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ROTATION EFFECTS ON MHD FLOW PAST AN ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

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ABSTRACT: An exact analysis of rotation effects on unsteady flow of an incompressible and electrically conducting fluid past a uniformly accelerated infinite vertical plate, under the action of transversely applied magnetic field has been presented. The plate temperature is raised linearly with time and the concentration level near the plate is also raised to C'_w. The dimensionless governing equations are solved using Laplace transform technique. The velocity profiles, temperature and concentration are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is also observed that the velocity increases with decreasing magnetic field parameter.
KEYWORDS: Rotation, accelerated, isothermal, vertical plate, heat transfer, mass diffusion, magnetic field

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. It has important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Gupta et al (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis al (1981).

MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh (1985). Mass transfer effects on flow past a uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Basant Kumar Jha and Ravindra Prasad (1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Singh (1984) studied MHD flow past an impulsively started vertical plate in a rotating fluid. Rotation effects on hydromagnetic free convective flow past an accelerated isothermal vertical plate was studied by Raptis and Singh (1981).

Hence, it is proposed to study the effects of rotation on the hydromagnetic free convection flow of an incompressible viscous and electrically conducting fluid past a uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function. Such a study is found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, magnetic suppression of molten semi-conducting materials and meteorology.

❖ GOVERNING EQUATIONS

Consider the unsteady hydromagnetic flow of an electrically conducting fluid induced by viscous incompressible fluid past a uniformly accelerated motion of an isothermal vertical infinite plate when the fluid and the plate rotate as a rigid body with a uniform angular velocity Ω′ about z′-axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially, the temperature of the plate and concentration near the plate are assumed to be T_∞ and C'_∞. At time t′ > 0, the plate starts moving with a velocity $u = u_0 t'$ in its own plane and the temperature from the plate is raised linearly with time and the concentration level near the plate are also raised to C'_w. Since the plate occupying the plane z′ = 0 is of infinite extent, all the physical quantities depend only on z′ and t′. It is assumed that the induced magnetic field is negligible so that $\vec{B} = (0, 0, B_0)$. Then the unsteady flow is governed by free-convective flow of an electrically conducting fluid in a rotating system under the usual Boussinesq's approximation in dimensionless form are as follows:

$$\frac{\partial U}{\partial t} - 2\Omega V = Gr\theta + GcC + \frac{\partial^2 U}{\partial Z^2} - MU \tag{1}$$

$$\frac{\partial V}{\partial t} + 2\Omega U = \frac{\partial^2 V}{\partial Z^2} - MV \tag{2}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} \tag{3}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \tag{4}$$

With the following initial and boundary conditions:

$$\begin{array}{llllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all} & y,\ t' \le 0 \\ t' > 0: & u = u_0 t', & T = T'_\infty + (T'_w - T'_\infty)\,A\,t', & C' = C'_w & \text{at} & y = 0 \\ & u \to 0, & T \to T_\infty, & C' \to C'_\infty & \text{as} & y \to \infty \end{array} \tag{5}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{1/3}},\ V = \frac{v}{(\nu u_0)^{1/3}}, \tag{6}$$

$$t = t'\left(\frac{u_0^2}{\nu}\right)^{1/3},\ Z = z\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\beta(T_w - T_\infty)}{u_0},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0}$$

$$M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{1/3},\ P_r = \frac{\mu C_p}{k},\ S_C = \frac{\nu}{D}$$

where, $A = \left(\frac{u_0^2}{\nu}\right)^{1/3}$.

The hydromagnetic rotating free-convection flow past an accelerated vertical plate is described by coupled partial differential equations (1) to (4) with the prescribed boundary conditions (5). To solve the equations (1) and (2), we introduce a complex velocity q = U + iV, equations (1) and (2) can be combined into a single equation:

$$\frac{\partial q}{\partial t} = Gr\theta + GcC + \frac{\partial^2 q}{\partial Z^2} - mq \tag{7}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & q = 0, & \theta = 0, & C = 0 & \text{for all} & Z,\ t \le 0 \\ t > 0: & q = t, & \theta = t, & C = 1 & \text{at} & Z = 0 \\ & q \to 0, & \theta \to 0, & C \to 0 & \text{as} & Z \to \infty \end{array} \tag{8}$$

where, $m = M + 2i\Omega$.

❖ SOLUTION PROCEDURE

The dimensionless governing equations (2), (3) and (7), subject to the initial and boundary conditions (8), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} q = & \left(\frac{t}{2}+c+cat+d\right)\left[\exp(2\eta\sqrt{mt})erfc(\eta+\sqrt{mt})+\exp(-2\eta\sqrt{mt})erfc(\eta-\sqrt{mt})\right] \\ & -\eta\sqrt{\frac{t}{m}}\left[\frac{1}{2}+ac\right]\left[\exp(-2\eta\sqrt{mt})erfc(\eta-\sqrt{mt})-\exp(2\eta\sqrt{mt})erfc(\eta+\sqrt{mt})\right] \\ & -2c\,erfc(\eta\sqrt{\Pr}) - c\exp(at)[\exp(2\eta\sqrt{(m+a)t})erfc(\eta+\sqrt{(m+a)t}) \\ & \quad +\exp(-2\eta\sqrt{(m+a)t})erfc(\eta-\sqrt{(m+a)t})] \\ & -2d\,erfc(\eta\sqrt{Sc}) + c\exp(at)[\exp(2\eta\sqrt{at\Pr})erfc(\eta\sqrt{\Pr}+\sqrt{at}) \\ & \quad +\exp(-2\eta\sqrt{at\Pr})erfc(\eta\sqrt{\Pr}-\sqrt{at})] \\ & -d\exp(bt)[\exp(2\eta\sqrt{(m+b)t})erfc(\eta+\sqrt{(m+b)t}) \\ & \quad +\exp(-2\eta\sqrt{(m+b)t})erfc(\eta-\sqrt{(m+b)t})] \\ & +d\exp(bt)[\exp(2\eta\sqrt{btSc})erfc(\eta\sqrt{Sc}+\sqrt{bt})+\exp(-2\eta\sqrt{btSc})erfc(\eta\sqrt{Sc}-\sqrt{bt})] \\ & -2act\left[\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr})-\frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\sqrt{\Pr}\ )\right] \end{aligned} \tag{9}$$

$$\theta = t\left\{\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr})-\frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\right\} \tag{10}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{11}$$

Where, $a = \dfrac{m}{\Pr-1}$, $b = \dfrac{m}{Sc-1}$, $c = \dfrac{Gr}{2a^2(1-\Pr)}$, $d = \dfrac{Gc}{2b(1-Sc)}$ and $\eta = \dfrac{Z}{2\sqrt{t}}$

In order to get the physical insight into the problem, the numerical values of q have been computed from (9). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$\begin{aligned} erf(a+ib) = & erf(a) + \frac{\exp(-a^2)}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right] \\ & + \frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}\left[f_n(a,b)+i\,g_n(a,b)\right]+\in(a,b) \end{aligned}$$

where, $f_n = 2a - 2a\cosh(nb)\cos(2ab) + n\sinh(nb)\sin(2ab)$

$g_n = 2a\cosh(nb)\sin(2ab) + n\sinh(nb)\cos(2ab)$

$\left|\in(a,b)\right| \approx 10^{-16}\left|erf(a+ib)\right|$

❖ RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, m and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 2.01 which correspond to ethyl benzene. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71) and water (Pr = 7.0). The numerical values of the velocity, temperature and concentration fields are computed for different physical parameters like Prandtl number, rotation parameter, magnetic field parameter, thermal Grashof number, mass Grashof number, Schmidt number and time.

The temperature profiles are calculated for water and air from equation (10) and these are shown in figure 1. at time t = 0.2. The effect of the Prandtl number plays an important role in temperature field. It is observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

Figure 2 represents the effect of concentration profiles for different Schmidt number (Sc = 0.16, 0.3, 0.6). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

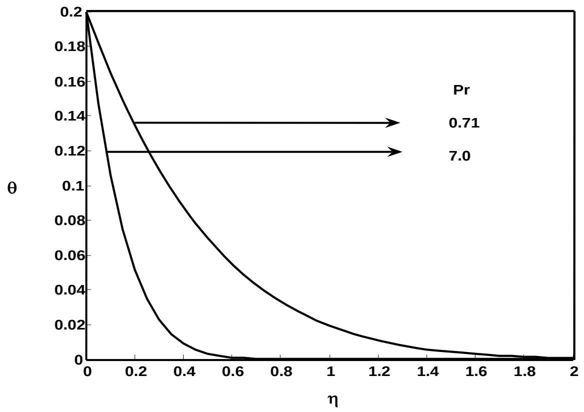

Figure 1. Temperature Profiles for different Pr

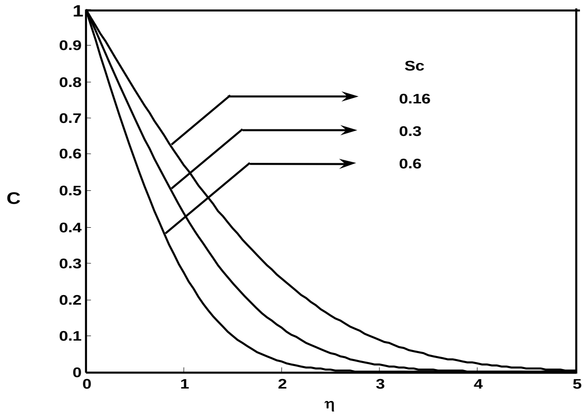

Figure 2. Concentration Profiles for different Sc

Figure 3 demonstrates the effects of different thermal Grashof number (Gr =2, 5), mass Grashof number (Gc = 2, 5), Ω =0.5, M = 1 and Pr = 7 on the primary velocity at time t = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. Figure 4 illustrates the effects of the magnetic field parameter on the velocity when (M = 0, 5, 10), Gr = Gc = 5, Ω =0.5, = 0.5, Pr = 7 and t = 0.4. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

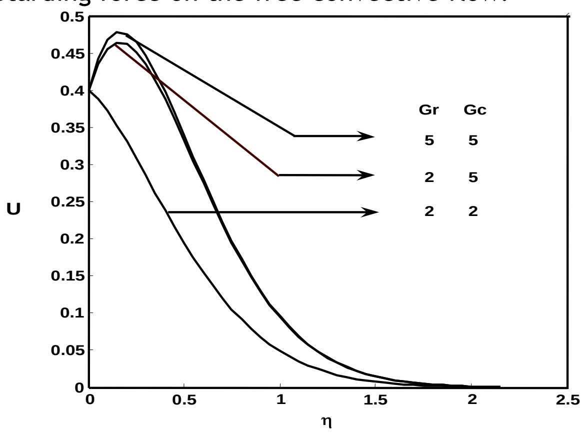

Figure 3. Primary Velocity Profiles for different Gr and Gc

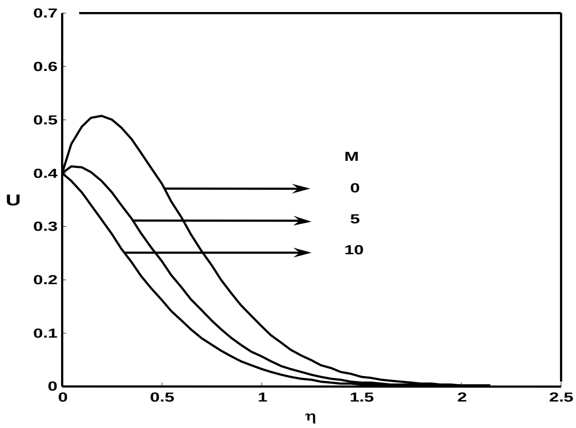

Figure 4. Primary Velocity Profiles for different M

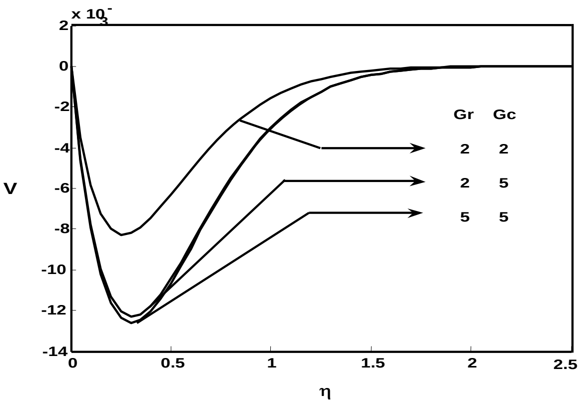

Figure 5. Secondary Velocity Profiles for different Gr and Gc

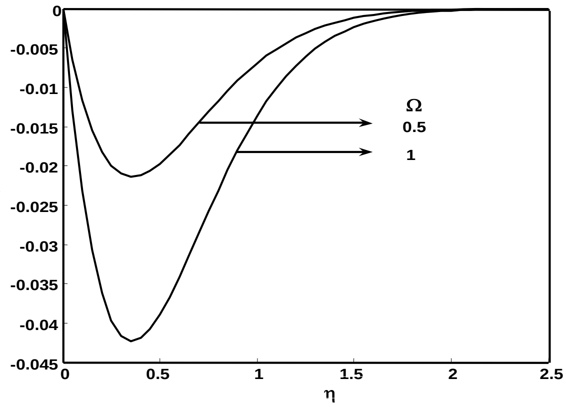

Figure 6. Secondary Velocity Profiles for different Ω

The secondary velocity profiles for different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 2, 5), = 0.5, Ω =0.5, M = 10, Pr = 7 and t = 0.4 are presented in figure 5. The trend shows that the velocity decreases with increasing values of thermal Grahof number or mass Grashof number.

The secondary velocity profiles for different rotation parameter (Ω = 0.5, 1), Gr = Gc = 5, Pr = 7, M = 5 and t = 0.4 are shown in figure 6. It is observed that the velocity increases with decreasing values of the rotation parameter.

❖ CONCLUSION

The theoretical solution of flow past a uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the rotation parameter or magnetic field parameter M.

❖ NOMENCLATURE, GREEK SYMBOLS

A - constant
C' - species concentration in the fluid
C - dimensionless concentration
C_w- wall concentration
C_∞ - concentration far away from the plate
C_p - specific heat at constant pressure
D - mass diffusion coefficient
Gc - mass Grashof number
Gr - thermal Grashof number
g - accelerrated due to gravity
k - thermal conductivity
Pr - Prandtl number
Sc - Schmidt number
T - temperature of the fluid near the plate
T_w - temperature of the plate
T_∞ - temperature of the fluid far away from the plate
t' - time
t - dimensionless time
u - velocity of the fluid in the x-direction
u_0- velocity of the plate
q - dimensionless velocity
x - spatial coordinate along the plate
y - coordinate axis normal to the plate
Z - dimensionless coordinate axis normal to the plate
β - volumetric coefficient of thermal expansion
β - volumetric coefficient of expansion with concentration*
μ - cofficient of viscosity
υ - kinematic viscosity
ρ - density of the fluid
τ- dimensionless skin-friction kg.
θ - dimmensionless temperature
η - similarity parameter
erfc - complementary error function

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