizations which generalize classics results are also provided.
Keywords:
Stochastic evolution semiflow, stochastic evolution cocycles, exponential stability

1. INTRODUCTION

The problem of existence of stochastic semiflows for semilinear stochastic evolution equation is a non-trivial one, mainly due to the well-established fact that finite-dimensional methods for constructing (even continuous) stochastic flow break down in the infinite dimensional setting of semilinear stochastic evolution equations (cf. [3], [7], [8]). For linear stochastic evolution equation with finite-dimensional noise, a stochastic semiflow (i.e. random evolution operator) was obtained in [1].

In [5], is prove the existence of perfect differentiable cocycles generated by mild solutions of a large class of semilinear stochastic evolution equations (sees) and stochastic partial differential equations (spdes).

In this article we consider the stochastic evolution cocycle over a stochastic evolution semiflow, and present the proprieties of exponential stability.

We considered (X, d) metric space and denote V and U real Hilbert spaces. Let B(V) be a Banach space of all linear and bounded maps $A: V \to V$. We denote the sets $T = \{(t,t_0) \in R^2, t \ge t_0 \ge 0\}$ and $Y = X \times V$. The mapping $P: Y \to Y$ given by P(x,v)=(x, P(x)v), $\forall (x,v) \in Y$ is an projector on Y, where P(x) is a projection on $Y_x = \{x\} \times V$, $x \in X$. Let B(X) denote its Borel σ-algebra.

Assume (Ω,F,**P**) is a complete probability space with a normal filter $\{F_t\}_{t \ge 0}$, i.e. F_0 contains the null sets in F and $F_t = \bigcap_{s>t} F_s$, for all $t \ge 0$, and let us consider a real valued $\{F_t\}$ - Wiener process $\{W(t)\}$, $t \ge 0$.

Definition 1. A stochastic process $W(t):(\Omega, F, P) \to U$ is a Wiener process if and only if $W(t) = \sum_{i=1}^{\infty} \beta_i(t) e_i$ where $\beta_i(t)$ are real Wiener process, independents, which are the mean $E(\beta_i(t)^2) = \lambda_i t$, where $\sum_{i=1}^{\infty} \lambda_i < \infty$, $\{e_i\}, i = 1,2,\cdots$ is an orthonormal system of vectors from U.

Definition 2. Let be X a Banach space. A stochastic evolution semiflow on X is a random field $\varphi: T \times X \times \Omega \to X$ satisfying the following proprieties:

(1) $\varphi(t,t,x,\omega) = x$, for all (t, x) from T×X and $\omega \in \Omega$;

(2) $\varphi(t,s,\varphi(s,t_0,x,\omega),\omega) = \varphi(t,t_0,x,\omega)$, $\forall (t,s),(s,t_0) \in T$, $\forall x \in X$, and $\omega \in \Omega$.

Definition 3. A stochastic evolution cocycles on V, over an stochastic evolution semiflow $\varphi : T\times X\times\Omega \to X$, is a random field $\Phi : T\times X\times\Omega \to B(V)$, with the following proprieties:

(1) $\Phi(t,t,x,\omega) = I,\ \forall(t,x)\in R_+\times X$, and $\omega\in\Omega$,

(2) $\Phi(t,s,\varphi(s,t_0,x,\omega),\omega)\Phi(s,t_0,x,\omega) = \Phi(t,t_0,x,\omega),\ \forall(t,s),(s,t_0)\in T,\ \ \forall x\in X$, and $\omega\in\Omega$.

Definition 4. An stochastic evolution cocycles Φ has uniformly exponential growth if exist the constants $M\geq 1,\ \lambda > 0$ such that

$$E\,\|\,\Phi(t,s,x,\omega)\,\|^2 \leq Me^{\lambda(t-s)},\ \forall(t,s)\in T,\ \forall x\in X \text{ and } \omega\in\Omega. \tag{1}$$

Definition 5. The stochastic evolution cocycles Φ is called strongly measurable if for every $(s,x,\omega,v)\in T\times X\times\Omega\times V$ the mapping $t\to\|\,\Phi(t,s,x,\omega)v\,\|$ is measurable on $[s,\infty)$.

Definition 6. The mapping, $C : T\times X\times V\times\Omega \to X\times V$, definite by

$$C(t,s,x,v,\omega) = \big(\varphi(t,s,x,\omega),\Phi(t,s,x,\omega)v\big), \tag{2}$$

where Φ is a stochastic evolution cocycles over an stochastic evolution semiflow φ, is called stochastic skew-evolution semiflow on Y.

2. UNIFORMLY EXPONENTIALLY STABILITY

Let be $F : [0,T]\times\Omega \to H$ an stochastic process, then $E(F) = \int_\Omega F(\Omega)\,dP(\omega)$ represent the mean of stochastic process F, where P is the probability measure. If $F\in C\big([0,T],L^2(\Omega,H)\big)$ then

$$\int_0^T E\,\|\,F(t)\,\|^2\ dt = E\int_0^T \|\,F(t)\,\|^2\ dt. \tag{3}$$

For an process Wiener W(t) in rapport with the filter $\{F_t\}$ we have

$$E\left\|\int_0^T F(t)\,dw(t)\right\|^2 = E\int_0^T \|\,F(t)\,\|^2\ dt. \tag{4}$$

Definition 7. The stochastic evolution cocycles Φ is said to be uniformly exponentially stable if for some positive constants $N\geq 1, \nu > 0$ one has

$$E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2 \leq Ne^{-\nu(t-s)}E\,\|\,\Phi(s,t_0,x,\omega)v\,\|^2, \tag{5}$$

for all $(t,s),(s,t_0)\in T,\ (x,v)\in X\times V,$ and $\omega\in\Omega$.

Lemma 8. The stochastic skew-evolution semiflow C=(φ, Φ), is uniformly exponentially stable if and only if a no decreasing function $f : [0,\infty) \to [1,\infty)$, with property $\lim_{t\to\infty} f(t) = \infty$, such that we have the relation:

$$f(t-s)E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2 \leq E\,\|\,\Phi(s,t_0,x,\omega)v\,\|^2, \tag{6}$$

for all $(t,s),(s,t_0)\in T,\ \ (x,v)\in X\times V$, and for all $\omega\in\Omega$.

Proof. *Necessity*. If C=(φ, Φ), is uniformly exponentially stable, result from Definition 7 that for $f(t) = N^{-1}e^{\nu t}$.

Sufficiency. Let be $t\geq s\geq t_0\geq 0$ and we denote t - s = [n]. Let N= f(1) > 1 and $\nu = \ln N$. That result:

$$E\,\|\,\Phi(s,t_0,x,\omega)v\,\|^2 \geq f(1)E\,\|\,\Phi(s+1,t_0,x,\omega)v\,\|^2 \geq \cdots \geq N^n E\,\|\,\Phi(s+n,t_0,x,\omega)v\,\|^2 \geq$$

$$\geq N^{n+1}E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2 \geq Ne^{n\nu}E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2 \geq Ne^{\nu(t-s)}E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2$$

and so

$$E\,\|\,\Phi(s,t_0,x,\omega)v\,\|^2 \geq Ne^{\nu(t-s)}E\,\|\,\Phi(t,t_0,x,\omega)v\,\|^2,$$

for all $(t,s),(s,t_0)\in T,\ \ (x,v)\in X\times V$, and for all $\omega\in\Omega$.

Thus the stochastic skew-evolution semiflow C=(φ, Φ), is uniformly exponentially stable.□

Theorem 9. Let be C=(φ, Φ), an skew-product semiflow with uniformly exponential growth, and is strong measurable. Then C is uniformly exponentially stable if and only if $\exists M \geq 0$ a constant, such that:

$$\int_t^{\infty} E\,\|\Phi(s,t_0,x,\omega)v\|^2\,ds \leq M\,E\,\|\Phi(t,t_0,x,\omega)v\|^2, \tag{7}$$

for all $(t,t_0)\in T$, $(x,v)\in X\times V$, and for all $\omega\in\Omega$.

Proof. *Necessity*. Let be an stochastic skew-evolution semiflow C=(φ, Φ), uniformly exponentially stable. Then, for $N\geq 1, \nu>0$ we have from Definition 7, that

$$E\,\|\Phi(s,t_0,x,\omega)v\|^2 \leq Ne^{-\nu(s-t)}E\,\|\Phi(t,t_0,x,\omega)v\|^2, \tag{8}$$

for all $(s,t),(t,t_0)\in T$, $(x,v)\in X\times V$, and for all $\omega\in\Omega$. Then from integration of this inequality result:

$$\int_t^{\infty} E\,\|\Phi(s,t_0,x,\omega)v\|^2\,ds \leq NE\,\|\Phi(t,t_0,x,\omega)v\|^2\int_t^{\infty} e^{\nu(t-s)}ds \leq ME\,\|\Phi(t,t_0,x,\omega)v\|^2,$$

for all $(t,t_0)\in T$, $(x,v)\in X\times V$, and $\omega\in\Omega$, where $M=N\nu^{-1}$.

Sufficiency. For $t\geq t_0+1$, and ω from Definition 4 we have

$$\frac{1-e^{-\omega}}{\omega}E\,\|\Phi(t,t_0,x,\omega)v\|^2 \leq \int_{t_0}^{t} e^{-\omega(t-s)}E\left|\left\langle v^*,\Phi(t,s,\varphi(s,t_0,x,\omega))\Phi(s,t_0,x,\omega)v\right\rangle\right|^2 ds \leq$$

$$\leq M'\|v^\bullet\|\int_{t_0}^{t} E\,\|\Phi(s,t_0,x,\omega)v\|^2\,ds \leq M'M\,\|v\|\|v^\bullet\|,$$

for all $(t,t_0)\in T$, $(x,v)\in X\times V$, and $\omega\in\Omega$. So we have the relation

$$E\,\|\Phi(t,t_0,x,\omega)v\| \leq K\,\|v\|,\quad \forall(t,t,x,v,\omega)\in T\times Y\times\Omega,$$

where $K=M'\left(e^{\omega}+M/c\right)$, $c=(1-e^{-\omega})/\omega$.

Thus result that

$$(t-t_0)E\,\|\Phi(t,t_0,x,\omega)v\|^2 = \int_{t_0}^{t} E\,\|\Phi(t,t_0,x,\omega)v\|^2\,ds =$$

$$= \int_{t_0}^{t} E\,\|\Phi(t,s,\varphi(s,t_0,x,\omega),\omega)\Phi(s,t_0,x,\omega)v\|^2\,ds \leq$$

$$\leq K\int_{t_0}^{t} E\,\|\Phi(s,t_0,x,\omega)v\|^2\,ds \leq KM\,\|v\|.$$

In conclusion we obtain the relation

$$(t-t_0+1)E\,\|\Phi(t,t_0,x,\omega)v\|^2 \leq K(M+1)\,\|v\|,$$

for all $(t,t_0)\in T$, $(x,v)\in X\times V$, and $\omega\in\Omega$.

Thus for function $f:[0,\infty)\to[1,\infty)$, with property $\lim_{t\to\infty} f(t)=\infty$, definite by

$$f(t)=\frac{t+1}{K(M+1)},$$

Result from Lemma 8, that C is uniformly exponentially stable.

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