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METHOD FOR CALCULATIONS OF SPUR GEAR DRIVES WITH ASYMMETRIC INVOLUTE–LANTERN MESHING

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ABSTRACT: An engineering method has been developed for the geometric calculation of gear drives with asymmetric involute-lantern meshing. The method for calculating the lantern circle radiuses has been clarified. The restrictive conditions for the geometric design of the gearing have been defined. Numerical examples have been solved. The article is a continuation of the previous work, published in the same magazine.
KEYWORDS: Spur gear, Asymmetric teeth, Involute – lantern meshing, Lantern circle, Contact ratio

❖ INTRODUCTION

In many fields of the machine-building and first of all for the high efficient gear pumps and compressors it is necessary that the meshed gears to have a small teeth number [Vulgakov, 1995], [Kotelnikov, 1973]. For the realization of a continuous transmission of motion between gears of a small number of spur teeth in [Alipiev, 2010] is proposed a new type of gear meshing called “involute-lantern meshing”. The proposed meshing is asymmetric and the gear teeth are convex from the one side and concave from the other side. Whence the convex profiles take part in the involute meshing and the opposite profiles - in a corrected lantern meshing [Alipiev, 1990]. The geometry of involute-lantern meshing for the private case where equal gears mesh is examined in [Alipiev, Antonov 2009]. For the general case, by meshing of different gears (of different teeth number) in the previous paper [Alipiev, 2011] are clarified the specific details, connected with the geometrical shape and character of gear meshing.

The aim of the present paper is to propose a simplified method for geometric calculation of gearings of involute-lantern meshing. The developed method should be based on common principles known from the traditional theory of involute meshing and to be suitable for direct use by engineers and designers.

❖ RADII OF LANTERN CIRCLES

The first problem that should be solved by the geometrical design of the proposed gearing is connected with defining the radii r_{p1} and r_{p2} (Figure 1) of the lantern circles of the teeth profiles. The determination of these radii appears as the most complex stage of the geometric calculation of the involute-lantern gearing. The complexity is caused by the necessity the lantern circle to contact simultaneously the convex and concave tooth side. Besides, in order to define the geometry of the concave profile of the one gear [Alipiev, 2011], preliminary it is necessary to be known the radius of the lantern circle of the other gear.

In Figure 1a are shown two characteristic positions of the involute-lantern gearing, defined as boundary positions of lantern meshing, with the help of which are explained the necessary conditions for the determination of radii of the lantern circles.

In the first boundary position the lantern circle (of a center C_2 and radius r_{p2}) of gear 2 contacts in point K with the last point of the concave profile g of gear 1. Simultaneously point K should be a point of contact of both lantern circles of radii r_{p1} and r_{p2}, and a point of contact of the lantern circle r_{p1} with the profile g.

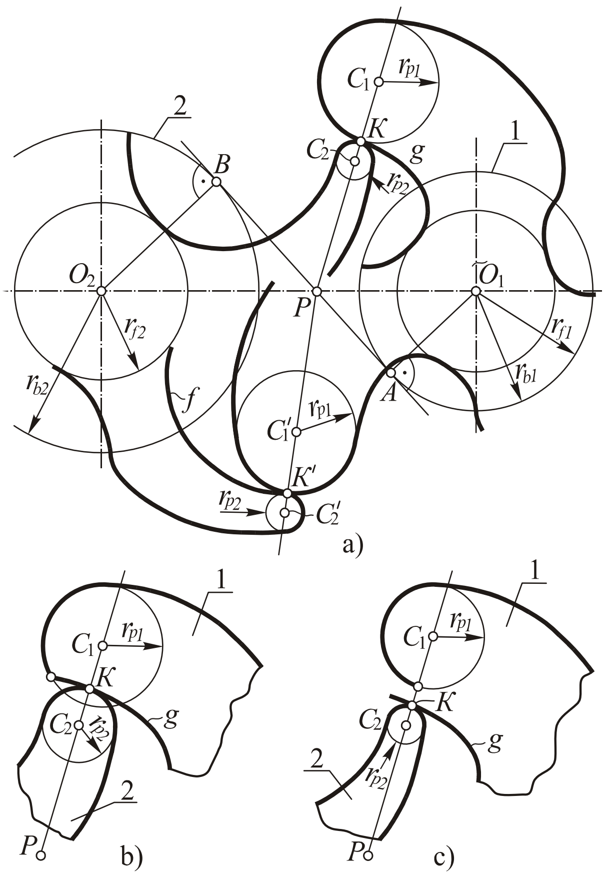


Figure 1. Boundary locations of the lantern meshing

So that the basic theorem of the gear meshing to be met, it is necessary the common normal of the conjugate profiles (the concave profile g and the circle of gear 2), in their contact point K to cross also the pitch point P. Based on these reasons directly it is defined also the first condition for correspondence between the radii of the lantern circles in the following way: in the first boundary location the lantern circles of both gears contact one another, and the connecting straight line of their centers C_1 and C_2 crosses the pitch point P, where point C_2 is situated between points C_1 and P. If this condition is not satisfied, the concave profile g and the lantern circle of radius r_{p1} will cross (Figure 1b), or will not touch (Figure 1c).

Analogously, the second condition for correspondence between the radii of the lantern circles, relating to the second boundary location is defined. In this location the lantern circle (of a radius $C_1^{/}$ and radius r_{p1}) of gear 1 (Figure 1a) in point $K^{/}$ contacts with the last point of the concave profile f of gear 2. The definition of the second condition for correspondence differs from the definition of the first one only because the locations of the lantern circles towards the pitch point P are exchanged, as a result of which point $C_1^{/}$ is found between points $C_2^{/}$ and P.

For the calculation of r_{p1} and r_{p2} a numerical method is developed, that is based on the above mentioned conditions for correspondence between the radii of the lantern circles. In this case the determination of r_{p1} and r_{p2} is simply defined, if the following are specified: the module m, teeth number z_1 and z_2 of gears and the transverse contact ration of the gearing ε_α. Using these initial values the pressure angle α_w is directly determined, using the formula [Alipiev, Antonov 2009]

$$\alpha_w = \mathrm{arctg}[2\pi\varepsilon_\alpha / (z_1 + z_2)]\,. \tag{1}$$

As the module m appears as a scale factor, it is more appropriate instead of the radii r_{p1} and r_{p2} to be calculated the dimensionless coefficients r_{p1}^* and r_{p2}^*, hence, from equations

$$r_{p1} = m\, r_{p1}^*\,, \qquad r_{p2} = m\, r_{p2}^* \tag{2}$$

to define the actual values of the radii of the lantern circles.

The results from the calculations using the developed numerical method, for each gear, having different teeth number of gears, can be represented by the drawing shown on Figure 2. From it, at specified value of ε_α, the coefficients r_{p1}^* and r_{p2}^* are determined, then from equations (2) are determined the radii of the lantern circles. For instance at $\varepsilon_\alpha = 1{,}04$, the coefficients of the radii of the lantern circles for the meshed gears ($z_1 = 4$, $z_2 = 5$) are: $r_{p1}^* = 0{,}654$ and $r_{p2}^* = 0{,}305$, respectively.

When designing involute-lantern gearings of equal teeth number of the meshed gears, it is appropriate to use the drawing shown on Fig. 3, in which $r_p^* = r_{p1}^* = r_{p2}^*$. From the drawing the dimensionless coefficient r_p^* of the radius of the lantern circle for both gears are directly determined, at specified transverse contact ratio and gear teeth number. From Figure 3 it is seen that the possible range in which the lantern circle radius is changed, increases together with the increase of gear teeth number. Besides, this radius is the largest when the transverse contact ratio is the smallest ($\varepsilon_\alpha = 1$). Increasing of ε_α the radius of the lantern circle decreases and at $r_p^* = 0$ the transverse contact ratio gets its maximum value.

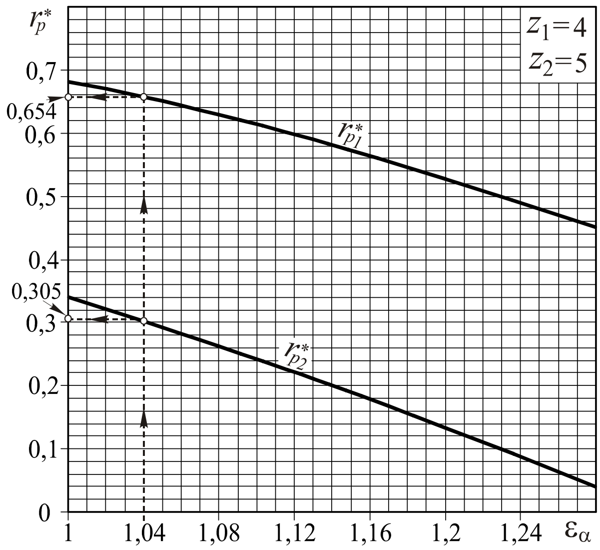


Figure 2. Radii of the lantern circles ($z_1 \neq z_2$)

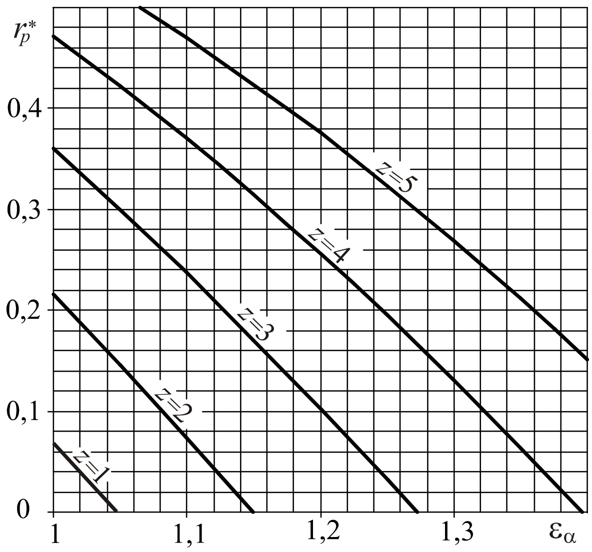


Figure 3. Radius of the lantern circle at $z = z_1 = z_2$

❖ METHOD OF GEOMETRICAL CALCULATIONS

The geometry of spur gear gearings of asymmetric involute–lantern meshing, formed by two different gears, is fully defined if the following four independent parameters are specified: m, z_1, z_2, ε_α. In some cases, in order to get whole values of the pressure angles, it is more convenient, instead of ε_α to specify α_w as an independent parameter. Then the pressure angle is calculated by the formula

$$\varepsilon_\alpha = (z_1 + z_2)/2\pi \tan \alpha_w , \tag{3}$$

got after transformation of equation (1). A combined variant is also possible, where having preliminary specified value of ε_α and formula (1), α_w is calculated, afterwards the got value for the pressure angle is rounded and by equation (3) the real value of ε_α is defined.

For the respective calculations is developed a method for geometric design of involute-lantern gearings. On Table 1 are given the sequence by which the geometrical calculations are made and the necessary formulas for the calculation process. In the last column of these tables are shown also the results from the solved numerical example. Using the got geometric dimensions on Figure 4 in scale is drawn the picture of meshing of the designed gearing.

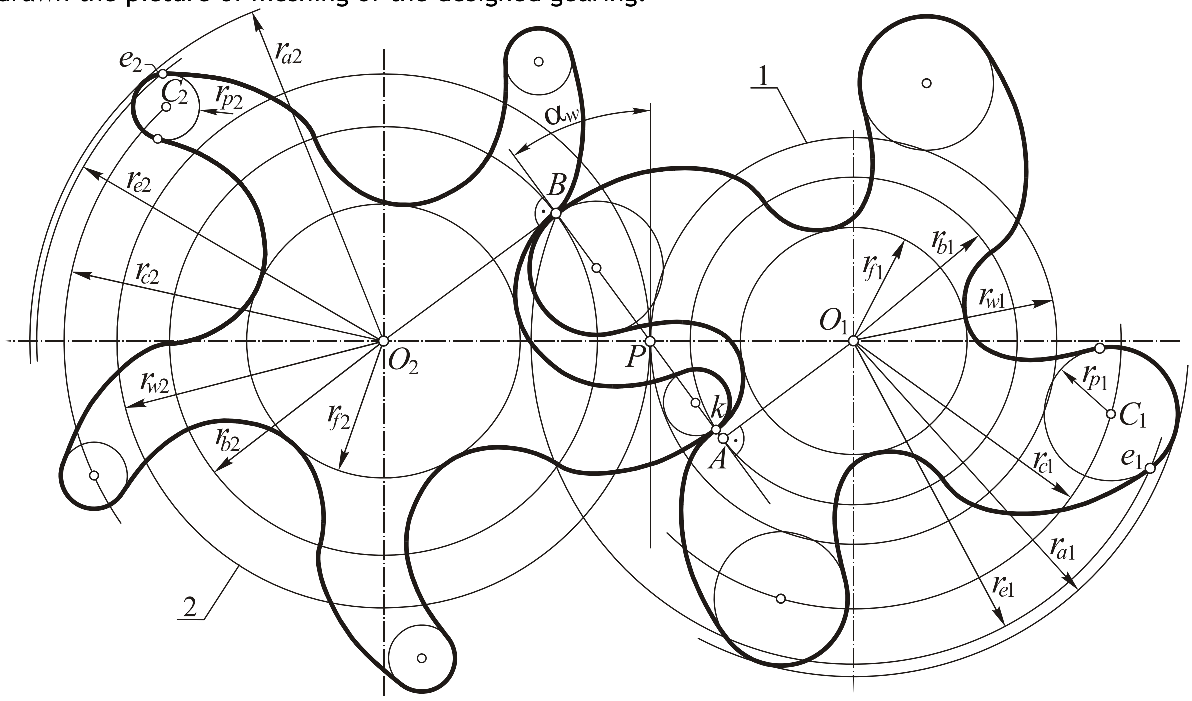


Figure 4. Involute - lantern meshing $z_1 = 4$, $z_2 = 5$

T a b l e 1. Dimensions of gears and gearing parameters

INITIAL VALUES OF THE GEOMETRICAL CALCULATIONS (Module: $m = 30$ mm; Teeth number of the driving gear: $z_1 = 4$; Teeth number of the driven gear: $z_2 = 5$; Involute pressure angle: $\alpha_w = 36°$)

	№	Calculated values	Sign	Calculating formula	Numerical example
DRIVING GEAR (PINION)	1	Diameter of the pitch circle	d_{w1}	$d_{w1} = m\,z_1$	120,000
	2	Diameter of the base circle	d_{b1}	$d_{b1} = m\,z_1 \cos\alpha_w$	97,08204
	3	Profile angle in the last point of the involute	α_{e1}	$\alpha_{e1} = \operatorname{arctg}(\frac{z_1+z_2}{z_1}\operatorname{tg}\alpha_w)$	58,5448°
	4	Diameter of location of the last point of the involute	d_{e1}	$d_{e1} = m\sqrt{z_1^2\cos^2\alpha_w + (z_1+z_2)^2\sin^2\alpha_w}$	186,041
	5	Ratio of the radius of the lantern circle	r_{p1}^*	Figure 2 (numerical definition)	0,6545
	6	Radius of the lantern circle	r_{p1}	$r_{p1} = m\,r_{p1}^*$	19,635
	7	Diameter of location of the centers of lanterns	d_{c1}	$d_{c1} = m\sqrt{z_1^2\cos^2\alpha_w + [(z_1+z_2)\sin\alpha_w - r_{p1}^*]^2}$	153,912
	8	Diameter of the addendum circle	d_{a1}	$d_{a1} = d_{c1} + 2r_{p1}$	193,182
	9	Diameter of the internal circle	d_{f1}	$d_{f1} = m(z_1+z_2) - d_{a2}$	66,126
	10	Ratio of the shape of the concave profile	λ_1	$\lambda_1 = d_{c1}/d_{w1}$	1,2826
	11	COORDINATES OF THE CONVEX PROFILE $\alpha_i\,[0 \div \alpha_{e1}]$		$X_{i1} = -0{,}5\,m\,z_1\cos\alpha_w\sin\delta_i/\cos\alpha_i$; $Y_{i1} = 0{,}5\,m\,z_1\cos\alpha_w\cos\delta_i/\cos\alpha_i$ where $\delta_i = \operatorname{inv}\alpha_i = \tan\alpha_i - \alpha_i$,	
	12	COORDINATES OF THE CONCAVE PROFILE		$\xi_{j1} = \frac{m}{2}\left\{(z_1+z_2)\sin\varphi_j - \lambda_2 z_2\sin(\frac{z_1}{z_2}\varphi_j+\varphi_j) - \frac{2r_{p2}^*\left[\lambda_2\sin(\frac{z_1}{z_2}\varphi_j+\varphi_j) - \sin\varphi_j\right]}{\sqrt{1-2\lambda_2\cos\frac{z_1}{z_2}\varphi_j+\lambda_2^2}}\right\}$ $\eta_{j1} = \frac{m}{2}\left\{(z_1+z_2)\cos\varphi_j - \lambda_2 z_2\cos(\frac{z_1}{z_2}\varphi_j+\varphi_j) - \frac{2r_{p2}^*[\lambda_2\cos(\frac{z_1}{z_2}\varphi_j+\varphi_j) - \cos\varphi_j]}{\sqrt{1-2\lambda_2\cos\frac{z_1}{z_2}\varphi_j+\lambda_2^2}}\right\}$	
DRIVEN GEAR (CROWN)	13	Diameter of the pitch circle	d_{w2}	$d_{w2} = m\,z_2$	150,000
	14	Diameter of the base circle	d_{b2}	$d_{b2} = m\,z_2\cos\alpha_w$	121,3525
	15	Pressure angle at the end point of the involute	α_{e2}	$\alpha_{e2} = \operatorname{arctg}(\frac{z_1+z_2}{z_2}\operatorname{tg}\alpha_w)$	52,5964°
	16	Diameter of location of the end point of the involute	d_{e2}	$d_{e2} = m\sqrt{z_2^2\cos^2\alpha_w + (z_1+z_2)^2\sin^2\alpha_w}$	199,782
	17	Ratio of the radius of the lantern circle	r_{p2}^*	Figure 2 (numerical definition)	0,3047
	18	Radius of the lantern circle	r_{p2}	$r_{p2} = m\,r_{p2}^*$	9,141
	19	Diameter of location of the centers of lanterns	d_{c2}	$d_{c2} = m\sqrt{z_2^2\cos^2\alpha_w + [(z_1+z_2)\sin\alpha_w - r_{p2}^*]^2}$	185,591
	20	Diameter of the addendum circle	d_{a2}	$d_{a2} = d_{c2} + 2r_{p2}$	203,873
	21	Diameter of the internal circle	d_{f2}	$d_{f2} = m(z_1+z_2) - d_{a2}$	76,818
	22	Ratio of the shape of the concave profile	λ_2	$\lambda_2 = d_{c2}/d_{w2}$	1,2373
	23	COORDINATES OF THE CONVEX PROFILE $\alpha_i\,[0 \div \alpha_{e2}]$		$X_{i2} = -0{,}5\,m\,z_2\cos\alpha_w\sin\delta_i/\cos\alpha_i$; $Y_{i2} = 0{,}5\,m\,z_2\cos\alpha_w\cos\delta_i/\cos\alpha_i$ where $\delta_i = \operatorname{inv}\alpha_i = \tan\alpha_i - \alpha_i$,	
	24	COORDINATES OF THE CONCAVE PROFILE		$\xi_{j2} = \frac{m}{2}\left\{(z_1+z_2)\sin\varphi_j - \lambda_1 z_1\sin(\frac{z_2}{z_1}\varphi_j+\varphi_j) - \frac{2r_{p1}^*\left[\lambda_1\sin(\frac{z_2}{z_1}\varphi_j+\varphi_j) - \sin\varphi_j\right]}{\sqrt{1-2\lambda_1\cos\frac{z_2}{z_1}\varphi_j+\lambda_1^2}}\right\}$	
PARAMETERS OF THE ARINGGE	30	Tooth ratio	u	$u = \frac{z_2}{z_1}$, where $z_2 > z_1$	1,25
	31	Centre distance	a_w	$a_w = m(z_1+z_2)/2$	135,000
	32	Length of the line of involute action	l_{AB}	$l_{AB} = 0{,}5\,m(z_1+z_2)\sin\alpha_w$	79,351
	33	Transverse contact ratio of the involute meshing	ε_α	$\varepsilon_\alpha = (z_1+z_2)\operatorname{tg}\alpha_w/2\pi$	1,04
	34	Transverse contact ratio of the lantern meshing	ε_o	$\varepsilon_o \approx [z_1\arccos(d_{w1}/d_{c1}) + z_2\arccos(d_{w2}/d_{c2})]/2\pi$	0,93

For the geometric design of asymmetric gears of involute-lantern meshing the following two restrictive conditions should be obligatorily satisfied: 1) the transverse contact ratio of the involute meshing should be larger than one; 2) the coefficients r_{p1}^* and r_{p2}^* of the radii of the lantern circles should have positive values.

The first requirement written by the inequation

$$\varepsilon_\alpha > 1 , \quad (4)$$

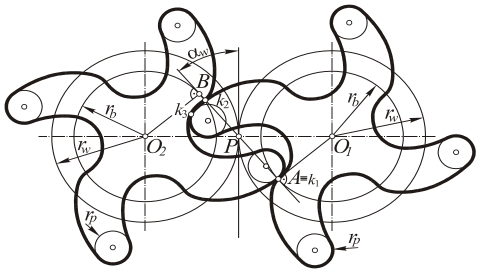
provides continuous motion transfer in the driving direction of movement and by the second requirement

$$r_{p1}^* > 0 , \qquad r_{p2}^* > 0 \quad (5)$$

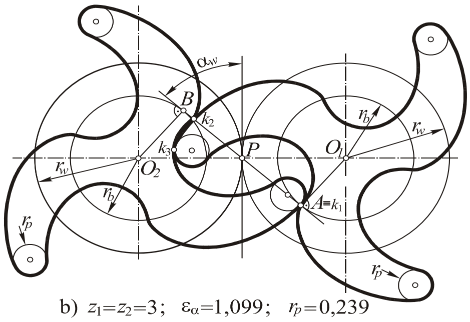
the ability of the concave tooth profile to cut a part of the convex involute profile is eliminated. In practice with the second restrictive requirement the crossing of the tooth flank profiles before the end point e_2 (or e_1) of the respective involute curve (Figure 4) is eliminated.

❖ RESULTS FROM THE GEOMETRIC DESIGN

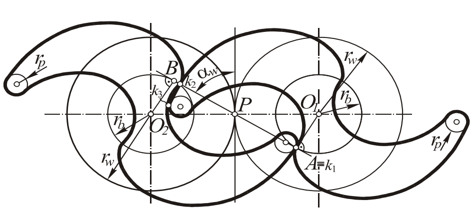
Using the designed method for geometric calculation of the involute-lantern meshing are designed many gearings. For the realization of a correlation between the calculated values and their geometric essence, on Figure 5 and Figure 6 is shown the gear meshing of these gearings. Despite, on Figure 5 are shown gearing for which the teeth number of the meshed gears is equal ($z=z_1=z_2$), and in Figure 6 – not equal ($z_1>z_2$).



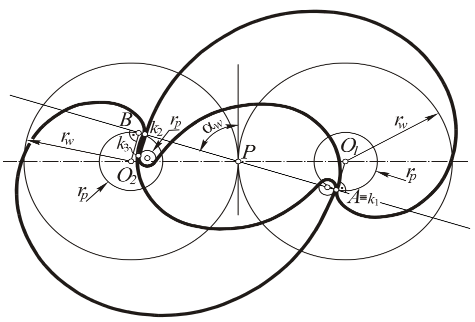
a) $z_1=z_2=4$; $\varepsilon_\alpha=1,08$; $r_p=0,392$



b) $z_1=z_2=3$; $\varepsilon_\alpha=1,099$; $r_p=0,239$

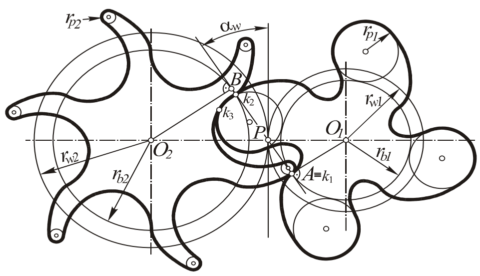


c) $z_1=z_2=2$; $\varepsilon_\alpha=1,06$; $r_p=0,132$

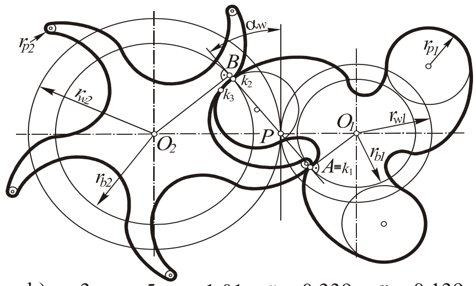


d) $z_1=z_2=1$; $\varepsilon_\alpha=1,02$; $r_p=0,039$

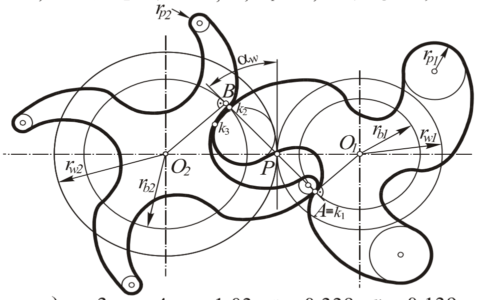
Figure 5 Involute-lantern gearings $z_1=z_2$



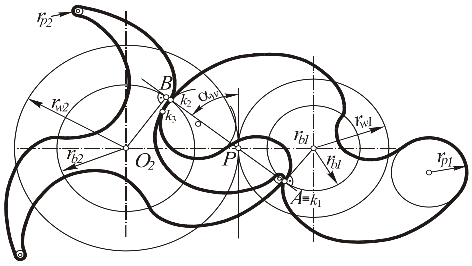
a) $z_1=4$; $z_2=6$; $\varepsilon_\alpha=1,04$; $r_{p1}=0,339$; $r_{p2}=0,139$



b) $z_1=3$; $z_2=5$; $\varepsilon_\alpha=1,01$; $r_{p1}=0,339$; $r_{p2}=0,139$



c) $z_1=3$; $z_2=4$; $\varepsilon_\alpha=1,03$; $r_{p1}=0,339$; $r_{p2}=0,139$



d) $z_1=2$; $z_2=3$; $\varepsilon_\alpha=1,01$; $r_{p1}=0,339$; $r_{p2}=0,139$

Figure 6 Involute-lantern gearings $z_1<z_2$

From the pictures of meshing it is established that the restrictive requirements (4) and (5) for all gearings are observed. Besides, from the meshing of the convex profiles it is seen that in the moment when a specified pair of involute curves contacts the initial point A of the line of action AB, at the same moment the previous pair of involute curves contacts the point k_2, lying on the line AB. The same profiles come out of contact in the last possible point B, on the line of action. As the involute profiles contact along the full length of the line of action AB, the transverse contact ratio in the driving direction of movement is equal to its maximally possible value at the respective pressure angle.

The designed gears of three and two teeth are produced and the meshing between the actual samples is shown on Figure 7 and Figure 8.

Figure 7. Involute-lantern gearing $z_1 = z_2 = 3$

Figure 8 Involute-lantern gearing $z_1 = z_2 = 2$

The involute-lantern gearing, formed by two gears of one tooth each, shown on Figure 5d may be considered exotic. The transverse contact ratio in this case is $\varepsilon_a = 1{,}02$, and the possible interval of its change is $\varepsilon_\alpha = 1 \div 1{,}045968$. As the gears have only one tooth, the necessary contact ($\varepsilon_\alpha > 1$) is provided from the presence in definite moments of two contact points k_1 and k_2 between the meshed involute profiles. In this gearing the pressure angle considerably increases ($\alpha_w = 72{,}6685^0$), and the radius of the lantern circle is relatively small ($r_p^* = 0{,}039$ mm). As a result the meshing between gears of one tooth is interesting only from a theoretical aspect as a gearing of the smallest teeth number.

From the gearings shown on Figure 6 it is found that when gears of different teeth number mesh, the radius of the lantern circle of the small gear is larger than the one of the large gear ($r_{p1} > r_{p2}$). Besides, by increasing the difference between the teeth number, r_{p1} sharply decreases. The conducted research shows that at larger differences between z_2 and z_1 the proposed meshing can't exist because the teeth of the larger gear are sharpened and a part of its involute profile is cut. In these cases it is appropriate to use modified involute-lantern meshing that will be considered in the next publications by the authors.

❖ CONCLUSION

The conducted research regarding the geometric design of spur-teeth gearings of involute-lantern meshing enables the drawing of the following important conclusions:

- □ The area of application of the proposed meshing are gearings formed by gears of a very small teeth number.
- □ The geometry of the involute-lantern gearing is simply defined by four independent parameters: m, z_1, z_1, α_w.
- □ The radii of the lantern circles r_{p1} and r_{p2}, contacting the concave teeth profiles are defined by a numerical method.
- □ The smallest different teeth number of gears of involute-lantern meshing, for which $\varepsilon_\alpha > 1$, is $z_1 = 2$ and $z_2 = 3$.
- □ The smallest equal teeth number of gears of involute-lantern meshing, for which $\varepsilon_\alpha > 1$, is $z_1 = z_2 = 1$.

❖ REFERENCES

[1.] Alipiev O.: Geometric design of spur gear drives with asymmetric involute–lantern meshing (Part I – geometry of meshing). Mechanics of Machines, 2011, (in print)

[2.] Alipiev O.: Spur gearing with internal gearing. Patent application 110302 / 06.01.2009. Bulgaria.

[3.] Alipiev O., S. Antonov: Asymmetric involute–lantern meshing formed by identical spur gears with a small number of teeth. International conference on gears, VDI-Berichte - 2108, Munich, Germany, 2010, p. 925-940

[4.] Alipiev O.: Geometry and gear cutting of epi- and hipocycloid gears for cyclo dears drives. Dissertation, Ruse, 1990, p. 208

[5.] Vulgakov E.: Theory of involute gears. Mashinostroyenie, Moscow, 1995, p. 320

[6.] Kotelnikov V.: The smallest number of spur external teeth, cut with a non-standard rack-type cutter. Works of universities - Mashinostroyenie, 6, (1973), Russia p. 52-56