

# FORGING OF SINTERED POLYGON DISC: AN UPPER BOUND APPROACH

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#### Abstract

The paper reports on an investigation into the various aspects of closed die cold forging of polygonal powder preforms, which have been compacted and sintered from atomized powder. It is found that for certain dimensional ratios of the preform the die pressure is minimum. An attempt has been made for the determination of the die pressures developed for given geometries of the disc during the closed die forging of polygonal powder preform by using an upper bound approach. Uniform frictional stresses are assumed on top and bottom interfaces and along the interfaces on sides. The solution can be extended to closed die forging of any polygonal shape. The results of hexagonal shape is taken for illustration and results so obtained are discussed critically to illustrate the interaction of various process parameters involved and are presented graphically. **Keywords:** Closed die forging, sintering, preform, interfacial friction law.

# **1. INTRODUCTION**

During the last few years metal-powder components have assumed an important position in industry, as they are being used successfully in a wide range of applications. Both the mechanical and the metallurgical properties of the metal powder components compare favorably with those of wrought materials [1]. Bulk processing of metal powder preforms is a convenient method of reducing or eliminating the porosity from the conventional powder metallurgy products. Process is attractive because it avoids large number of operations, high scrap losses and high-energy consumption associated with the conventional manufacturing processes such as casting, machining etc. In this new technology sintered porous powder preforms are used as starting materials in metal forming processes. Metal powder products manufactured by this new technology are comparable and in some cases even superior to those of cast and wrought products.

Because of constraints on sides the flow pattern in the closed die forging is quite different in the earlier papers [2-4]. It is important to use sufficient metal in the forging blank so that the die-cavity is completely filled by the metal powder preform. It is difficult to put the exact amount of material in the die-cavity. In this paper the load is calculated when the die is nearly full, because the load is maximum at complete filling of the die. At the point of complete filling the load can be infinite.

Although a considerable amount of work has been reported recently as the various technological aspects of the industrial processing of metal-powder preforms [5-7], no systematic attempt has been made so far to study the processing load and deformation characteristics during forging of polygonal disc in a closed die. It is expected that the present work will be of great importance for the assessment of die load during the forging of metal powder preforms.

#### 2. INTERFACIAL FRICTION LAW

In an investigation of the plastic deformation of metal-powder preforms, it is evident that with the application of compressive hydrostatic stress the pores will close and the relative density will increase, whereas the application of tensile hydrostatic stress the pores will grow and the relative density will decrease. The density distribution also does not seem to be uniform throughout, being high in the central region and low at the edges. The density distribution will be more uniform for smaller coefficient of friction and for a greater initial relative density [8].

Friction condition between deforming tool and work piece in metal forming are of the greatest importance concerning a number of factors such as force and made of deformation, properties of the finished specimen and resulting surface roughness. During the sinter forging process it is very important to keep special consideration on the interfacial friction, as this will determine the success or failure of the operation. The relative velocity between the work piece material and the die surface, together with high interfacial pressure and or deformation modes, create a condition of composite





friction which is due to adhesion and sliding [9]. The pattern of the metal flow during the forging of a metal powder preform is such that there exists two zones, an inner one where no relative movement between work piece and die occurs (the sticking zone) and an outer zone where sliding occurs. Therefore, the appropriate friction laws for particular condition is:

$$\tau = \mu \left[ p + \rho_0 \phi_0 \right] \tag{1}$$

where  $\iota$  is shear stress and  $\mu$  is the coefficient of friction. The first term  $\mu p$  being due to sliding and second term  $\mu \rho_0 \phi_0$  being due to adhesion, which later arises from change of the relative density of the preform during the process.

### **3. PLASTIC DEFORMATION OF SINTERED PREFORM**

In an investigation of the plastic deformation of sintered deformation of sintered metal powder preforms, it is clear that change in volume occurs due to porosity. A preform with a high relative density yields at a relative high stress whereas a low relative density preform yields at a relatively low stress. Even hydrostatic stress can cause the sintered metal powder preforms to yield, as the yield surface is closed on the hydrostatic stress axis. Tabata, Masaki and Abe [10] proposed the following yield criterion for porous metal powder preforms:

$$\rho^{k} = \sqrt{3 J_{2}} \pm 3\eta \sigma_{m} \tag{2}$$

where  $\rho$  = relative density of the preform, k = constant equal to 2 in yield criterion,  $J_2$  = Second invariant of deviatoric stress,  $\eta$  = Constant and a function of  $\rho$  only and  $\sigma_m$  = Hydrostatic stress.

Figure (1) shows that the yield surface for a porous metal powder preform given by the above equation consists of two cones, indicates the height of the cones increasing with increasing  $\rho$ . When  $\rho = 1$ , i.e. a pore-free metal, the cone becomes a cylinder and the equation reduces to the Von Mises yield criterion.

The negative sign is taken for  $\sigma_m \leq 0$  and the

positive sign is taken for  $\sigma_m > 0$ . The values of  $\eta$  and k were determined experimentally from simple compression and tension test of sintered copper-powder preforms [10] as  $\eta = 0.54(1 - \rho)^{1.2}$  for  $\sigma_m \le 0$ ;



Figure (1) Yield surface for porous metal powder preform

 $\eta = 0.55(1-\rho)^{0.83}$  for  $\sigma_m > 0$  and k=2. For the axisymmetric condition the yield criterion reduces to

 $\sigma_1 = \frac{\rho^k \sigma_0}{(1-2\eta)} + \frac{(1+\eta)}{(1-2\eta)} \sigma_2$  (Appendix A). whilst for the axisymmetric condition the compactibility

equation becomes  $\mathcal{E}_r = \frac{(2\eta - 1)}{2(1+\eta)} \ln \frac{t_2}{t_1}$  (Appendix B) where  $t_1$  and  $t_2$  are the instantaneous thickness at

subsequent reduction of the preform.

# 4. VELOCITY FIELD & STRAIN RATE

# 4.1 Velocity Field

By drawing normal to the sides of hexagon from in-centre, the disc is divided into six regions, which are symmetrical as far as flow is concerned. In the region GAHO (Figure 2) and so also in other five regions, we take that during compression the horizontal component of velocity of all particles is directed towards the corner. Taking the corner as the origin of cylindrical coordinates the velocity components  $U_{\theta}$  and  $U_z$  are given as below:

$$U_{ heta}=0$$

$$U_z = -\frac{U}{t}z$$



(3)

(4)





In the deformation of powder preform the volume constancy has not been assumed. During the forging of powder preform, the pores get closer and continuously volume changing occurs. So in this case, mass constancy of the preform is to be assumed and thus the normal-strain components must satisfy the following compressibility equation for porous materials [11]:

$$\dot{\varepsilon}_r + \dot{\varepsilon}_{\theta} + \frac{(1-2\eta)}{2(1+\eta)} \dot{\varepsilon}_z = 0$$
(5)

From the mass continuity of material flow it is easily inferred that the rate of flow across section QR (Figure 1) towards the apex A is equal to the rate at which the material is compressed over the area QRTS. The area TTP and SSP being equal, (ST) is arc with centre at A) we may take the rate of compression over area QRTS instead of area QRTS to simplify the analysis without any error. Area of the element QRTS is given by

$$\frac{1}{2} \left( \frac{R_0}{\cos \theta} d\theta + r d\theta \right) \left( \frac{R_0}{\cos \theta} - r \right) = \frac{1}{2} \left( \frac{R_0^2}{\cos^2 \theta} - r^2 \right) d\theta$$
(6)

where r = radius of the generic point from origin at the center,  $R_0 = Half$  the side of a hexagonal disc

Equating the mass flow across the section QR to the rate of pressing over the area QRTS', (considering constant mass flow) we can get:

$$\boldsymbol{U}_{r} = \frac{(1-2\eta)U}{2(1+\eta)rt} \left( \frac{\boldsymbol{R}_{0}^{2}}{\cos^{2}\theta} - \boldsymbol{r}^{2} \right)$$
(7)

The velocity field given by (3), (4) and (7) must satisfy the boundary conditions:  $U_r = 0$  at  $r = R_0/\cos\theta$ . 4.2 Strain Rate

At the top surface  $U_z = -U =$  velocity of ram. On the side surface the flow is only along the surface towards the respective corner (apex). The strain rate field is obtained as:

$$\mathbf{\hat{e}}_{r} = \frac{(1-2\eta)U}{2(1+\eta)t} \left[ \frac{R_{0}^{2}}{r^{2}\cos^{2}\theta} + 1 \right]$$
(8)
$$\mathbf{\hat{e}}_{z} = -\frac{U}{t}$$
(10)

$$\mathbf{\hat{\varepsilon}}_{\theta} = -\frac{(1-2\eta)U}{2(1+\eta)t} \left( \frac{\mathbf{R}_{0}^{2}}{r^{2}\cos^{2}\theta} - 1 \right)$$

$$(9) \qquad \mathbf{\hat{\varepsilon}}_{r\theta} = -\frac{(1-2\eta)U}{2(1+\eta)t} \left( \frac{\mathbf{R}_{0}^{2}}{\mathbf{r}^{2}}\operatorname{sec}^{2}\theta \cdot \tan\theta \right)$$

$$(11)$$

The above strain rates satisfy the compressibility equation (5) for powder preform.

# 5. DIE LOAD

as:

For plastic deformation of a metal powder the external power  $J^{*}$  supplied by the platens is given  $J^{*} = W_{i} + W_{f} + W_{a} + W_{t}$  (12) The first term on the right hand side denotes the rate of internal energy dissipation W<sub>i</sub>, the

The first term on the right hand side denotes the rate of internal energy dissipation  $W_i$ , the second term denotes the frictional shear energy losses  $W_f$ , the third term denotes the energy dissipation due to inertia forces  $W_a$ , and the last term covers power supplied by predetermined body tractions  $W_t$ . In this case forces due to inertia is negligibly small and no external surface traction is stipulated. Therefore,  $W_a = W_t = 0$ . Now the external power  $(\overset{\circ}{J})$  supplied by the press for entire hexagonal shape through the platen is  $\overset{\circ}{J} = \int_{F_i} U_i \, ds = NPU = N p_{av} U R_0^2 \tan(2\pi/N) = W_i + W_{fl} + W_{f2}$ ; (13)

Where N = number of sides in a polygon,  $W_{fl}$  is the rate of energy dissipation at the bottom and top of the preform and  $W_{f2}$  is the rate of energy dissipation at the N flat faces of the preform.

5.1 Rate of Dissipation of Deformation Energy (W<sub>i</sub>)

$$W_{i} = \frac{2}{\sqrt{3}} \sigma_{0}^{*} \int_{V} \left[ \frac{1}{2} \left( \sum_{\varepsilon_{r}}^{2} + \varepsilon_{\theta}^{2} + \varepsilon_{z}^{2} + \varepsilon_{r\theta}^{2} \right) \right]^{\frac{1}{2}} dV$$
(14)

Simplification of yield criterion for axisymmetric condition gives:  $\sigma_0^* = \frac{\rho^* \sigma_0}{(1-2\eta)}$ ; and  $dv = t \cdot rd\theta \cdot dr$ .

After putting the value of strain rates from equations (8) - (11) into (14), we get (for all N- faces),  $R_{\odot}$ 

$$W_{i} = N \frac{4\rho^{*} \sigma_{0} UC}{\sqrt{3} \sqrt{2}(1-2\eta)} \int_{0}^{\frac{2\pi}{N}} \left[ \sec^{3} \theta \ln r \left( 1 - \frac{\cos^{2} \theta}{2} \right) \right]^{\frac{1}{2}} \int_{0.1}^{\frac{1}{2}} d\theta \quad \text{where } C = \frac{(1-2\eta)}{2(1+\eta)}$$



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Lower limit for r is taken as  $r_0$  (corner radius of the die between side surface) = 0.1 to facilitate the calculations. It is verified that there is very little difference in the value of  $W_i$  for  $r_0 = 0.1$  to  $r_0 = 0.75$ .

$$W_{i} = N \cdot \sqrt{\frac{2}{3}} \frac{\rho^{k} \sigma_{0} UC R_{0}^{2}}{(1-2\eta)} A$$

$$\text{where } A = \left[ \left( \ln \frac{R_{0}}{\cos \theta} - \ln 0.1 \right) \sqrt{1 - \frac{\cos^{2} \theta}{2}} \left\{ \tan \theta \cdot \sec \theta + \ln(\sec \theta + \tan \theta) \right\} \right]_{0}^{\frac{2\pi}{N}}$$

$$(15)$$

For hexagonal shape:  $W_i = 6 \cdot \sqrt{\frac{2}{3}} \frac{\rho^k \sigma_0 UC R_0^2}{(1-2\eta)} \cdot A$ ,

where  $A = \left[ \left( \ln \frac{R_0}{\cos \theta} - \ln 0.1 \right) \sqrt{1 - \frac{\cos^2 \theta}{2}} \left\{ \tan \theta \cdot \sec \theta + \ln(\sec \theta + \tan \theta) \right\} \right]_0^{\frac{\pi}{3}}$ 

5.2 Frictional Dissipation on the Bottom & Top Surfaces (Wfi)

$$W_{f_1} = \int_{S} \tau |\Delta v| ds \tag{16}$$

where  $\tau = \mu \left[ p + \rho_0 \phi_0 \right]; \left| \Delta v \right| = \frac{(1 - 2\eta)U}{2(1 + \eta)rt} \left( \frac{R_0^2}{\cos^2 \theta} - r^2 \right), \text{ and } \Delta v \right| = \text{magnitude of velocity discontinuity}$ 

on surface s of zone of plastic deformation

$$W_{f1} = 2 N \cdot \frac{U(1 - 2\eta)\mu \left(p + \rho_0 \phi_0\right)}{2(1 + \eta)} \int_{0}^{2\pi} \int_{0}^{\frac{\pi}{\cos\theta}} \left(R_0^2 \sec^{-2\theta} - r^2\right) d\theta \cdot dr$$

$$W_{f1} = 2 N \cdot \frac{U(1 - 2\eta)\mu \left(p + \rho_0 \phi_0\right)}{2(1 + \eta)} \left(\frac{1}{3} |\tan \theta \cdot \sec \theta + \ln (\sec \theta + \tan \theta)|_{0}^{2\pi}\right) - \left|\frac{1}{9} \left(\frac{1}{3} |\tan \theta \cdot \sec \theta + \ln (\sec \theta + \tan \theta)|_{0}^{2\pi}\right)\right|$$

$$= \left|\frac{0 \cdot 1 \tan \theta}{R_0}\right|_{0}^{\pi} + \left|\frac{1}{9} |_{0}^{\pi} \cdot \frac{0 \cdot 001}{3R_0^{3}}\right|$$

$$= \left|\frac{1}{9} \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\mu \left(p + \rho_0 \phi_0\right)}{2(1 + \eta)} R_0^{\pi} \left(\frac{1}{3} |\tan \theta \cdot \sec \theta + \ln (\sec \theta + \tan \theta)|_{0}^{\pi}\right)\right|$$

$$= \left|\frac{1}{9} \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\mu \left(p + \rho_0 \phi_0\right)}{2(1 + \eta)} R_0^{\pi} \left(\frac{1}{3} |\tan \theta \cdot \sec \theta + \ln (\sec \theta + \tan \theta)|_{0}^{\pi}\right)\right|$$

$$= \left|\frac{1}{9} \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\mu \left(p + \rho_0 \phi_0\right)}{2(1 + \eta)} R_0^{\pi} \left(\frac{1}{3} |\tan \theta \cdot \sec \theta + \ln (\sec \theta + \tan \theta)|_{0}^{\pi}\right)\right|$$

$$= \left|\frac{1}{9} \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\mu \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\mu \left(\frac{1 - 2\eta}{2(1 + \eta)}\right)\right|$$

For hexagonal shape:  $W_{f_1} = 12 \cdot \frac{U(1-2\eta)\mu(p+\rho_0\phi_0)R_0^3}{2(1+\eta)} \begin{bmatrix} 3 \\ -\left|\frac{0.1\tan\theta}{R_0}\right|_0^{\frac{\pi}{3}} + |\theta|_0^{\frac{\pi}{3}} \cdot \frac{0.001}{3R_0^3} \end{bmatrix}$ 

# 5.3 Frictional Dissipation at the Six Flat Surfaces ( $W_{f2}$ )

$$W_{f2} = N \int_{0}^{t} \tau |U_{z}|^{2} R_{0} dz; \quad W_{f2} = \frac{2 N U \mu R_{0}}{t} (p + \rho_{0} \phi_{0}) \frac{|z^{2}|}{2} \Big|_{0}^{t}; \quad W_{f2} = N U R_{0} \mu t (p + \rho_{0} \phi_{0})$$
(18)

For hexagonal shape:  $W_{f2} = 6.U R_0 \mu t (p + \rho_0 \phi_0)$ 

Now putting the value of  $W_i$ ,  $W_{f1}$  and  $W_{f2}$  from (15), (17) and (18) respectively in (13), we can get the value of  $\frac{p_{av}}{\sigma_0}$ . For illustration, the value of  $\frac{p_{av}}{\sigma_0}$  for hexagonal shape is calculated as:

$$\frac{p_{av}}{\sigma_{0}} = \begin{bmatrix} 1 - \frac{(1 - 2\eta)\mu(1 + x)R_{0}}{\tan(\pi/3).t} \begin{cases} \frac{1}{3} |\tan\theta.\sec\theta + \ln(\sec\theta + \tan\theta)|_{0}^{\frac{\pi}{3}} \\ - \left|\frac{0.1\tan\theta}{R_{0}}\right|_{0}^{\frac{\pi}{3}} + |\theta|_{0}^{\frac{\pi}{3}} \cdot \frac{0.001}{3R_{0}^{3}} \end{cases} \\ - \frac{\mu t(1 + x)}{\tan(\pi/3).R_{0}} \end{bmatrix}^{-1} \times \sqrt{\frac{2}{3}} \frac{\rho^{\kappa} \cdot \sigma_{0}}{(1 - 2\eta)\tan(\pi/3)}$$
(19)

#### 6. RESULTS AND DISCUSSION

For the solution for equation (19) we have to make two deviations. First is to take the lower limit for r, the generic radius, as equal to 0.1 instead of zero. This is done to facilitate the calculation of  $W_i$ . If we take r equal to zero then log (0) becomes minus infinity. This lower limit for r does not impair the solution of the problem, as there is little difference in the value of  $W_i$  when we take lower limit equal to 0.25, 0.5 and 0.75, particularly for the conditions of lubrication prevailing in actual processing. The





second deviation is that the curve made by corner radius is concave if we look from the center of the hexagon but since we have taken the corner as origin for the study of flow the curve is convex. Again there is negligibly small difference from the actual case (as the preform does not completely fill up the corners). A certain radius is always there near the corners of the hexagonal disc.



FIGURE (3) VARIATION OF RELATIVE AVERAGE PRESSURE WITH D/t FOR DIFFERENT INITIAL RELATIVE DENSITY OF PREFORM



FIGURE (5) VARIATION OF RELATIVE AVERAGE PRESSURE WITH PERCENTAGE REDUCTION IN HEIGHT OF THE HEXAGONAL DISC AT DIFFERENT VALUE OF INITIAL RELATIVE DENSITY



FIGURE (7) VARIATION OF RELATIVE DENSITY WITH RELATIVE AVERAGE PRESSURE ON PUNCH (HEXAGONAL PREFORM)







FIGURE (4) EFFECT OF THE CONDITION OF LUBRICATION ON THE RELATIVE AVERAGE PRESSURE FOR VARIOUS GEOMETRIES OF THE HEXAGONAL DISC



FIGURE (6) INFLUENCE OF D/1 RATIO ON RELATIVE AVERAGE PRESSURE ON THE PUNCH WITH PERCENTAGE REDUCTION IN PREFORM HEIGHT



FIGURE (8) VARIATION OF RELATIVE AVERAGE PRESSURE WITH D/t FOR DIFFERENT NO. OF SIDES IN A POLYGON

The relative forging pressure for different value of initial relative density shows a minima and hence there is an optimum value of D/t (Figure 3). The minima is clearly observed at higher initial relative density of the preform. This value is of course different conditions of lubrication (Figure 4), i.e. they depend upon the coefficient of factors,  $\mu$ . The relative forging pressure increases with percentage reduction in height of preform (Figure 4). Again, it increases with increasing value of D/t (Figure 6). The relative density of the preform increases with the forging pressure and it is asymptotic (Figure 7), i.e. nearly equal to 1. It is observed that the relative density of the preform increases sharply during initial phase of loading. It is due to the fact that initially large amount of load is





consumed in compaction and then after gaining sufficient relative density, it is deformed and gives a uniform proportionate curve. Again from figure (8), the relative average pressure increases with as number of sides increases in a polygon. Also from figure (9), the relative average pressure increases as the number of sides in a polygon increases.

#### 7. CONCLUSION

During forging of metal powder preforms through closed dies, it is seen that compaction and compression both take place simultaneously. Initially the closing of pores dominates the compression process. The larger amount of applied load is utilized in densification and lesser amount is consumed for compression. The density of the preform increases with an increase in the forging load. If we can decrease the value of D the side of the hexagonal nut, we will be saving on material as well as forging cost. The above determined observations will be helpful to designing the product of hexagonal shape i.e. mainly hexagonal nuts. If we can decrease the value of D the side of the hexagonal nut, we will be saving on material as well as forging cost. Since the nuts are used in large quantities the saving may come to huge amounts. Similarly, certainly it will give the guidelines for developing the new sintered products of other shapes also. These results can be expanded for other shapes. This solution can be also expanded with minor modification to the problem of forging of bolts also.

Appendix A. Writing the second invariant of deviatoric stress and hydrostatic stress in terms of principal

stresses in equation (2) leads to  $\rho^{*}\sigma_{0} = \left[ \left( \sigma_{1} - \sigma_{2} \right)^{2} + \left( \sigma_{2} - \sigma_{3} \right)^{2} + \left( \sigma_{3} - \sigma_{1} \right)^{2} \right] / 2 \right]^{\frac{1}{2}} \mp \eta \left( \sigma_{1} + \sigma_{2} + \sigma_{3} \right)$  (A-1) where k is constant and equal to 2

For the axisymmetric condition-  $\sigma_1 = \sigma_3 = \sigma_{rr}$  and  $\sigma_1 = -p$ . Hence  $(\sigma_1 - \sigma_2) \mp (2\sigma_1 + \sigma_2)\eta = \rho^k \sigma_0$  (A-2)

For the positive sign  $\sigma_1 = \frac{\rho^k \sigma_0}{(1+2\eta)} + \frac{(\eta-1)}{(1+2\eta)} \sigma_2$  (A-3) **Appendix B.** According to Tabata, Masaki and Abe [10] the principal strain increments are given as

 $d_{\mathcal{E}_{i}} = d\lambda \left(\frac{3(\sigma_{i} - \sigma_{m})}{2\sqrt{3} J} \pm \eta\right)$  (i = 1, 2, 3) (B-1); where  $d\lambda = \frac{\sqrt{2}}{3} \sqrt{(d_{\mathcal{E}_{1}} - d_{\mathcal{E}_{21}})^{2} + (d_{\mathcal{E}_{2}} - d_{\mathcal{E}_{3}})^{2} + (d_{\mathcal{E}_{3}} - d_{\mathcal{E}_{1}})^{2}}$ (B-2)

The volumetric strain increment is given as

 $d_{\mathcal{E}_{V}} = d_{\mathcal{E}_{1}} + d_{\mathcal{E}_{2}} + d_{\mathcal{E}_{3}} = \pm 3\eta \cdot d\lambda = \pm \sqrt{2\eta} \left[ (d_{\mathcal{E}_{1}} - d_{\mathcal{E}_{2}})^{2} + (d_{\mathcal{E}_{2}} - d_{\mathcal{E}_{3}})^{2} + (d_{\mathcal{E}_{3}} - d_{\mathcal{E}_{1}})^{2} \right]^{\frac{1}{2}}$ (B-3) For axisymmetrical compression:  $\sigma_1 = \sigma_3 = \sigma_r = \sigma_\theta$  and  $\sigma_2 = \sigma_\theta$  (B-4)

Hence,  $d_{\mathcal{E}_r} = \left[\frac{1}{2} - \eta\right] d\lambda = d_{\mathcal{E}_{\theta}}$  and  $d_{\mathcal{E}_z} = -(1+\eta) d\lambda$  (B-5). Putting the values from (B-4) and (B-5) into (B-3)

(considering the negative sign):  $2d_{\mathcal{E}_r} + d_{\mathcal{E}_Z} = -2\eta d_{\mathcal{E}_r} + 2\eta d_{\mathcal{E}_Z}$  (B-6)

Rearranging leads to  $d_{\mathcal{E}_r} = \frac{(2\eta - 1)}{2(1+\eta)} d_{\mathcal{E}_z}$ . Considering  $d_{\mathcal{E}_z} = \frac{dt}{t}$  and putting it into (B-7):  $d_{\mathcal{E}_r} = \frac{(2\eta - 1)dt}{2(1+\eta)t}$  and upon integrating,  $\mathcal{E}_r = \frac{(2\eta - 1)}{2(1+\eta)} \log_e \frac{t_2}{t_c}$  (B-8)

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