ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
ISSN: 1584 - 2673

ANALYSIS OF REGRESSION MODEL OF FUNCTIONAL DEPENDENCY IN IMPACT FORCE FROM HEIGHT AND WEIGHT OF RAM FOR CONVEYOR BELT

[1,2] Miriam ANDREJIOVÁ, Anna PAVLISKOVÁ

[1,2] Technical University of Košice, Faculty of Mechanical Engineering,
Department of Applied Mathematics, Košice, SLOVAKIA

ABSTRACT:
The article is about regression model of functional dependency on impact force from height of falling and weight of ram for selected conveyor belt. First part of article is devoted to point and interval estimate of model's parameters. In the next part we test statistical significance of regression model and model's parameters.
KEYWORDS: regression model, parameter estimates, test for significance, partial residuals plots

1. INTRODUCTION

In the Department of logistics and production systems of the Technical University in Košice was established the laboratory, which serves for the simulation and modelling constructional parts of transporting devices, including conveyor belts and modern experimental device used for testing conveyor belts from aspect of their resistance against breakdowns. The last years were performed impact tests for various type of conveyor belts. In all cases were applied rubber belts with multicomponent textile structure.

Based on experimental data was created regression model of function dependency on impact force F_R from height h of falling and ram weight m in dependence on head of ram.

Mathematic model was created from regression function

$$y_i = \beta_0 + \beta_1 \cdot x_{i1} + \beta_2 \cdot x_{i2} + \beta_3 \cdot x_{i3} + \beta_4 \cdot x_{i4} + \beta_5 \cdot x_{i5} + \varepsilon_i \,, \tag{1}$$

where y_i is dependent random variables of impact force, $\beta_0, \beta_1, \ldots, \beta_5$ are model parameters and ε is random error. Variables x_i, $i = 1, 2, 3, 4, 5$ were selected as follows

$$x_1 = h, x_2 = m, x_3 = h^2, x_4 = m^2, x_5 = h \cdot m \,. \tag{2}$$

Height of ram falling was changed from height of 0.2m to height of 2.6m with differential 0.2m. Weight of ram itself is 50kg and in next measurements weight was changed with differential from 10kg up to 100kg in the case of spherical head of ram and in case of using pyramid shaped head of ram weight was changed from 50kg up to 80kg.

The regression model was estimated by the sample regression model

$$\hat{y} = b_0 + b_1 \cdot h + b_2 \cdot m + b_3 \cdot h^2 + b_4 \cdot m^2 + b_5 \cdot h \cdot m \,, \tag{3}$$

where $b_0, b_1, \ldots, b_n$ are estimations of unknown parameters $\beta_0, \beta_1, \ldots, \beta_n$ and $\hat{y}_i$ is adjustable or foreseen value of impact force.

In following part of article we are concerned only with regression model for case of impact Force, provided by using spherical head of ram for conveyor belt P630/3.

FIGURE 1. Testing board with a hydraulic system and spherical head of ram

2. ESTIMATE OD MODEL'S PARAMETERS

Point estimate of functional dependency model for impact force from height of falling and weight of spherical head of ram for conveyor belt P630/3 is expressed by equation

$$F_R = -11{,}0285 + 2{,}5929 \cdot h + 0{,}2974 \cdot m - 0{,}6693 \cdot h^2 - 0{,}0014 \cdot m^2 + 0{,}1129 \cdot h \cdot m \,. \qquad (4)$$

In figure is created graph of empirical (measured) values and theoretical (calculated) values of impact force for ram with spherical head.

Quantification of lower and upper limits of interval estimation/estimate of parameters will be expressed by formula

$$b_j \pm s_{b_j} \cdot t_{1-\frac{\alpha}{2}}(n-k-1), \;\; j = 0, 1, 2, 3, 4, 5 \,. \qquad (5)$$

where $t_{0,975}(66) = 2{,}29369$ (resp. $t_{0,995}(66) = 2{,}90447$).

TABLE1. Parameter estimates

	Parameter estimate	Lower 95%	Upper 95%	Lower 90%	Upper 90%
intercept	-11,0285	-12,9457	-9,11134	-12,6305	-9,42658
h	2,59294	1,81964	3,366241	1,946793	3,239088
m	0,297389	0,248669	0,346109	0,25668	0,338098
h²	-0,66927	-0,84022	-0,49832	-0,81211	-0,52643
m²	-0,00141	-0,00172	-0,0011	-0,00167	-0,00115
h.m	0,112928	0,105824	0,120032	0,106993	0,118864

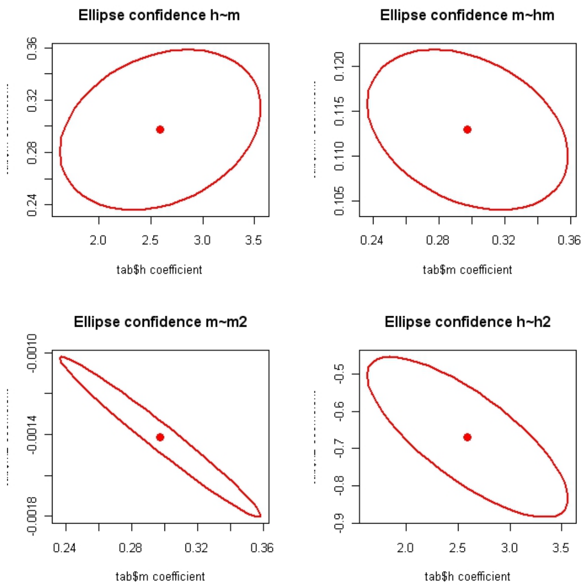

FIGURE 2. Ellipse confidence

We can use interval parameter estimate of regression model also for statistical testing of models parameter significance. If null is in confidence interval of parameter then parameter is statistically insignificant.

Ellipse confidence (Fig. 2) is for some pairs of two regression coefficients of model.

3. TEST FOR SIGNIFICANCE OF REGRESSION MODEL

The test for significance of regression in the case of multiple linear regression analysis is carried out using the analysis of variance.

On significance level $\alpha = 0{,}05$ (resp. $\alpha = 0{,}01$) we test the significance of given of estimated regression model. We use F- test, by witch we test null hypothesis

$$H_0: \; \beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 \,,$$

against $H_1 : \beta_i \neq 0$ for at least one i.

(resp. H_0 : *All regression coefficients are null* in contrary to alternative hypothesis H_1 : *where at least one regression coefficient is non null.*

The test statistic is

$$F = \frac{MS_M}{MS_R} = \frac{(n-k-1) \cdot \sum_{i=1}^{n} (\hat{y}_i - \overline{y})^2}{k \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}, \qquad (6)$$

where MS_M is the regression mean square and MS_R is the error mean square. If the null hypothesis is true, then the statistic F follows the $F-$ distribution with $(k; n-k-1)$ degrees of freedom. The null hypothesis is rejected, if the calculated statistic F is such that $F > F_{1-\alpha}(k; n-k-1)$.

TABLE 2. Analysis of Variance

Variability	Sum of squares	Degrees of freedom	Mean Squares	F statistic
Model	$SS_M = 3553{,}596$	$df_M = 5$	$MS_M = 71{,}7192$	$F = 6805{,}131$
Residual	$SS_R = 6{,}89296$	$df_R = 66$	$MS_R = 0{,}10444$	
Total	$SS_T = 3560{,}489$	$df_T = 71$		

The critical value for this test is $F_{0,95}(5;66) = 2.354$ (resp. $F_{0,99}(5;66) = 3.308$). According to this (and p-value $p = 4{,}41.10^{-88} < \alpha$), we reject null hypothesis on significance level α and we could assume, that at least one coefficient is significant and regression model is statistically significant of level $\alpha = 0{,}05$ (resp. $\alpha = 0{,}01$).

4. TEST SIGNIFICANCE OF REGRESSION PARAMETERS

Now we test the statistical significance of estimates of individual regression coefficients β_j in regression model.

The null hypothesis to test significance of a particular regression coefficient is $H_0 : \beta_j = 0$ against $H_1 : \beta_j \neq 0$. The test statistic

$$t = \frac{b_j}{s_{b_j}}. \tag{7}$$

has the Student's t-distribution with $(n-k-1)$ degrees of freedom, where b_j is parameter estimate and s_{b_j} is standard error. We reject null hypothesis at a significance level α, if $|t| > t_{1-\frac{\alpha}{2}}(n-k-1)$, where k is number of regression model parameters.

TABLE3. Parameter estimates and t-test

Parameter	Parameter Estimate	Standard Error	t statistic	p-value
b_0	-11,0285	0,96024	-11,485*	$2{,}17.10^{-17}$
b_1	2,59294	0,38732	6,695*	$5{,}64.10^{-09}$
b_2	0,59739	0,02440	12,187*	$1{,}49.10^{-18}$
b_3	-0,66927	0,08562	-7,817*	$5{,}64.10^{-11}$
b_4	-0,00141	0,00016	-9,043*	$3{,}65.10^{-13}$
b_5	0,11293	0,00356	31,739*	$1{,}08.10^{-41}$

In the table 3 are showed parameter estimates, standard error, t statistic for partial parameters and evaluation of contribution of explanatory variables in regression model.

Since the p-value is less than significance level α ($p < \alpha$), we do not reject null hypothesis and it is concluded that all regression coefficients are none null (*) and they have significant effect on impact force.

5. PARTIAL RESIDUALS PLOTS

They are denoted as component + residual plots, which are used mainly for identification various type of nonlinearity in case of incorrect proposed regression model. Linear continuance with zero residuals in all partial graphs indicates to the accurate choice of suggested regression model.

Linearity in all partial residuals graphs confirms the accurate choice of suggested regression model (Fig. 3).

6. MULTIPLE DETERMINATION AND CORRELATION COEFFICIENT

The coefficient of multiple determinations shows the percentage of variations of dependent variable Y which is described by common influence of independent variables which are involved in this model. The coefficient of multiple determinations is $r^2 = 0{,}9981$, and the coefficient of multiple correlations is $r = 0{,}99905$.

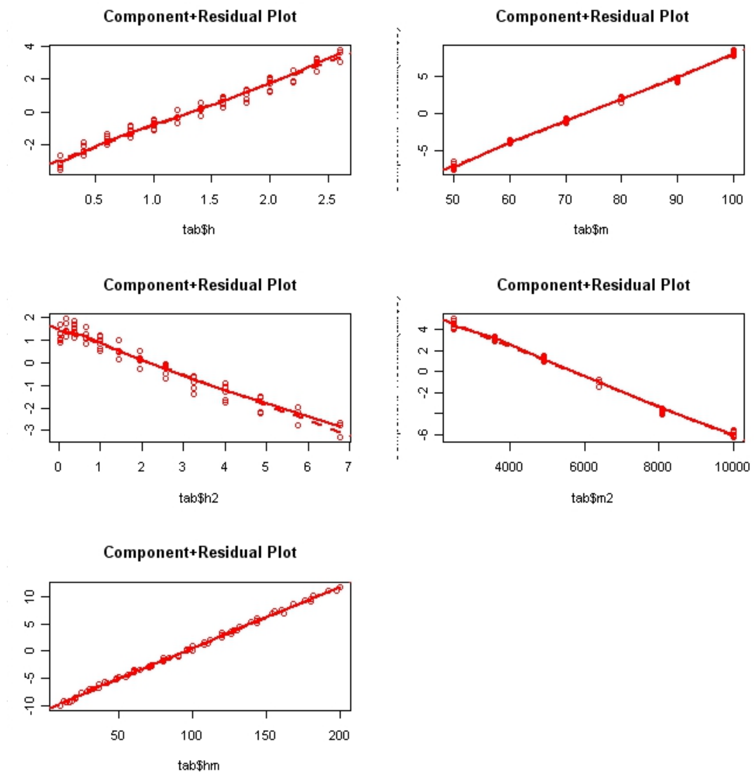

FIGURE 3. Partial residuals plots

We should take care of the number of independed variables and of sample size and we calculated too the adjusted coefficient of multiple determination

$$r_{adj}^2 = 1 - \left(1 - r^2\right) \cdot \frac{n-1}{n-k-1},$$

where n is the sample of size and k is number of independent variables. The adjusted coefficient of multiple determinations is $r_{adj}^2 = 0{,}9979$.

We test the statistical significance of multiple correlation coefficient ρ by performing test. The null hypothesis is $H_0 : \rho = 0$ against alternative hypothesis $H_1 : \rho \neq 0$.

The test statistic is $F = \dfrac{(n-k-1)\cdot r^2}{k \cdot \left(1 - r^2\right)}$ and the null hypothesis is rejected on significance level α, if $F > F_{1-\alpha}(k; n-k-1)$.

Since value of test statistic is $F = 6934{,}168 > F_{0{,}95}(5; 66) = 2.354$ (resp. $F_{0{,}99}(5; 66) = 3.308$), the null hypothesis can be rejected and it can be assumed, that on significance level $\alpha = 0{,}05$ (resp. $\alpha = 0{,}01$) between described variable and explanatory variables is statistically significant linear dependency.

7. THE CONCLUSIONS

From test results of statistic significance of model and individual coefficients of regression model as well as from partial residual graphs resulted that, suggested model is accurate. We can explain by this model variability of impact force effected in 99.81%. resp. 0.19% are caused by factors non-implied in model or other explanatory variables and accidental influential. In the next part we recommend to research/examine fulfilment of conditions for accidental component and diagnosing extreme and influential values of model.

ACKNOWLEDGEMENTS
This work was supported by the Slovak Grant Agency for Science VEGA 1/0543/10.

REFERENCES / BIBLIOGRAPHY

[1.] GRINČOVÁ, A. – BEREŽNÝ, Š. – MARASOVÁ, D.: *Tvorba regresného modelu na základe experimentálnych skúšok dopravných pásov proti prierazu. In: Acta Montanistica Slovaca. vol. 1 (2009), p. 113-120. ISSN 1335-1788.*
[2.] GRINČOVÁ, A. – BEREŽNÝ, Š.: *Matematické modely správania sa opravných pásov. In: Science & military, Roč. 4, č.1 (2009), p. 5-9. Internet: < www.aos.sk/casopisy/science/index.html >*
[3.] HEBÁK, P. – HUSTOPECKÝ, J.: *Vícerozměrné statistické metody (2). Praha. Informatorium, 2005. p. 240. ISBN 80-7333-036-9.*
[4.] MELOUN, M – MILITKÝ, J.: *Statistická analýza experimentálních. Praha. ACADEMIA, 2004. 953 p. ISBN 80-200-1254-0*
[5.] ŠOLTÉS, E.: *Regresná a korelačná analýza s aplikáciami. Bratislava. Iura Edition, 2007. 287 p. ISBN* 978-80-8078-163-7.