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CALCULATION OF INTERHARMONICS OF POWER ELECTRONIC CONVERTERS – USING OF HARMONIC ANALYSIS

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ABSTRACT: Paper deals with problems of using Fourier progression for harmonic analysis of unfavourable effects in power grid made by power semiconductor converters. Detailed theoretical analyze has been made and useful mathematical equations for interharmonics calculation were derived. Several simulations and experimental measurements for analytical calculation verification have been realized.

KEYWORDS: Fourier Series, EMC, Harmonic Analysis, Interharmonics, Power Converters

❖ INTRODUCTION

The power quality is primarily influenced by the electric appliances connected to the power grid. If a linear load such as resistive heater is connected to the power grid, the resulting current will be a sine wave and, therefore, only the fundamental frequency will be introduced. However, if the load is non-linear, drawing short pulses of current within each cycle, the current shape will be distorted (non-sinusoidal) and higher frequency current components (e.g. characteristic and non-characteristic harmonics, interharmonics - see Fig.1 and 2) will occur in the frequency spectrum of phase current.

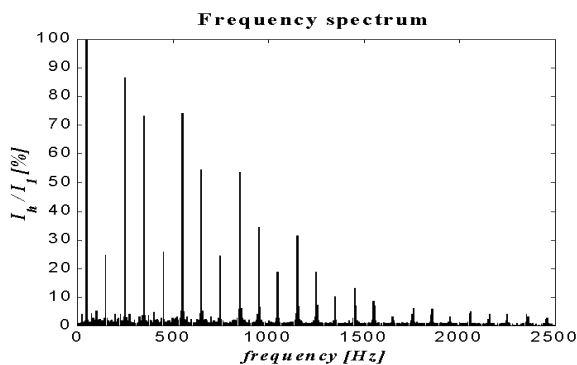


Figure 1. Frequency spectrum of phase current taken by uncontrolled 3phase bridge rectifier (e.g. frequency converter)

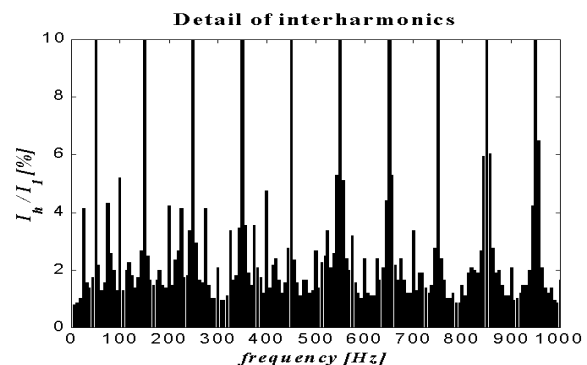


Figure 2. Detail of frequency spectrum of phase current with focus on interharmonic components (defined below)

Thus, the resulting current will be composed of the fundamental and higher frequency components. These frequency components are transferred to the power grid, where they can cause distortion of supply voltage, disturbance of connected equipment (e.g. ripple control devices, compensation units), etc [1,2].

This paper looks mainly at the problem how to calculate these harmonic components. According to standards the low-frequency interference is considered on a frequency range 2.5 kHz and the frequency components can be defined as follows:

Harmonic $f = h * f_1$ where h is an integer > 0

DC $f = 0$ Hz ($f = h * f_1$ where $h = 0$)

Interharmonic $f * h * f_1$ where h is an integer > 0

Sub-harmonic $f > 0$ Hz and $f < f_1$

Where f_1 is the fundamental power system frequency (50 Hz)

❖ HARMONIC ANALYSIS

For calculation of the frequency components in the power grid we can use Fourier progression which is defined for periodical functions. We can show its use at the following example of taken phase current of the 3-phase uncontrolled diode rectifier (see Fig.3). The typical waveform of a taken phase current under ideal operating conditions (symmetrical power supply, ideal power semiconductor devices, indefinite short circuit power etc.) is shown in Fig.4. The non-sinusoidal waveform of a phase current creates higher frequency current components. For the harmonic components calculation of phase current it is necessary to simplify the phase current wave as is shown in Fig.4.

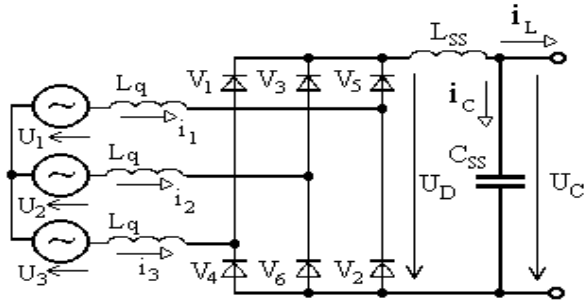


Figure 3. Three-phase bridge rectifier configuration

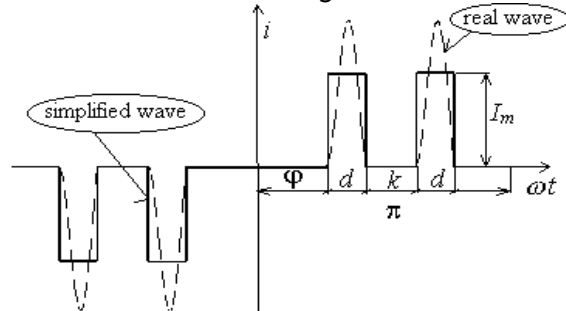


Figure 4. Real and simplified phase current wave

Amplitude I_m is constituted so that the area of both currents will be identical for the same parameter d (where d is a diode conduction time). From the figure it is obvious that used simplification is rough in commensurate with the value of parameter d . The error of used simplification decreases with the decreasing of parameter d and for small value d corresponds to reality.

Using the well-known quotation for Fourier analysis [1,3] we can calculate coefficients a_h and b_h :

$$f(t) = \frac{a_0}{2} + \sum_{h=1}^{\infty} [a_h \cos(h\omega t) + b_h \sin(h\omega t)]. \tag{1}$$

For Fourier coefficients valid:

$$a_0 = \frac{2}{T} \int_0^T f(t) dt, \quad a_h = \frac{2}{T} \int_0^T f(t) \cos(h\omega t) dt, \quad b_h = \frac{2}{T} \int_0^T f(t) \sin(h\omega t) dt, \quad h = 1, 2, \dots$$

Since the current waveform from Fig.4 is symmetrical odd function, coefficients a_h are zero and we can solve coefficients b_h only:

$$b_h = \frac{2}{\pi} \int_0^{\pi} i_f(\omega t) \sin(h\omega t) d\omega t \tag{2}$$

After editing we will get:

$$b_h = -\frac{4I_m}{h\pi} \left[\sin\left(\frac{hk}{2}\right) - \sin\left(\frac{hk}{2} + hd\right) \right] \cdot \sin\left(\frac{h\pi}{2}\right)$$

For symmetrical power network is valid $d+k=600$ and relation (2) we can convert to:

$$b_h = \frac{8I_m}{h\pi} \cdot \sin\frac{hd}{2} \cdot \cos\frac{h\pi}{6} \cdot \sin\frac{h\pi}{2} \tag{3}$$

The Back expression of current i by Fourier progression is:

$$i_f(\omega t) = \sum_{h=1}^{\infty} \frac{8I_m}{h\pi} \sin\frac{hd}{2} \cdot \sin\frac{h\pi}{2} \cdot \cos\frac{h\pi}{6} \cdot \sin(h\omega t) \tag{4}$$

For higher current harmonics amplitudes is valid:

$$I_h = \frac{1}{h} I_1 \cdot \frac{\sin\frac{hd}{2}}{\sin\frac{d}{2}} \tag{5}$$

$$I_1 = \frac{8I_{fm}}{\pi} \cdot \sin\frac{d}{2} \cdot \cos\frac{\pi}{6} = 2,205 \cdot I_{fm} \cdot \sin\frac{d}{2}$$

where

When we use the relation (4), we find out that only harmonics of a definite order (5., 7., 11., 13. etc.) will appear on a frequency spectrum. These harmonic orders are called characteristic harmonics and their amplitudes are solved by an equation (5) so-called "1 over h rule". Under real conditions,

unbalanced power source - amplitude or phase non-symmetry, the considered problem becomes more complicated and in the frequency spectrum we can find also non-characteristic components. In contrast to characteristic harmonics for calculation amplitudes of non-characteristic harmonics we can not use equation (5) and we have to apply numerical Fourier analysis (DFT or FFT). Power source non-symmetry causes distortion of phase currents and drift of basic harmonic wave of phase current against phase voltage [2,3,4,5].

Excepting characteristic and non-characteristic harmonics discussed in the previous paragraph, we can also find interharmonic components in frequency spectrum of consumed current (see Figure 1). The interharmonics occur as a consequence of dynamic changes of circuit parameters (power supply voltage dips, load variation etc.). The interharmonic current magnitudes are relatively small in comparison with characteristic and non-characteristic harmonic components, but they may impact the proper function of neighbouring appliances (e.g. interference of ripple control and tuned filters). However frequency of the interharmonic is not integer multiple of the fundamental grid frequency 50Hz, therefore we cannot use standard Fourier quotations (1). For an explanation occurrence of the frequency components in the power grid it is necessary to derive new multipurpose quotations which allow calculating interharmonic components.

If function $f(t)$ is defined and integrable in the close interval $[-l, l]$ so valid $f \in L(-l, l)$:

$$f(t) = \frac{1}{2} a_0 + \sum_{n=1}^{+\infty} \left(a_n \cos \frac{n\pi\omega t}{l} + b_n \sin \frac{n\pi\omega t}{l} \right) \quad (6)$$

For Fourier coefficients valid:

$$a_n = \frac{1}{l} \int_{-l}^l f(t) \cos \frac{n\pi\omega t}{l} d\omega t \quad (n=0,1,2,\dots) \quad , \quad b_n = \frac{1}{l} \int_{-l}^l f(t) \sin \frac{n\pi\omega t}{l} d\omega t \quad (n=1,2,\dots) \quad (7)$$

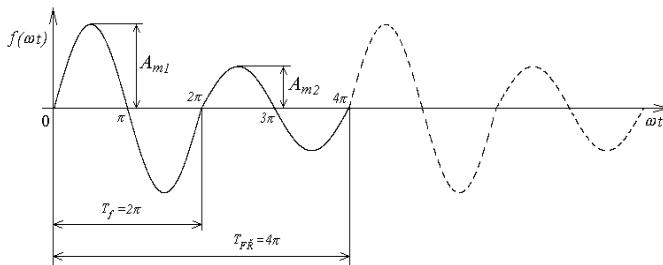


Figure 5. Harmonic waveform of function $f(\omega t)$ for interharmonic components analysis

periodical function in Fig.5 with period $T_f=2\pi$ (50Hz). It changes amplitude each period from value A_{m1} to A_{m2} and so on. From the function behaviour it is evident, that Fourier progression of the function have double period $T_{FR}=4\pi$ (25Hz).

In relation to difference between function period and Fourier progression period we can expect relation between frequency of first harmonic of time function f_1 and frequency of first harmonic of Fourier progression time function f_{FR1} :

$$f_1 = \frac{1}{T_f} = \frac{1}{2\pi}, \quad f_{FR1} = \frac{1}{T_{FR}} = \frac{1}{4\pi} \Rightarrow f_1 = 2 f_{FR1} \quad (9)$$

Therefore we can expect frequency components as multiple of 25 Hz although the function waveform has period 50 Hz as shown in Figures 6-8 (frequency component 50 Hz is 100% and it is second component in the figures).

❖ SIMULATION AND EXPERIMENTAL RESULTS

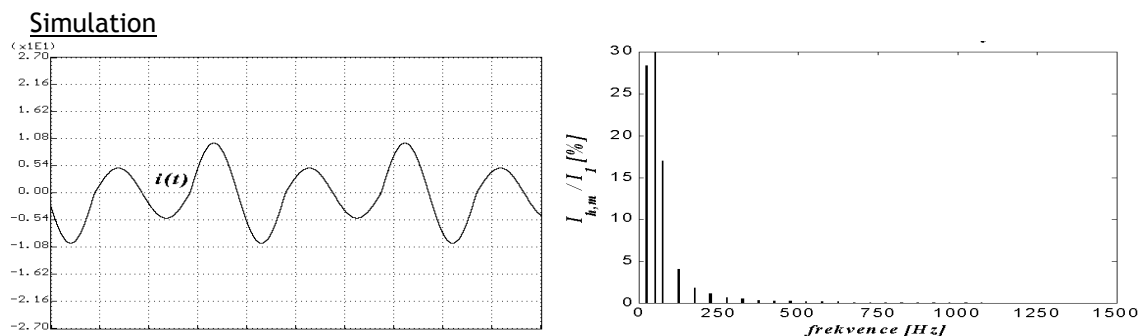


Figure 6. Simulation results of function waveform harmonic analysis

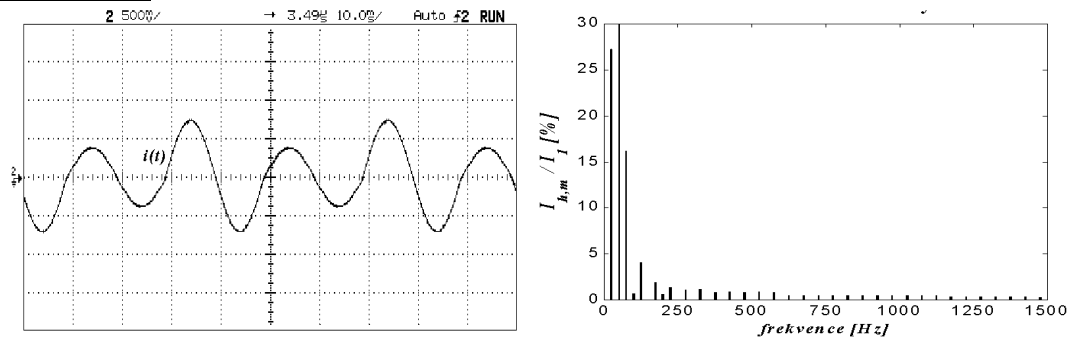
Measurement

Figure 7. Measurement results of function waveform harmonic analysis

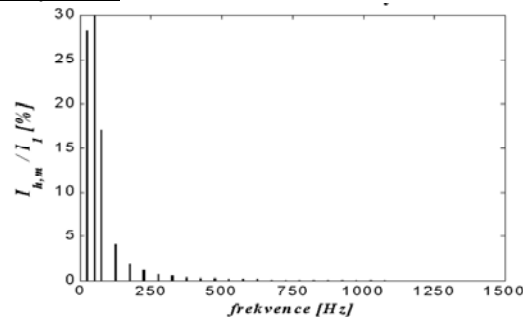
Analytical calculation - equation

Figure 8. Measurement results of function waveform harmonic analysis

❖ CONCLUSIONS

Detailed theoretical analyze has been made and useful mathematical equations for interharmonics calculation were derived. Several simulations and experimental measurements for analytical calculation verification have been realized. Results from simulation and measuring confirm that frequency of first harmonic of time function f_1 and frequency of first harmonic of Fourier progression time function f_{FR1} should not be the same thing.

❖ ACKNOWLEDGMENT

This research work has been made within research project of Czech Science Foundation No. GACR 102/09/1164.

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