MHD AND CHEMICAL REACTION EFFECTS ON FREE CONVECTION FLOW WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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Abstract
An analytical study is performed to examine the effects of chemical reaction and a uniform transverse magnetic field on the unsteady free convection and mass transform flow of a viscous incompressible electrically-conducting fluid past an exponentially accelerated infinite vertical plate. The plate temperature and the concentration level near the plate are raised linearly with time. The magnetic lines of force are assumed to be fixed relative to the plate. Expressions for the velocity field and skin friction are obtained by the Laplace transform technique. The influence of the various parameters, entering into the problem, on the velocity field and skin friction is extensively discussed with the help of graphs.

Keywords: MHD, Chemical reaction, free convection, mass diffusion, vertical plate, Laplace transform method.

1. INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magneto hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its applications in MHD pumps, MHD bearings etc. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. The thermal physics of hydro magnetic problems with mass transfer is of interest in power engineering and metallurgy.

Diffusion rates can be altered tremendously by chemical reactions. The effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate is proportional to the n\textsuperscript{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role in chemical process industries such as food processing and polymer production.

Chambre and Young [1] have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Gupta [2] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [3] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen kumar [4]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [5]. Basant kumar Jha [6] studied MHD free convection and mass transform flow through a porous medium. Later Basant kumar Jha et al. [7] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Das et al. [8] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [9]. The
dimensionless governing equations were solved by the usual Laplace Transform technique. Muthucumaraswamy and Senthil Kumar [10] studied heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. Recently Muthucumaraswamy et al. [11] studied mass transfer effects on exponentially accelerated isothermal vertical plate.

The object of the present paper is to study the effects chemical reaction and a uniform transverse magnetic field (fixed relative to the plate) on the free convection and mass transform flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique.

2. MATHEMATICAL ANALYSIS

The unsteady flow of an incompressible and electrically conducting viscous fluid past an infinite vertical plate with variable temperature and mass diffusion has been considered. A magnetic field (fixed relative to the plate) of uniform strength \( B_0 \) is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed to be in \( x' \)-direction which is taken along the vertical plate in the upward direction. The \( y' \)-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature \( T_\infty \) in the stationary condition with concentration level \( C_\infty \) at all points. At time \( t' > 0 \), the plate is exponentially accelerated with a velocity \( u = u_0 e^{a't} \) in its own plane and the plate temperature and the level of concentration near the plate are raised linearly with time \( t \). It is assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. The induced magnetic field and viscous dissipation are assumed to be negligible. Then by usual Boussinesq’s approximation, the unsteady flow is governed by the following equations.

\[
\frac{\partial u'}{\partial t'} = g \beta (T' - T'_\infty) + g \beta' (C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} (u' - u_0 e^{a't})
\]

(1)

\[
\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2}
\]

(2)

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_i (C' - C'_\infty)
\]

(3)

with the following initial and boundary conditions \( t' \leq 0 \) : \( u' = 0, \ T' = T'_\infty, \ C' = C'_\infty \) for all \( y', \ t' > 0 \) : \( u' = u_0 e^{a't}, \ T' = T'_w + (T'_w - T'_\infty) At', \ C' = C'_w + (C'_w - C'_\infty) At' \) at \( y' = 0, \ u' = 0, \ T' \to T'_\infty, \ C' \to C'_\infty, \) as \( y' \to \infty \).

(4)

where \( A = \frac{u_0^2}{v} \). Equation (1) is valid when the magnetic lines of force are fixed relative to the plate.

On introducing the following non-dimensional quantities:

\[
u = \frac{u'}{u_0}, \ t = \frac{t'u_0^2}{v}, \ y' = \frac{y'u_0}{v}, \ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \ G_r = \frac{g \beta v (T'_w - T'_\infty)}{u_0^2}, \ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \ G_m = \frac{g \beta v (C'_w - C'_\infty)}{u_0^2}, \ P_r = \frac{\mu C_p}{\kappa}, \ S_c = \frac{v}{D}, \ M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \ K = \frac{K_i v}{u_0^2}, \ a = \frac{a'}{u_0^2}
\]

(5)

in equations (1) to (4), leads to

\[
\frac{\partial u}{\partial t} = G_r \theta + G_m C + \frac{\partial^2 u}{\partial y'^2} - M (u - e^{a't})
\]

(6)

\[
\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y'^2}
\]

(7)

\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y'^2} - K C
\]

(8)
with the initial and boundary conditions \( t \leq 0 : u = 0 , \theta = 0 , C = 0 \) for all \( y , t > 0 : u = e^{\alpha t} , \theta = t , C = t \) at \( y = 0 , u = 0 , \theta = 0 , C = 0 \) as \( y \to \infty \) (9)

All the physical parameters are defined in the nomenclature. The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace transform technique and the solutions are derived as follows.

\[
\theta = \left[ t + \frac{y^2}{2} \right] \text{erfc} \left[ \frac{y \sqrt{S_c}}{2 \sqrt{t}} \right] - y \sqrt{t} \frac{P_r}{\pi} e^{-\frac{y^2}{4t}}
\]

\[
C = \frac{t}{2} \left[ e^{-y \sqrt{KS}} \text{erfc} \left( \frac{y \sqrt{S_c}}{2 \sqrt{t}} + \sqrt{Kt} \right) + e^{-y \sqrt{KS}} \text{erfc} \left( \frac{y \sqrt{S_c}}{2 \sqrt{t}} - \sqrt{Kt} \right) \right]
\]

\[
\frac{y \sqrt{S_c}}{4K} \left[ e^{-y \sqrt{KS}} \text{erfc} \left( \frac{y \sqrt{S_c}}{2 \sqrt{t}} - \sqrt{Kt} \right) - e^{-y \sqrt{KS}} \text{erfc} \left( \frac{y \sqrt{S_c}}{2 \sqrt{t}} + \sqrt{Kt} \right) \right]
\]

\[
u = \frac{ae^{\alpha t}}{2(a + M)} \left[ e^{-y \sqrt{a + M}} \text{erfc} \left( \frac{y \sqrt{a}}{2 \sqrt{t}} - \sqrt{(a + M)t} \right) + e^{y \sqrt{a + M}} \text{erfc} \left( \frac{y \sqrt{a}}{2 \sqrt{t}} + \sqrt{(a + M)t} \right) \right]
\]

\[
+ \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{M - b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} - \sqrt{(M - b)t} \right) + e^{y \sqrt{M - b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} + \sqrt{(M - b)t} \right) \right]
\]

\[
+ \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{M - f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} - \sqrt{(M - f)t} \right) + e^{y \sqrt{M - f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} + \sqrt{(M - f)t} \right) \right]
\]

\[
- \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{M - b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} - \sqrt{(b)t} \right) + e^{y \sqrt{M - b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} + \sqrt{(b)t} \right) \right]
\]

\[
- \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{M - f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} - \sqrt{(f)t} \right) + e^{y \sqrt{M - f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} + \sqrt{(f)t} \right) \right]
\]

\[
+ \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} - \sqrt{(b)t} \right) + e^{y \sqrt{b}} \text{erfc} \left( \frac{y \sqrt{b}}{2 \sqrt{t}} + \sqrt{(b)t} \right) \right]
\]

\[
+ \frac{G_e e^{-bt}}{2d} \left[ e^{-y \sqrt{f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} - \sqrt{(f)t} \right) + e^{y \sqrt{f}} \text{erfc} \left( \frac{y \sqrt{f}}{2 \sqrt{t}} + \sqrt{(f)t} \right) \right]
\]

\[
+ \frac{M e^{-Mt}}{a + M} \text{erfc} \left( \frac{y \sqrt{t} P_r}{2 \sqrt{t}} \right) + \text{erfc} \left( \frac{y \sqrt{t} P_r}{2 \sqrt{t}} \right) \left[ \frac{G_r - G_b}{d} \left( t + \frac{y^2 P_r}{2} \right) \right]
\]

\[
+ \frac{yG_b}{d} \sqrt{\frac{tP_r}{\pi}} e^{-\frac{y^2}{4t}} \frac{M (e^{\alpha t} - e^{-Mt})}{a + M}
\]

where \( b = \frac{M}{P_r - 1} \), \( f = \frac{KS_c - M}{S_c - 1} \), \( d = -b M \), \( e = f (KS_c - M) \)
3. SKIN-FRICTION

We now study skin-friction from velocity field. It is given in non-dimensional form as

\[ \tau = \left. \frac{du}{dy} \right|_{y=0} \]  

(13)

Then from equations (12) and (13), we have

\[
\tau = \frac{ae^{\alpha t}}{a + M} \left[ \sqrt{a + M} \mathrm{erf} \left( (a + M)t + \frac{1}{\sqrt{\pi t}} e^{(a+M)t} \right) \right] + \frac{Ge^{\beta t}}{d} \left[ \sqrt{M-b} \mathrm{erf} \left( (M-b)t + \frac{1}{\sqrt{\pi t}} e^{(M-b)t} \right) \right] \\
+ \frac{G_m e^{-k t}}{e} \left[ \sqrt{(K-f)S_e} \mathrm{erf} \left( (K-f)t \right) + \frac{S_e}{\sqrt{\pi t}} e^{-K t} \right] - \frac{G_r e^{-k t}}{e} \left[ \sqrt{K} \mathrm{erf} \left( Kt \right) + \frac{S_e}{\sqrt{\pi t}} e^{-K t} \right] \\
- \frac{G_m f}{e} \left[ \sqrt{M} \mathrm{erf} \left( Mt \right) + \frac{1}{\sqrt{\pi t}} e^{-Mt} \right] + \frac{G_r f}{e} \left[ \sqrt{M} \mathrm{erf} \left( Mt \right) + \frac{1}{\sqrt{\pi t}} e^{-Mt} \right] - \frac{G_m f}{e} \left[ \sqrt{K} \mathrm{erf} \left( Kt \right) + \frac{S_e}{\sqrt{\pi t}} e^{-K t} \right] \\
+ \frac{Me^{-Mt}}{(a + M)^{3/2}} + \frac{G_r}{d} \left[ \frac{P_t}{\sqrt{\pi t}} \right] - \frac{2G_r b \sqrt{P_t}}{d \sqrt{\pi}} 
\]  

(14)

4. RESULTS AND DISCUSSION

In order to get the physical insight into the problem, we have plotted velocity profiles for different physical parameters M (Magnetic field parameter), K (Chemical reaction parameter), Sc (Schmidt number), Pr (Prandtl number), Gr (Thermal Grashof number), Gm (Mass Grashof number), a (Accelerating parameter) and t (Time) in figures (1) to (13) for the cases of heating (Gr<0) and cooling (Gr>0) of the plate. The heating and cooling take place by setting up free convection current due to temperature and concentration gradient. The value of Prandtl number Pr is chosen as 7, which represents water.

Figures (1) and (2) represent the velocity profiles due to variations in M (Magnetic field parameter) in cases of cooling and heating of the plate at t=0.2 and t=0.4 respectively. It is found that the velocity increases with an increase in M for the cases of both cooling and heating of the plate. It is also found that at time t=0.2, the velocity is maximum near the plate and then decreases away from the plate for the cases of both cooling and heating of the plate. But at time t=0.4, the velocity increases near the plate and becomes maximum and then decreases away from the plate for the case of cooling. But in the case of hating the velocity is maximum near the plate and then decreases away from the plate.
Figures (3) and (4) illustrate the influences of K (Chemical reaction parameter) on the velocity field in cases of cooling and heating of the plate at t=0.4 and t=0.6 respectively. In the case of cooling the velocity is observed to decrease with an increase in K. But the reverse effect is observed in the case of heating of the plate. It is also observed that at time t=0.4, the velocity increases near the plate and becomes maximum and then decreases away from the plate for the case of cooling of the plate. But in the case of heating the velocity is maximum near the plate and then decreases away from the plate. At time t=0.6, the velocity increases near the plate and becomes maximum and then decreases away from the plate for the case of cooling. But in the case of heating the reverse effect is observed.

The velocity profiles for different values of t (time) are presented in figure (5) for the cases of both cooling and heating of the plate. It is observed that the velocity increases with an increase in t (time) for the cases of both cooling and heating of the plate.

Figures (6) and (7) reveal velocity variations with Sc (Schmidt number) in cases of cooling and heating of the plate at t=0.4 and t=0.6 respectively. In the case of cooling of the plate, the velocity is found to increase with decreasing Schmidt number Sc. But the opposite phenomenon is found in the case of heating of the plate.

Figures (8) and (9) display the effects of Pr (Prandtl number) on the velocity field for the cases of cooling and heating of the plate at t=0.2 and t=0.4 respectively. It is found that the velocity decreases with an increase in prandtl number Pr for the case of cooling of the plate. Physically, it is possible because fluids with high prandtl number have high viscosity and hence move slowly. The reverse effect is observed in the case of heating of the plate.
Figures (10) and (11) reveal velocity variations with Gr (Thermal Grashof number) and Gm (Mass Grashof number) in cases of cooling and heating of the plate at $t=0.2$ and $t=0.4$ respectively. It is observed that greater cooling of surface (an increase in Gr) and increase in Gm results in an increase in the velocity. It is due to the fact increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. But the opposite phenomenon is observed in the case of heating of the plate.

Figures (12) and (13) represent the velocity profiles for different values of $a$ (accelerating parameter) in cases of cooling and heating of the plate at $t=0.2$ and $t=0.4$ respectively. It is evident from figures that the velocity increases with an increase in $a$ in cases of both cooling and heating of the plate.

The skin friction is presented against time $t$ for different parameters in figures (14) to (20). Figure (14) represents the effects of $M$ (Magnetic parameter) on Skin friction against $t$ in cases of cooling and heating of the plate. In the case of cooling, the skin friction is found to decrease with an increase in $M$ (Magnetic parameter) up to certain time $t$ and then increase with an increase in $M$ (Magnetic parameter). But the reverse effect is found in the case of heating of the plate. It is also found...
that the skin friction decreases in the case of cooling and increases in the case of heating of the plate with an increase in time (t).

Figure (15) represents the skin friction against t (time) for different values of Sc (Schmidt number) in cases of cooling and heating of the plate. In the case of cooling of the plate, the skin friction is observed to increase with an increase in Sc. But the opposite phenomenon is observed in the case of heating of the plate.

Figure (16) and (17) display the effects of Gr (Thermal Grashof number) and Gm (Mass Grashof number) on Sk (Skin friction) against t (time) in cases of cooling and heating of the plate. From figure (16), in the case of cooling of the plate the skin friction is found to decrease with an increase in Gr. But the opposite phenomenon is found in the case of heating of the plate. From figure (17), it is found that the skin friction increases for the case of cooling and decreases for the case of heating with an increase in Gm up to certain time (t) and the reverse effect is observed later.
Figure (18) represents the skin friction against t (time) for different values of a (Accelerating parameter) in cases of cooling and heating of the plate. It is observed that the skin friction increases with an increase in a (Accelerating parameter) for the cases of both cooling and heating of the plate.

Figure (19) illustrates the effects of K (Chemical reaction parameter) on the skin friction Sk against time t in cases of cooling and heating of the plate. In the case of cooling, the skin friction is found to decrease up to certain stage and then increase with an increase in K up to certain time (t) and increases later with an increase in K. But the reverse effect is observed in the case of heating of the plate.

Figure (20) illustrates the influences of Pr (Prandtl number) on the skin friction against t (time) in cases of cooling and heating of the plate. It is noticed that the skin friction Sk increases with an increase in Pr for the case of cooling but decreases for the case of heating.

Appendix

Nomenclature

- $C_{\infty}$: Concentration in the fluid far away from the plate
- $C_r$: Concentration of the plate
- $A_y$: Coordinate axis normal to the plate
- $y$: Dimensionless concentration
- $\mu$: Coefficient of viscosity
- $\nu$: Kinematic viscosity
- $T_\infty$: Temperature of the fluid far away from the plate
- $T_r$: Temperature of the plate
- $T_\infty'$: Temperature of the fluid near the plate
- $T_r'$: Temperature of the plate
- $K$: Thermal conductivity of the fluid
- $G_r$: Thermal Grashof number
- $t$: Time
- $u$: Velocity of the fluid in the $x'$-direction
- $u_0$: Velocity of the plate
- $a$: Accelerating parameter
- $D$: Chemical Molecular diffusivity
- $g$: Acceleration due to gravity
- $K$: Chemical reaction parameter
- $M$: Magnetic field parameter
- $t$: Dimensionless time
- $\rho$: Density of the fluid
- $\theta$: Dimensionless temperature
- $\sigma$: Electric conductivity
- $\text{erf} C$: Complementary error function
- $\text{erf} C'$: Error function
- $\beta$: Volumetric coefficient of thermal expansion
- $\beta'$: Volumetric coefficient of expansion with concentration
- $w$: Conditions on the wall
- $\infty$: Free stream conditions
- $\eta$: Electric conductivity
- $\nu_0$: Kinematic viscosity
- $\theta$: Dimensionless skin friction
- $\mu$: Coefficient of viscosity

Greek symbols

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Subscripts

- $\infty$: Conditions on the wall
- $\infty$: Free stream conditions

Acknowledgement

I would like to acknowledge Dr. S. Vijaya Kumar Varma, Professor of Mathematics, S.V.University, Tirupati (A.P), India for fruitful discussion on the subject of this paper.

REFERENCES


