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LOAD CARRYING CAPACITY AND TIME HEIGHT RELATION FOR SQUEEZE FILM BETWEEN ROUGH POROUS RECTANGULAR PLATES

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ABSTRACT: An attempt has been made to analyze the squeeze film behavior between rough porous rectangular plates. The surfaces are assumed to be transversely rough. The stochastic random variable characterizing the roughness is assumed to be asymmetric with non zero mean and variance. The modified Reynolds' equation is then solved with appropriate boundary conditions to obtain the expressions for pressure and load carrying capacity leading to the calculation of response time. The results are presented in graphical form. It is observed that the composite roughness of the surfaces affects the performance of the bearing system adversely. However, this investigation tends to suggest that a proper choice of the aspect ratio may compensate this adverse effect up to certain extent in the case of negatively skewed roughness. Hence, while designing the bearing system the roughness must be given due consideration.
KEYWORDS: Reynolds' equation, random roughness, squeeze film, pressure, load carrying capacity

❖ INTRODUCTION

The transient load carrying capacity of a fluid film between two surfaces having a relative normal velocity plays an important role in frictional devices such as clutch plates in automatic transmissions. Wu (1, 2) analyzed the squeeze film performance when one of the surfaces was porous faced for mainly, two types of geometries namely, annular and rectangular. Prakash and Vij (3) investigated several bearing configurations such as circular, annular, elliptic, rectangular and conical. Besides, infinitely long rectangular plates were discussed. In this article a comparison was made between the squeeze film behavior of various geometries of equivalent surface area and it was found that the circular plates had the highest transient load carrying capacity (other parameters remaining same).

But due to elastic thermal and uneven wear effects the configurations encountered in practice are normally far from smooth. Besides, it is an established fact that the bearing surfaces after having some run-in and wear develop roughness. The effect of surface roughness was discussed by several investigators [Davies (4), Burton (5), Michell (6), Tonder (7), Tzeng and Saibel (8)]. Christensen and Tonder (9, 10, 11) studied the roughness from mathematical modeling point of view and proposed a comprehensive general analysis for finding out the effects of transverse as well as longitudinal surface roughness. This analysis of Christensen and Tonder (9, 10, 11) formed the basis for investigating the effect of surface roughness in a number of articles [Ting (12), Prakash and Tiwari (13, 14), Prajapati (15, 16), Guha (17), Gupta and Deheri (18), Andharia, Gupta and Deheri (19, 20)]. Recently, Patel. Deheri and Patel (21) discussed the effect of surface roughness between infinitely long rectangular plates in the presence of a magnetic fluid lubricant. Here it has been proposed to study the effect of transverse surface roughness on the configuration of Prakash and Vij (3) concerning rectangular plates.

❖ THE ANALYSIS

The bearing configuration which is given below, consists of two rectangular plates with dimensions a and b (a > b). The upper plate has a porous facing of thickness H which is backed by a solid wall. It moves normally towards an impermeable and flat lower plate with uniform velocity $\dot{h}_o = \frac{dh_o}{dt}$ where h_o is the central film thickness.

The bearing surfaces are assumed to be transversely rough. The thickness $h(x)$ of the lubricant film is $h(x) = \bar{h}(x) + h_s$ where $\bar{h}(x)$ is the mean film thickness and h_s is deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. Here h_s is considered to be stochastic in nature and governed by the probability density function $f(h_s)$, $-c \le h_s \le c$ where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry of the random variable h_s are defined by the relationships:

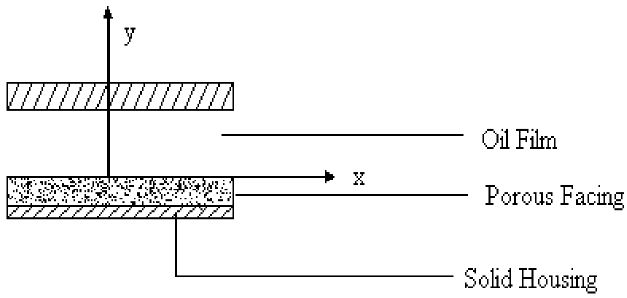

Figure 1. Configuration of the bearing

$$\alpha = E(h_s),$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by:

$$E(R) = \int_{-c}^{c} R f(h_s)\, dh_s$$

Under usual assumptions of hydrodynamic lubrications the associated Reynolds' equation for the pressure distribution can be obtained as [Prakash and Vij (3), Patel, Deheri and Patel (21)]

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{12\mu \dot{h}_0}{(h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon + 12\psi)} \tag{1}$$

Solving this equation under the boundary conditions

$$p(\pm a/2, z) = 0 \text{ and } p(x, \pm b/2) = 0 \tag{2}$$

we obtain the pressure distribution in non-dimensional form as

$$P = -\frac{h^3 p}{\mu \dot{h}_0 ab}$$

$$= \frac{6k}{\pi^3 A}\left[\left\{\frac{1}{4} - \left(\frac{z}{b}\right)^2\right\}\pi^3 + 8\sum_{n=1,3,\ldots\ldots}^{\infty} \frac{(-1)^{\frac{1}{2}(n+1)}\cos\left(\frac{n\pi z}{b}\right)\cosh\left(\frac{n\pi z}{b}\right)}{n^3\cosh\left(\frac{n\pi}{2k}\right)}\right] \tag{3}$$

where: $A = 1 + 3\sigma^{*2} + 3\alpha^* + 3\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^* + 12\psi$

Then the load carrying capacity w of the bearing can be expressed in dimensionless form as

$$W = -\frac{h^3\ w}{\mu \dot{h}_0 a^2 b^2}$$

$$= \frac{192k^2}{\pi^4 A}\left[\frac{\pi^4}{192k} - \frac{1}{\pi}\sum_{n=1,3,\ldots\ldots}^{\infty} \frac{1}{n^5}\tanh\left(\frac{n\pi}{2k}\right)\right] \tag{4}$$

Lastly, the time Δt taken by the upper plate to reach a film thickness h_2 starting from an initial film thickness h_1 can be determined in non-dimensional form from the equation

$$\Delta T = \frac{W\ h_0^2\ \Delta t}{\mu a^2 b^2}$$

$$= -h_0^2\, W \int_{\bar{h}_1}^{\bar{h}_2} \frac{1}{\bar{h}^3 + 3\sigma^{*2}\bar{h} + 3\bar{h}^2\alpha^* + 3\bar{h}\alpha^{*2} + 3\sigma^{*2}\alpha^* + \alpha^{*3} + \varepsilon^* + 12\psi}\, d\bar{h} \tag{5}$$

where: $\bar{h}_1 = \frac{h_1}{h_o}$, $\bar{h}_2 = \frac{h_2}{h_o}$ and $\bar{h} = \frac{h}{h_o}$

❖ RESULTS AND DISCUSSIONS

The pressure distribution is determined from equation (3) while equation (4) gives the profile of the load carrying capacity. The time height relation is determined from equation (5). It is clearly seen that these expressions depend on several parameters such as k, ψ, σ*, α* and ε*. This study reduces to the squeeze film behavior between rectangular plates analyzed by Prakash and Vij (3) considering the roughness parameters σ*, α* and ε* to be zero. Besides, this investigation compares well with the reports of Prakash and Vij (3). In addition, this article underlines the central role of the aspect ratio in minimizing the adverse effect of transverse surface roughness.

Figures (2) – (3) present the distribution of load carrying capacity with respect to the aspect ratio k for various values of the roughness parameters σ*, α* and ε* respectively while Figure (5) depicts the variation of load carrying capacity with respect to the aspect ratio k for different values of the porosity. It is observed from these figures that the load carrying capacity increases with increasing values of the aspect ratio. Further, porosity, the standard deviation of the roughness, positive variance and positive skewness decrease the load carrying capacity. It is worthwhile to note that the negatively skewed roughness increases the load carrying capacity. Likewise, the negative variance registers an increase in load carrying capacity. Figures (3) and (4) make it clear that the impact of the variance is quite sharp as compared to that of skewness.

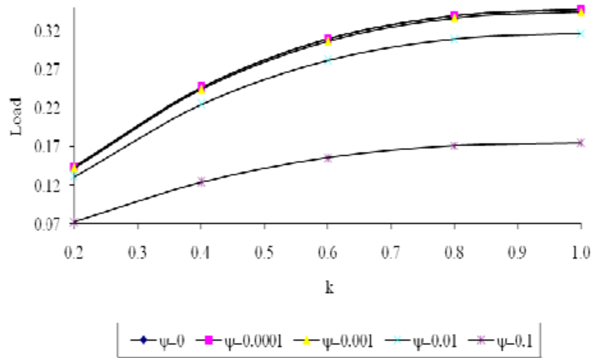

Figure : 2 Variation of load carrying capacity with respect to k and ψ.

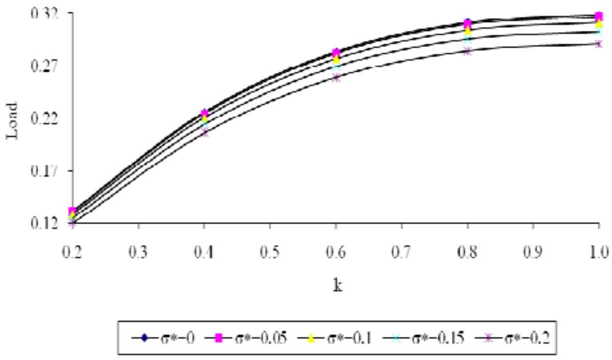

Figure : 2 Variation of load carrying capacity with respect to k and σ*.

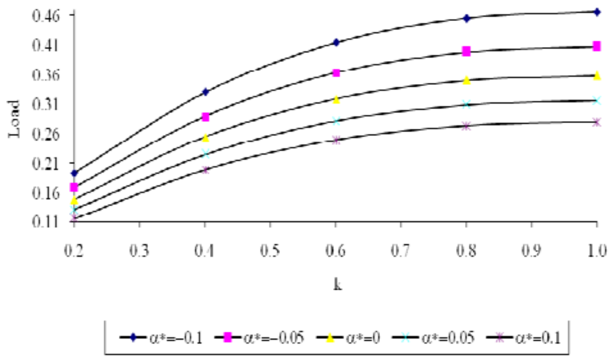

Figure : 4 Variation of load carrying capacity with respect to k and α*

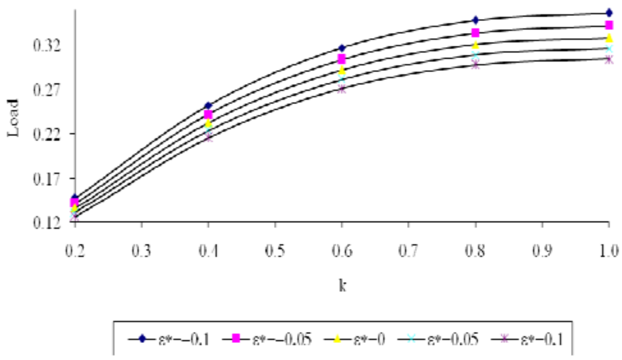

Figure : 5 Variation of load carrying capacity with respect to k and ε*

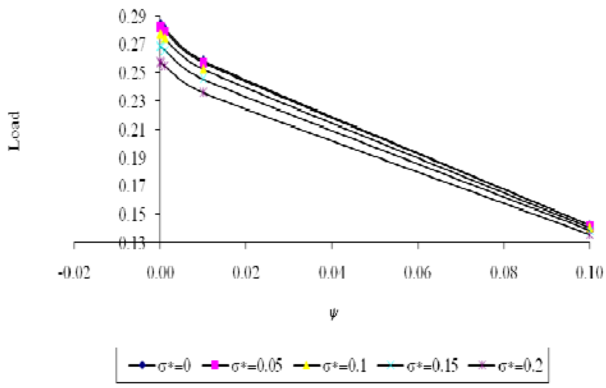

Figure : 6 Variation of load carrying capacity with respect to ψ and σ*

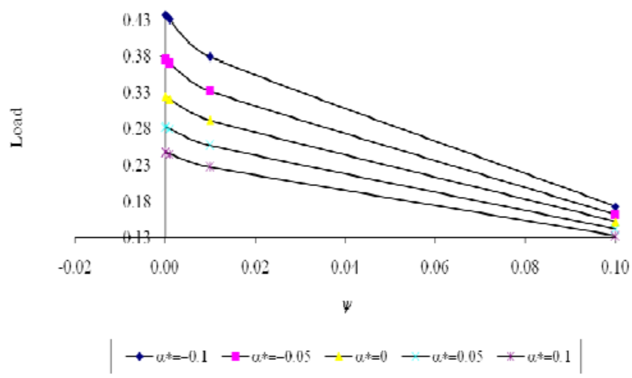

Figure : 7 Variation of load carrying capacity with respect to ψ and α*

In Figures (6) – (8) we have the distribution of load carrying capacity with respect to the porosity parameter ψ for various values of the roughness parameters σ^*, α^* and ε^* respectively. All these figures indicate that the porosity has a strong negative effect on the performance of the bearing system in the sense that even in the case of negatively skewed roughness the load carrying capacity considerably decreases. It is easily observed from Figures: (13) – (15) and Figure (2) that porosity effects are negligible up to $\psi \approx 0.001$.

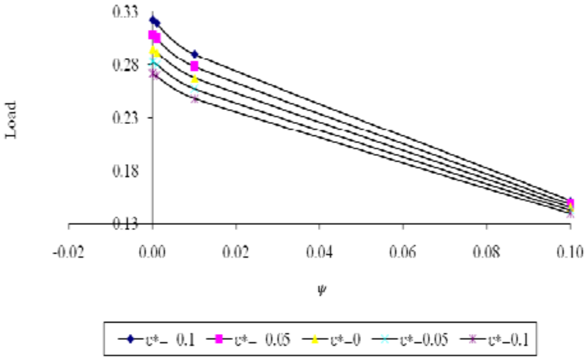

Figure : 8 Variation of load carrying capacity with respect to ψ and ε^*

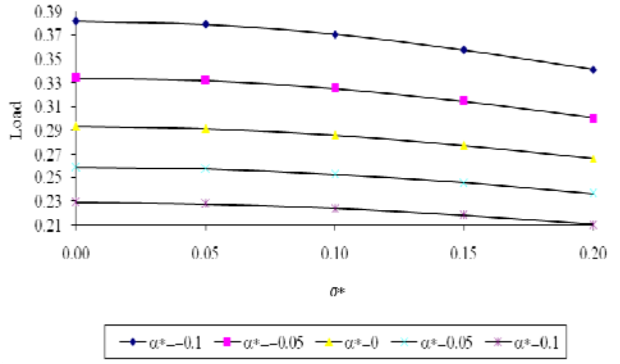

Figure : 9 Variation of load carrying capacity with respect to σ^* and α^*

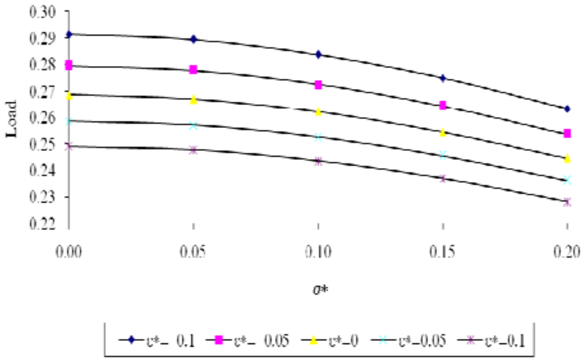

Figure : 10 Variation of load carrying capacity with respect to σ^* and ε^*

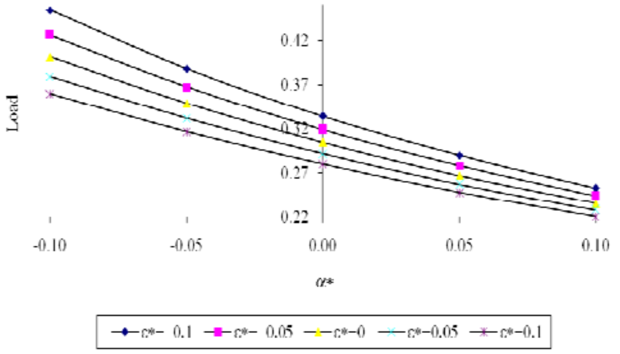

Figure : 11 Variation of load carrying capacity with respect to α^* and ε^*

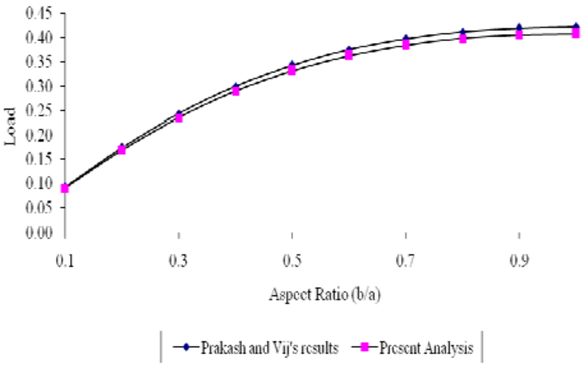

Figure : 12 Comparison of variation of load carrying capacity

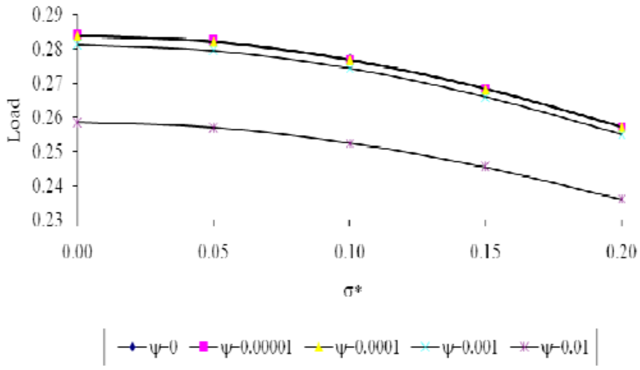

Figure : 13 Variation of load carrying capacity with respect to σ^* and ψ

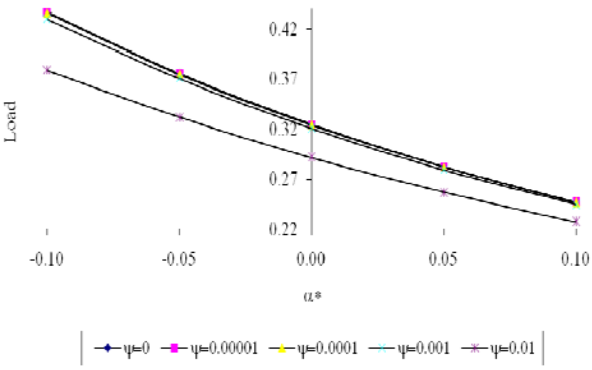

Figure : 14 Variation of load carrying capacity with respect to α^* and ψ

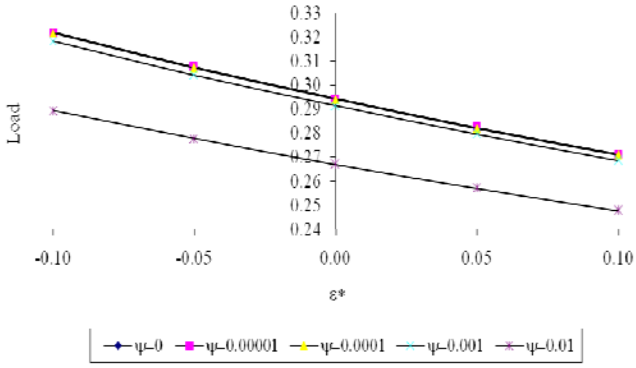

Figure : 15 Variation of load carrying capacity with respect to ε^* and ψ

Figures (9) – (11) give the profile for the load carrying capacity for the combined effect of roughness parameters. It is noticed that the combined effect of the standard deviation and the positive variance is considerably adverse while the combined effect of negative variance and negative skewness is significantly positive. Figures (12) – (15) compares the present result well with that of Prakash and Vij (3) concerning porous rectangular plates.
Further, it is noticed from equation (5) that response time follows the path of the load carrying capacity so far as trends are concerned. However, the time required to squeeze out the whole fluid is not infinite as is the case with non-porous bearings as oil bleeds into the porous matrix to allow the continuity of the flow.
It is clear that transverse surface roughness affects the bearing system adversely. However, this investigation suggests that a proper selection of the aspect ratio may enhance the performance of the bearing system in the case of negatively skewed roughness. Thus, while designing the bearing system the roughness must be given due consideration.

❖ ACKNOWLEDGEMENTS

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❖ NOMENCLATURE

h	*Film thickness*
H	*Thickness of the porous facing*
k	*The aspect ratio (b/a)*
p	*Pressure in the film region*
P	*Non-dimensional film pressure*
w	*Load carrying capacity*
W	*Non-dimensional load carrying capacity*
x,y,z	*Cartesian coordinates*
h_1	*Initial film thickness*
h_2	*Film thickness after time* Δt
Δt	*Time required for the film thickness to increase to a value* h_2 *from* h_1.
ΔT	*Non-dimensional response time*
μ	*Absolute viscosity of the lubricant*
ϕ	*Permeability of the porous facing*
ψ	*Porosity parameter* ($\phi H/h^3$)
α^*	*Non-dimensional variance* (α/h)
σ^*	*Non-dimensional standard deviation* (σ/h)
ε^*	*Non-dimensional skew ness* (ε/h^3)

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