ANNALS OF FACULTY ENGINEERING HUNEDOARA – INTERNATIONAL JOURNAL OF ENGINEERING
ISSN: 1584 - 2673

# INFLUENCE OF PRIMARY AND SECONDARY PARAMETERS ON THERMAL CONDUCTIVITY MODELING FOR TWO PHASE MATERIALS

1. A.P. SENTHIL KUMAR, 2. V.PRABHU RAJA, 3. P. KARTHIKEYAN

1,2 Faculty of Mechanical Engineering Department of Mechanical Engineering
3. Faculty of Automobile Engineering Department of Automobile Engineering,
PSG College of Technology, Coimbatore, INDIA

**ABSTRACT**
In this article, the resistance based unit cell models are developed to estimate the effective thermal conductivity of the two-phase materials including the effect of various inclusions. The algebraic equations are derived using unit cell based isotherm approach for two dimensional spatially periodic medium. The geometry of the medium is considered as a matrix of touching and non-touching in-line square, octagon and hexagon cylinders. The models are used to predict the thermal conductivity of numerous two-phase materials including the influence of primary and secondary parameters (maximum conductivity ratio of 1000, concentration ranging between 0 and 1 and contact ratio varying from 0 to 1). The contact ratio is deterministic parameter when conductivity ratio is high. The concentration is deterministic parameter when conductivity ratio approaching to one. For lower conductivity ratios ($\alpha$ < 1), the non-dimensional thermal conductivity is insensitive to the contact ratios, but it is sensitive to the higher conductivity ratios ($\alpha$ >1).
**Keywords:** Effective thermal conductivity; Concentration; Conductivity ratio; Unit-cell approach; Inclusions; Two-phase materials

## 1. INTRODUCTION

The knowledge of the effective thermal conductivity of two phase materials is becoming increasingly important in the technologist developments and in many applications. As a consequence has been considerable effect in the past to develop suitable analytical model. The problem of determination of effective thermal conductivity describing heat transfer in two phase materials is serious interest for many technological applications. The importance of two phase materials like ceramics, granular materials, emulsions and metal foams lies in their applications in high performance cryogenic insulations, packed beds, heterogeneous catalysts and catalytic reactors, composite materials, power generation and powder metallurgy.

Two phase materials are essentially composed of two distinct parts with different chemical or physical structures. They consist of continuous and dispersed phases and may be grouped in to in several distinct systems based on their characteristics. The two-phase systems are generally made of solid-fluid composition with variety of shapes and sizes. Two phase systems broadly are classified as porous, suspension, emulsion, sintered, and solid-solid systems. Many natural substances such as rocks, soils, biological tissues, and man made materials such as cements, foams and ceramics can be considered as porous systems. Some of the suspending system includes graphite, aluminum, and copper with water. Two –phase systems such as cello size in polypropylene glycol, water in petroleum solvent can be considered as emulsion system. The materials such as glass, alumina, and silica are considered as a sintered system. The solid-solid systems are considered as metal alloys.

Maxwell's work [1] predicting the magnetic permittivity of a dilute suspension of spheres is the earliest reported work in the modeling of transport properties of two-phase media, but it is valid only for very low concentration of the dispersed phase. The Hashin Z. Shtrikman [2] models are the minimum and maximum bounds for predicting the thermal conductivity of the two phase system. These are the most restrictive bounds proposed and every model should incorporate these bounds as a minimum and maximum. The upper and lower limits to the conductivity of two-phase materials based on parallel and series resistances were given by Wiener [3]. Zehner and Schlunder [4] proposed a model considering the effect of particle contact as well as the effect of secondary parameters such as thermal radiation, pressure dependence, particle flattening, shape and size distribution for cylindrical unit cell containing spherical inclusions. An important deficiency in the model is that the deformation of the flux field is taken only as a function of concentration, not as a function of the conductivity ratio. Krischer [5] was among the first to describe the unit cube thermal conductivity model. The particle is represented as a cube within a large unit cube. Kunii and smith [6] proposed a unit cell model consisting of spherical particles contacting at a point and assumed that the temperature gradient is in the direction of point contact. The heat conduction near the outer region of the unit cell is through the fluid phase around it. The equivalent thermal resistance of the composite layer is based on the thermal resistances of the fluid phase in series with the solid phase. A lumped parameter model for predicting the thermal conductivity of the porous medium was described by Hsu et al. [7]. They considered various porous geometric configurations under constant heat flux conditions with one-dimensional heat conduction in a unit cell. It was reported that the stagnant thermal conductivity based on the in-line touching cube model with a contact ratio of 0.13 showed good agreements with experimental data of a packed sphere bed. Samantray et al. [8] proposed a comprehensive conductivity model by considering the primary parameters based on unit cell and field solution approaches. Later, the validity of the model was extended to predict the effective conductivity of various binary metallic mixtures with a high degree of accuracy Karthikeyan and Reddy [9]. Reddy and Karthikeyan [10] developed the collocated parameter model based on the unit cell approach for predicting the effective thermal conductivity of the two-phase materials.

The aim of this paper is considering various primary and secondary parameters such as concentration of dispersed phase, conductivity ratio and contact ratio for predicting the effective thermal conductivity of two-phase materials. In this paper, a resistance based unit cell model is proposed with parallel isotherms to estimate the thermal conductivity of two-phase materials with various inclusions. In this approach, the influence of concentration, conductivity ratio and contact ratio are investigated.

## 2. DEVELOPMENT OF CORRELATION FOR PREDICTING AN EFFECTIVE THERMAL CONDUCTIVITY

The development of resistance based unit cell model model for estimating the effective thermal conductivity based on the material micro and nano-structure is extremely important for thermal design and analysis of two-phase systems is reported by Reddy and Karthikeyan [10]. The electric resistance analogy leads to algebraic expressions for stagnant thermal conductivity of the two-phase materials. The main feature of resistance method is to assume one-dimensional heat conduction in a unit cell. The unit cell is divided into three parallel layers, namely, solids, fluid or composite layers normal to the temperature gradient. The effective thermal conductivity of two-phase system is determined by considering equivalent electrical resistances of parallel and series in the unit cell model. The thermal conductivity of the composite layer is obtained using the series model.

The development of effective thermal conductivity modeling for two-phase materials for square cylinder has been developed and reported by Reddy and Karthikeyan [10]. The effective thermal conductivity of two-dimensional square cylinder is calculated for parallel isotherm conditions as follows:

Total resistance offered by the square cylinder in the unit cell is given as [10]:

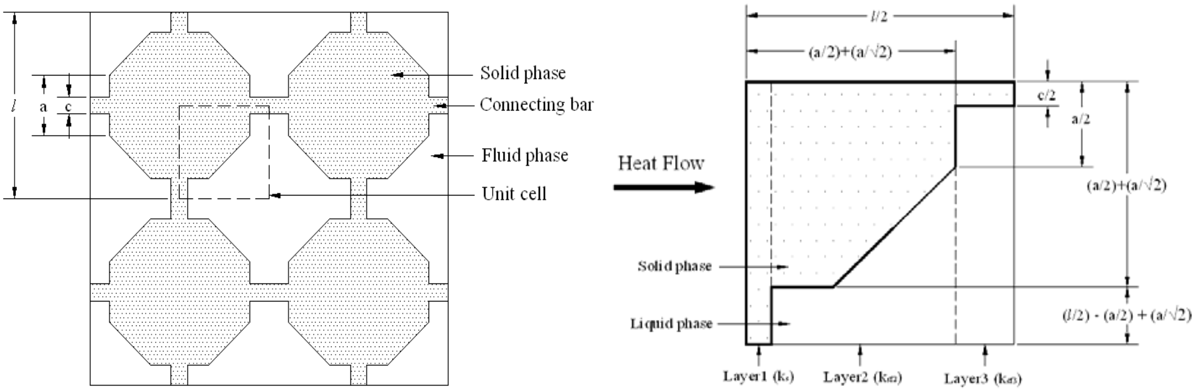
$$R_{total} = R_1 + R_{eff2} + R_{eff3} \tag{1}$$

$$R_{total} = \left[\frac{1}{k_f\, l}\right]\left[\frac{\varepsilon\lambda}{\alpha} + \frac{\varepsilon(1-\lambda)(\alpha+\varepsilon(1-\alpha))}{\alpha[\varepsilon(\alpha-1)(1-\varepsilon)+1]} + \frac{(1-\varepsilon)(\alpha+(1-\alpha)\varepsilon\lambda)}{\alpha\{(\varepsilon\lambda)(1-\varepsilon\lambda)(\alpha-1)+1\}}\right] \tag{2}$$

The non-dimensional thermal conductivity of two-dimensional square cylinder is given as [10]:

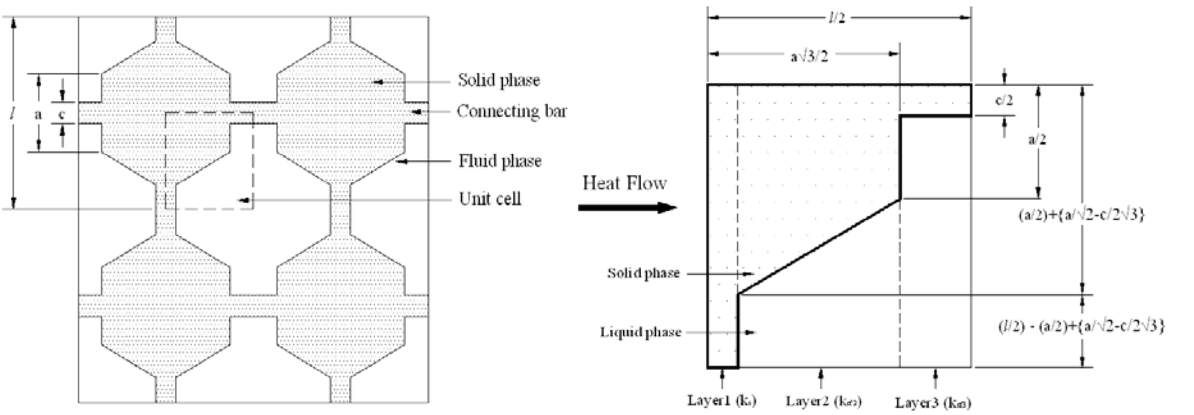
$$K = \frac{k_{eff}}{k_f} = \left[\frac{\varepsilon\lambda}{\alpha} + \frac{\varepsilon(1-\lambda)(\alpha+\varepsilon(1-\alpha))}{\alpha[\varepsilon(\alpha-1)(1-\varepsilon)+1]} + \frac{(1-\varepsilon)(\alpha+(1-\alpha)\varepsilon\lambda)}{\alpha\{(\varepsilon\lambda)(1-\varepsilon\lambda)(\alpha-1)+1\}}\right]^{-1} \tag{3}$$

The effective thermal conductivity of the two dimensional medium can be estimated by considering an octagon and hexagon cylinders with cross-section ‘a x a’ having a connecting bar width of ‘c’ as shown in Figs.1 [a]-2[a]. The stagnant thermal conductivity of the two-dimensional periodic medium is the finite contact between the spheres by connecting plates with ‘c/a’ denoting the contact parameter. Because of the symmetry of the plates, one fourth of the cross-section has been considered as a unit cell and is shown in Figs.1 [b]-2[b].




(a) (b)

Fig.1. Two-dimensional spatially periodic two-phase system: (a) Touching Octagon cylinder
(b) Unit cell of Octagon cylinder




(a) (b)

Fig.2. Two-dimensional spatially periodic two-phase system: (a) Touching Hexagon cylinder
(b) Unit cell of Hexagon cylinder

The unit cell consists of three rectangular layers normal to the direction of heat flow. The thermal conductivity of the solid and fluid layer is obtained based on a series model. For octagon cylinder, first rectangular layer is fully occupied by the solid with a dimension of ($l$/2) (c/2) and other two rectangular layers consists of solid and fluid phases with a dimension of ($l$/2) {[a/2 + a/√2]-c/2} and ($l$/2) ($l$/2-(a/2+a/√2)) respectively. The effective thermal conductivity of two-dimensional octagon cylinder is calculated for parallel isotherm conditions as follows:

$$R_{total} = R_1 + R_{eff2} + R_{eff3} \tag{4}$$

Total resistance offered by the octagon cylinder in the unit cell is given as:

$$R_{total} = \frac{\varepsilon\lambda}{\alpha} + \left[\frac{\frac{1}{\alpha}\left[\left(\frac{(1\text{-}\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]\times\frac{2\varepsilon\left([1\text{-}\lambda]+\sqrt{2}\right)}{\frac{k_{sf2}}{k_f}\left\{\left[2\left(1\text{-}\left(\varepsilon+\varepsilon\sqrt{2}\right)\right)\right]+\left[\varepsilon(1\text{-}\lambda)+\sqrt{2}\right]\right\}}}{\frac{1}{\alpha}\left[\left(\frac{(1\text{-}\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]+\frac{2\varepsilon\left([1\text{-}\lambda]+\sqrt{2}\right)}{\frac{k_{sf2}}{k_f}\left\{\left[2\left(1\text{-}\left(\varepsilon+\varepsilon\sqrt{2}\right)\right)\right]+\left[\varepsilon(1\text{-}\lambda)+\sqrt{2}\right]\right\}}}\right] + \left[\frac{\left[1-\left(\varepsilon+\varepsilon\sqrt{2}\right)\right]\left[\alpha+(1-\alpha)\varepsilon\lambda\right]}{\alpha\left\{(1-\varepsilon\lambda)+\varepsilon\lambda\left[\alpha+(1-\alpha)\varepsilon\lambda\right]\right\}}\right] \tag{5}$$

The non-dimensional thermal conductivity of two-dimensional octagon cylinder is given as:

$$K = \frac{k_{eff}}{k_f} = \left[\frac{\varepsilon\lambda}{\alpha} + \left[\frac{\frac{1}{\alpha}\left[\left(\frac{(1\text{-}\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]\times\frac{2\varepsilon\left([1\text{-}\lambda]+\sqrt{2}\right)}{\frac{k_{sf2}}{k_f}\left\{\left[2\left(1\text{-}\left(\varepsilon+\varepsilon\sqrt{2}\right)\right)\right]+\left[\varepsilon(1\text{-}\lambda)+\sqrt{2}\right]\right\}}}{\frac{1}{\alpha}\left[\left(\frac{(1\text{-}\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]+\frac{2\varepsilon\left([1\text{-}\lambda]+\sqrt{2}\right)}{\frac{k_{sf2}}{k_f}\left\{\left[2\left(1\text{-}\left(\varepsilon+\varepsilon\sqrt{2}\right)\right)\right]+\left[\varepsilon(1\text{-}\lambda)+\sqrt{2}\right]\right\}}}\right] + \left[\frac{\left[1-\left(\varepsilon+\varepsilon\sqrt{2}\right)\right]\left[\alpha+(1-\alpha)\varepsilon\lambda\right]}{\alpha\left\{(1-\varepsilon\lambda)+\varepsilon\lambda\left[\alpha+(1-\alpha)\varepsilon\lambda\right]\right\}}\right]\right]^{-1} \tag{6}$$

Similarly, for hexagon cylinder, first rectangular layer is fully occupied by the solid with a dimension of ($l$/2) (c/2) and other two rectangular layers consists of solid and fluid phases with a dimension of ($l$/2) [a√3/2 -c/√2] and ($l$/2) [$l$/2-(a√3/2)] respectively. The effective thermal conductivity of two-dimensional hexagon cylinder is calculated for parallel isotherm conditions as follows:

$$R_{total} = R_1 + R_{eff2} + R_{eff3} \tag{7}$$

Total resistance offered by the hexagon cylinder in the unit cell is given as:

$$R_{total} = \frac{\varepsilon\lambda}{\alpha} + \left[ \frac{\dfrac{2\sqrt{3}\varepsilon\left(\sqrt{3}-\lambda\right)}{\left[\left(\sqrt{3}-\lambda\right)+2\sqrt{3}\right]\times\dfrac{k_{sf2}}{k_f}\times\left[2\sqrt{3}\left(1-\left(\varepsilon+\left(\varepsilon-\dfrac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon\left(\sqrt{3}-\lambda\right)\right]}}{\dfrac{1}{\left[\left(\sqrt{3}-\lambda\right)+2\sqrt{3}\right]}+\dfrac{\varepsilon}{\dfrac{k_{sf2}}{k_f}\times\dfrac{1}{\alpha}\times\left[2\sqrt{3}\left(1-\left(\varepsilon+\left(\varepsilon-\dfrac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon\left(\sqrt{3}-\lambda\right)\right]}} \right] + \frac{\dfrac{\left(1-\varepsilon\sqrt{3}\right)}{\dfrac{k_{sf3}}{k_f}\left[\varepsilon\lambda\left(1-\varepsilon\lambda\right)\right]}}{\left[\dfrac{1}{\varepsilon\lambda}+\dfrac{1}{\left\{\dfrac{1}{\alpha}\times\left(\dfrac{k_{sf3}}{k_f}\right)\left(1-\varepsilon\lambda\right)\right\}}\right]} \tag{8}$$

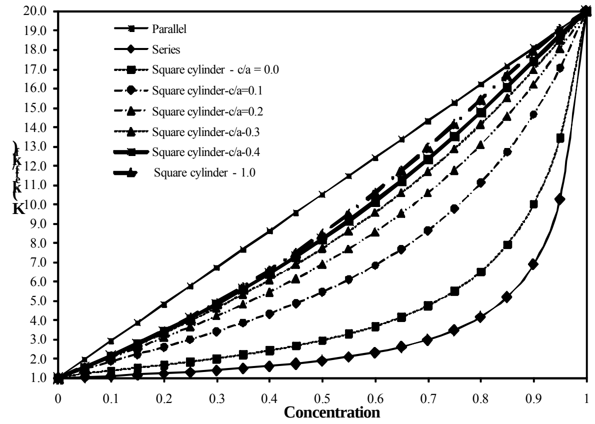
The non-dimensional thermal conductivity of two-dimensional hexagon cylinder is given as:

$$K = \frac{k_{eff}}{k_f} = \left[ \frac{\varepsilon\lambda}{\alpha} + \left[ \frac{\dfrac{2\sqrt{3}\varepsilon\left(\sqrt{3}-\lambda\right)}{\left[\left(\sqrt{3}-\lambda\right)+2\sqrt{3}\right]\times\dfrac{k_{sf2}}{k_f}\times\left[2\sqrt{3}\left(1-\left(\varepsilon+\left(\varepsilon-\dfrac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon\left(\sqrt{3}-\lambda\right)\right]}}{\dfrac{1}{\left[\left(\sqrt{3}-\lambda\right)+2\sqrt{3}\right]}+\dfrac{\varepsilon}{\dfrac{k_{sf2}}{k_f}\times\dfrac{1}{\alpha}\times\left[2\sqrt{3}\left(1-\left(\varepsilon+\left(\varepsilon-\dfrac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon\left(\sqrt{3}-\lambda\right)\right]}} \right] + \frac{\dfrac{\left(1-\varepsilon\sqrt{3}\right)}{\dfrac{k_{sf3}}{k_f}\left[\varepsilon\lambda\left(1-\varepsilon\lambda\right)\right]}}{\left[\dfrac{1}{\varepsilon\lambda}+\dfrac{1}{\left\{\dfrac{1}{\alpha}\times\left(\dfrac{k_{sf3}}{k_f}\right)\left(1-\varepsilon\lambda\right)\right\}}\right]} \right]^{-1} \tag{9}$$

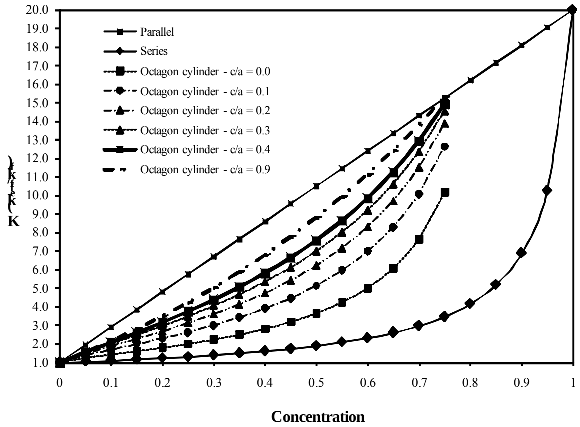
## 3. RESULTS AND DISCUSSIONS

The effective thermal conductivity of a two-phase system mainly depends on concentration, conductivity ratio, size, shape and contact ratio. The effect of concentration (υ) on the non-dimensional thermal conductivity of two-dimensional geometry's for low and high conductivity ratio's (α = 20, 100, 1000) have been investigated and are respectively shown in Figs. [3-5]. The present model with various inclusions lies between parallel and series lines for conductivity ratio (α) = 20 and contact ratio (λ) = 0-0.4 are shown in Fig.3. The contact ratio is having major influence on effective thermal conductivity of two-phase materials when contact ratio varying 0 to 0.3, but there is no impact beyond 0.4. For octagon cylinder, the present correlation is applicable for concentration varying from 0 to 0.7, for further increment in the concentration; the non-dimensional thermal conductivity is increasing beyond the upper bound. Similarly for hexagon cylinder the present equation is applicable, if the concentration varying from 0 to 0.8. Both the models are not applicable for concentration beyond 0.8, because the limitations in the shape of the models. Similarly for high conductivity ratio (α = 100, 1000), the deviation between the bounds are significance, this is due to increasing solid phase conductivity [Figs.4-5].

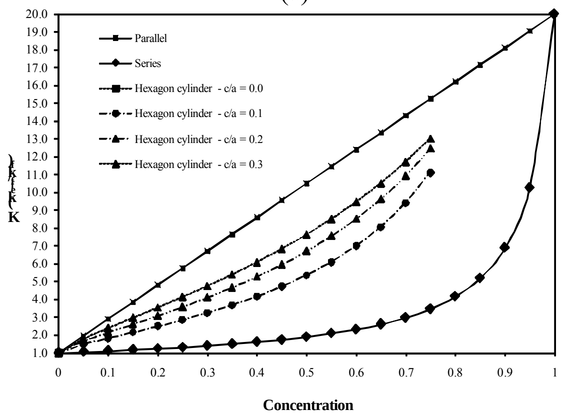
The predicted non-dimensional thermal conductivity increases with the conductivity ratio and contact ratios are shown in Figs 6-8. For lower (υ= 0.1 and 0.3) concentrations, the deviation between all models are almost same. For higher concentration (υ = 0.8) and higher conductivity ratios, the deviation is more within the models. For λ = 0.1, the variation in the non-dimensional thermal conductivity has been considerable. It can be seen that the contact ratio (λ) is the deterministic parameter when the conductivity ratio (α) is high where as concentration is deterministic parameter when α is approaching to one. Similarly, for lower conductivity ratios (α < 1), the non-dimensional thermal conductivity is insensitive to the contact ratios, but it is sensitive to the higher conductivity ratios (α >1). From the iso-conductance point, α = 1, the non-dimensional thermal conductivity approaches to unity for all the models with the same slope. The present model shows a good trend for the concentrations 0.3 and 0.8. For low values of conductivity ratio (α), thermal conductivity estimations of all the models are comparable, but they deviate when the conductivity ratio approaches to 1000.




(a)

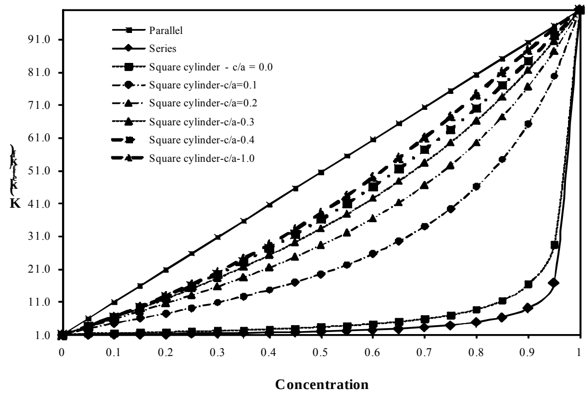



(b)

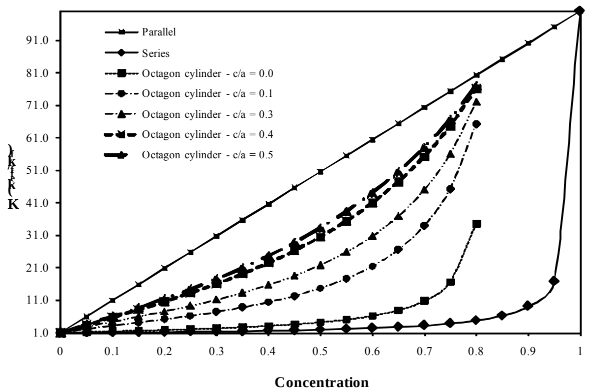



(c)

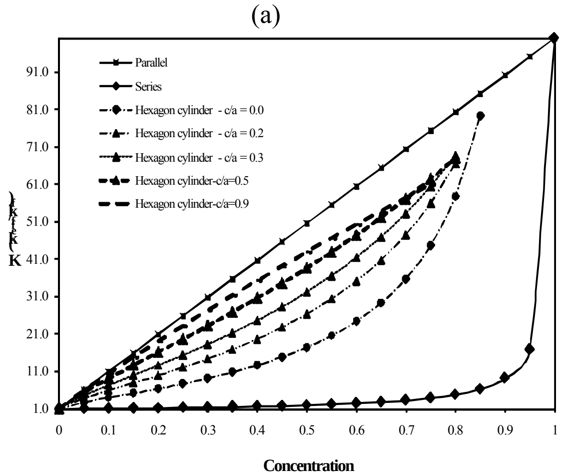
Fig.3 (a-c). Variation of non-dimensional thermal conductivity with concentration for various inclusion for α = 20




(a)

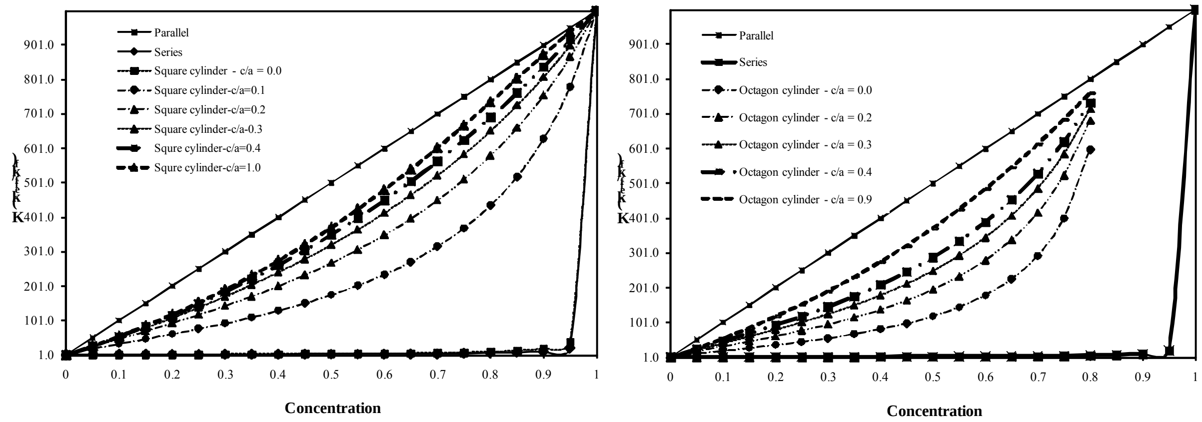



(b)

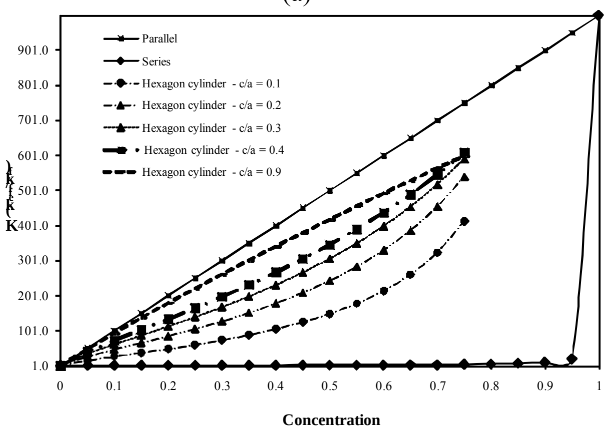



(c)

Fig.4 (a-c). Variation of non-dimensional thermal conductivity with concentration for various inclusion for α = 100

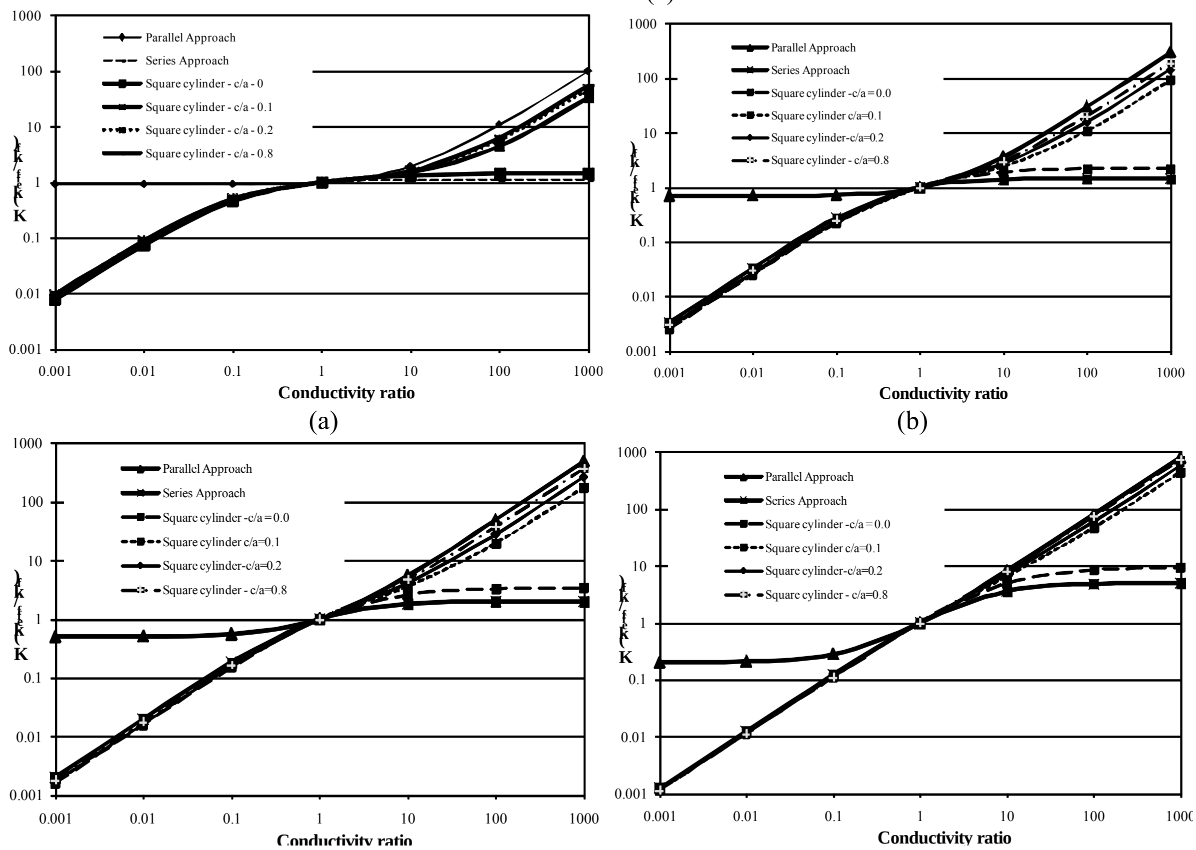



(a) (b)




(c)

Fig.5 (a-c). Variation of non-dimensional thermal conductivity with concentration for various inclusion for α = 1000




(a) (b) (c) (d)

Fig. 6. Variation of non-dimensional thermal conductivity with contact ratios for square cylinder for (a) concentration - (υ) = 0.1, (b) υ = 0.3, (c) υ = 0.5, (d) υ = 0.8

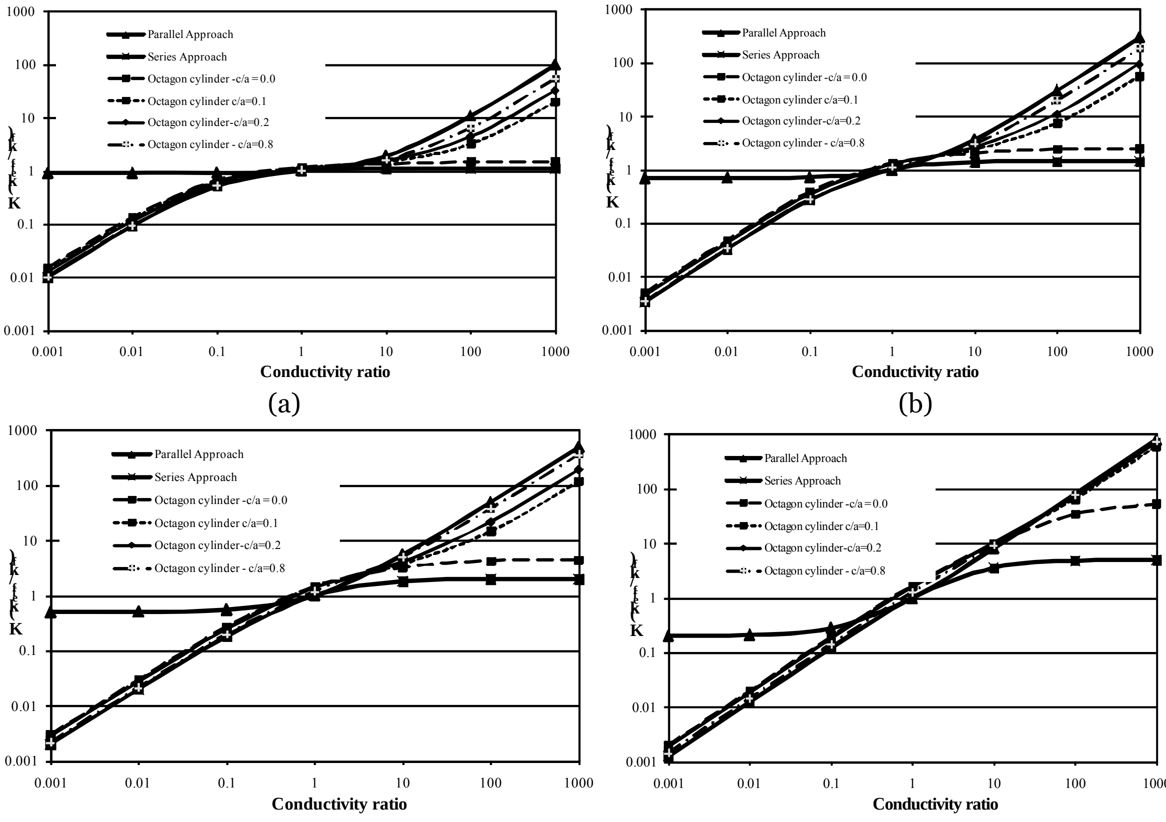



Fig. 7. Variation of non-dimensional thermal conductivity with contact ratios for octagon cylinder for (a) concentration - (υ) = 0.1, (b) υ = 0.3, (c) υ = 0.5, (d) υ = 0.8

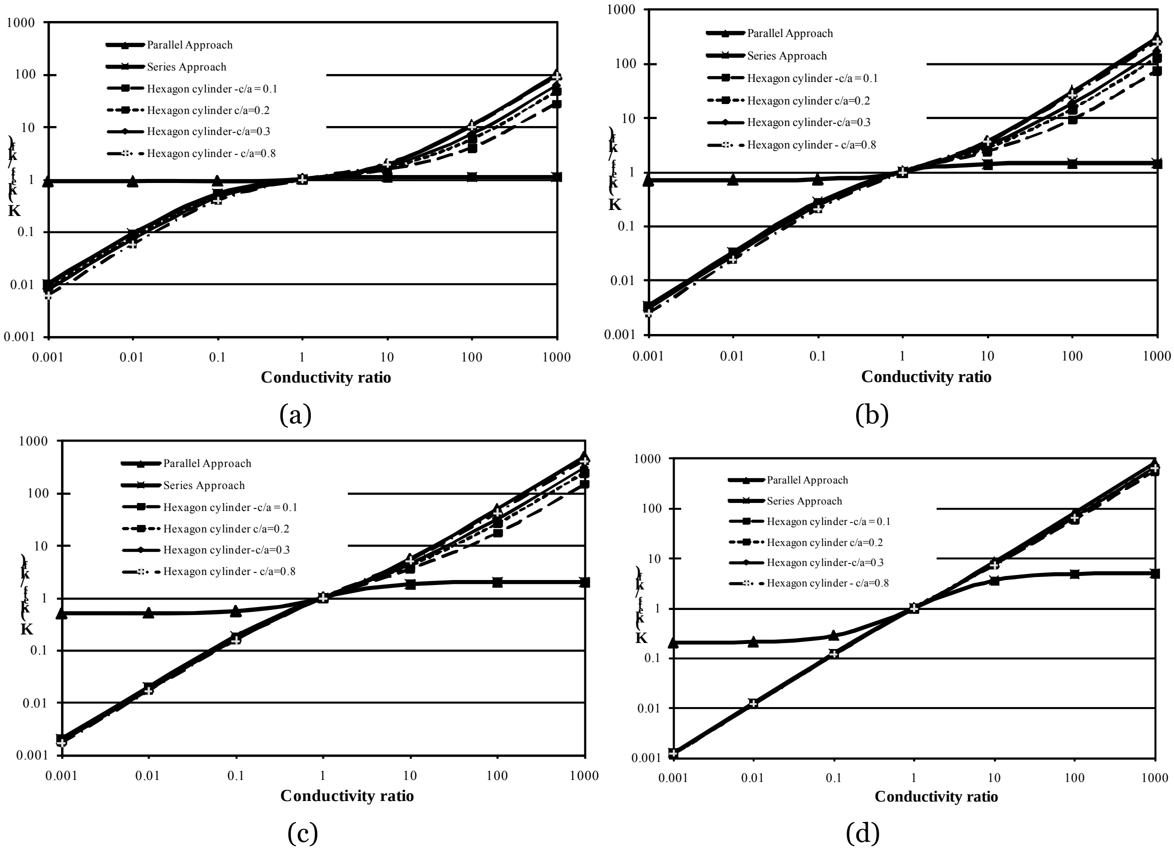



Fig. 8. Variation of non-dimensional thermal conductivity with contact ratios for hexagon cylinder for (a) concentration - (υ) = 0.1, (b) υ = 0.3, (c) υ = 0.5, (d) υ = 0.8

## 4. CONCLUSIONS

The resistance based unit cell models are developed with the effect of various inclusions for estimating the effective thermal conductivity of the two-phase materials. The effects of concentration, conductivity and contact ratios on the non-dimensional thermal conductivity of various inclusions have been investigated. The present models can be extensively used for predicting the effective thermal conductivity of two-phase materials used in the engineering applications.

**Nomenclature**

| | |
|---|---|
| a | Length of the octagon and hexagon cylinders |
| c | Width of the connecting plate in the octagon and hexagon cylinders |
| K | Non-dimensional thermal conductivity of the two-phase materials ($k_{eff}/k_f$) |
| $k_{eff}$ | Effective thermal conductivity of two-phase materials, ($Wm^{-1}K^{-1}$) |
| $k_f$ | Fluid or continuous thermal conductivity, ($Wm^{-1}K^{-1}$) |
| $k_s$ | Solid or dispersed thermal conductivity, ($Wm^{-1}K^{-1}$) |
| $k_{sf}$ | Equivalent thermal conductivity of a composite layer, ($Wm^{-1}K^{-1}$) |
| R | Thermal resistance, ($m^2K^{-1}W^{-1}$) |
| *l* | Length of the unit cell, (m) |

**Greek Symbols**

| | |
|---|---|
| α | Conductivity ratio ($k_s/k_f$)` |
| ε | Length Ratio (a/l) |
| λ | Contact ratio (c/a) |
| υ | Concentration |

**Subscripts**

| | |
|---|---|
| eff | Effective |
| exp | Experimental |
| pre | Predicted |

## REFERENCES


[1.] *Maxwell, J.C. A Treatise on Electricity and Magnetism, pp.365, Clarendon Press, Oxford, UK. (1873)*

[2.] *Hashin, Z. Shtrikman, S., A Variational Approach to the Theory of the Effective Magnetic Permeability of Multiphase Materials, Journal of Applied Physics, 33, pp. 3125- 3131, (1962).*

[3.] *Wiener, O. Lamellare doppelbrechung, Phys.Z., 5 ,332-338, (1904).*

[4.] *Zehner, P. Schlunder, E.U., On the Effective Heat Conductivity In Packed Beds With Flowing Fluid at Medium and High Temperatures, Chemical Engineering Technology,42 pp.933 – 941, (1970).*

[5.] *Krischer,O. Die wissenschaftlichan grundlagen der trocknungstechnik, 1st ed., Springer-Verlag, Heidelberg, Germany. (1956)*

[6.] *Kunii, D., Smith,J.M. Heat Transfer Characteristics In Porous Rocks, American Institute of Chemical Engineering Journal, 6 pp.71-78, (1960).*

[7.] *Hsu CT, Cheng P, Wong KW A lumped parameter model for stagnant thermal conductivity of spatially periodic porous media, ASME-Journal of Heat Transfer 117:264-269. (1995).*

[8.] *Samantray, P.K, Karthikeyan, P., Reddy, K.S. Estimating Effective Thermal Conductivity of Two-Phase Materials, International Journal of Heat and Mass Transfer, 49 , 21-22, pp. 4209-4219,(2006).*

[9.] *Karthikeyan, P., Reddy, K.S. Effective Conductivity Estimation of Binary Metallic Mixtures, International Journal of Thermal Sciences, 46, pp.419-425, (2007).*

[10.] *Reddy, K.S. Karthikeyan, P. Estimation of Effective Thermal Conductivity of Two-Phase Materials Using Collocated Parameter Model, Heat Transfer Engineering, 30, pp.998-1011. (2009).*