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THEORETICAL ANALYSIS OF FLOW OVER THE SIDE WEIR USING RUNGE KUTTA METHOD

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ABSTRACT: In this study highlights the possibility of increasing the discharge of side channel using inclined side weir with different crest height upstream of flow, five models were used and flow profile were studied along the side weir. Devised an equation to calculate the coefficient of discharge for the rectangular sharp crested weir, and used for calculated the discharge passing over the side weir for all cases of inclined side weir and compare it with the laboratory-measured data. The laboratory data proved the correct of theoretical equation and observed to increase the discharge of the inclined weir by (40%) compared to standard horizontal weir.
KEYWORDS: side weir, coefficient of discharge, Runge Kutta

❖ PREFACE

Flow over side weir is a typical case of spatially varied flow. Side weirs are control structures widely used in irrigation and drainage systems as a means of espies for flood protection works.

The hydraulic behavior of flow over side weir in a rectangular channel has been the subject of many investigations. Subramanya and Awasthy [1], studied the variation of coefficient of discharge with Froude number in subcritical and supercritical condition, from the experimental investigations they found that in super critical condition for side weir of zero height coefficient of discharge varies linearly with the Froude number.

Ranga Raju et. al. [2], related the discharge coefficient of a broad crested rectangular side weir to the main channel Froude number and hesd weir crest width ratio.

Ramamuthy and Carballada's [3], analysis inclined bed slope and frictional effects on side weir.

Other investigators like El-Khshab and Smith [4], Swamee et. al. [5], Ojha and Subbaiah [6], Yilmaz [7] and Rao and Pillai [8] have contributed both experimentally and analytically of the study of flow over side weirs for different flow conditions.

In the present study an alternative concept of elementary discharge coefficient along the side weir is introduced with different angles of side weir lip opposite flow direction. A methodology based on the numerical solution of two ordinary differential equation is proposed for discharge and flow depth, from experimental data collected from (Mwafaq & Ahmed, [9]).

❖ THEORETICAL CONSIDERATION

General expressions for water surface profile along the side weir derived by making use of energy relationships on the basis of the constant specific energy assumption as in the analytical solution. The specific energy of the flow is, (Henderson, [10])

$$E = y + \frac{v^2}{2g} \tag{1}$$

in which E=constant specific energy along the channel; y=mean depth of flow; v=mean cross sectional velocity of flow at any point in the channel; and g=gravitational constant; then

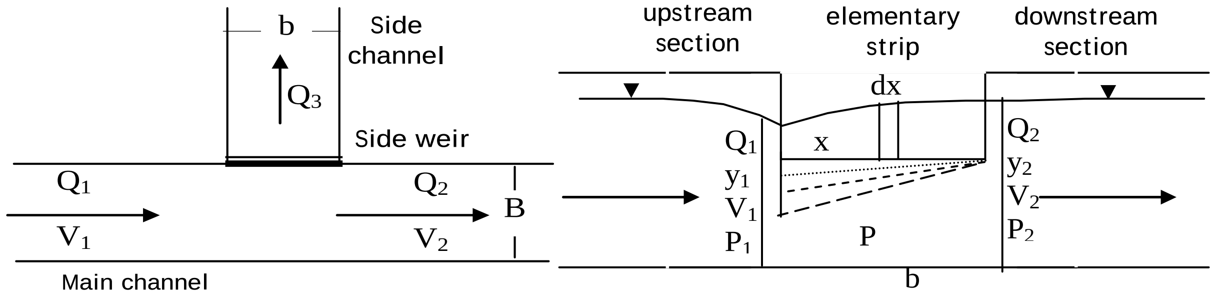
$$E = y + \frac{Q^2}{2gA^2} \tag{2}$$

where: Q=channel discharge, and A= cross sectional area (b*y)

Eq.(2) can be written as (Chaw, [11])

$$Q = by\sqrt{2g(E-y)} \tag{3}$$

For side weir channels, the rate flow Q varies with discharge along the main channel. Although the direction of the side weir is parallel to the flow, the equation of a conventional weir is assumed for discharge dQ through an elementary strip of length dx along the side weir, Fig.1, (Chow, [11]),



(a) (b)

Figure 1. Definition sketch; (a) plan; (b) elevation

$$\frac{dQ}{dx} = -\frac{2}{3} cd \sqrt{2g}(y-p)^{3/2} \tag{4}$$

in which p= weir height; x= distance measured along the side weir; and C_d =weir coefficient

The flow over side weir is a typical case of spatially varied flow with decreasing discharge. The differential equation for flow is (May et.al, [12])

$$\frac{dy}{dx} = \frac{s_o - s_f - \frac{Q}{gA^2}\frac{dQ}{dx}}{1 - F_r^{\,2}} \tag{5}$$

in which S_o=channel bed slope; S_f = friction slope; dQ/dX= discharge variation along the channel length; F_r=Froude number

Considering the discharge through an elementary strip of length along the side weir dQ/dx is given from eq. (4)

Using Manning equation for rectangular channel of bed width b and combining eq. (4 & 5) yield the following equation

$$\frac{dy}{dx} = \frac{s_o - \frac{Q^2 n^2}{b^2 y^{10/3}}(1+\frac{2y}{b})^{4/3} + \frac{2\sqrt{2}}{3} cd (y-p)^{3/2} \frac{Q}{b^2 y^2 \sqrt{g}}}{1 - \frac{Q^2}{g b^2 y^3}} \tag{6}$$

in which n=Manning's roughness coefficient

DISCHARGE COEFFICIENT

To solve equations (4) and (6) a functional relation ship of C_d is required, it can be assumed to be a functional of head to weir height ratio for sharp crested rectangular side weir. Swamee [13] and Swamee et. al. [5] introduced an equation for computation of discharge coefficient for sharp crested weir as

$$cd = k_1 \left[\left(\frac{k_2}{k_3 + (y-p)/p} \right)^{k_4} + \left(\frac{(y-p)/p}{((y-p)/p)+1} \right)^{k_5} \right]^{-k_6} \tag{7}$$

in which k_1:k_6= constant

In the present study used the following equation for calculated coefficient of discharge

$$C_d = c_1 \left[\left(\frac{C_2}{C_3 + (y-p)/p} \right)^{2} + \left(\frac{(y-p)/p}{((y-p)/p)+1} \right)^{-10} \right]^{C_4} \tag{8}$$

in which C_1:C_4= constant

EXPERIMENTS

The experimental study was carried out at Hydraulic Laboratory of the Water Resources Engineering Department, College of Engineering, Mosul University.

In a rectangular flume, fig.(2), with 30cm wide, 45cm height and 10m long, a side channel connected to 90^o at 5.5m from the main channel entrance, 15cm width, 30cm height and 2m long, the discharge at the main channel was measured by a rectangular sharp crested weir made from Plexiglas located at the end of the channel at (30*15*1)cm dimensions while the discharge at branch (side) channel was measured by volumetric calculation .

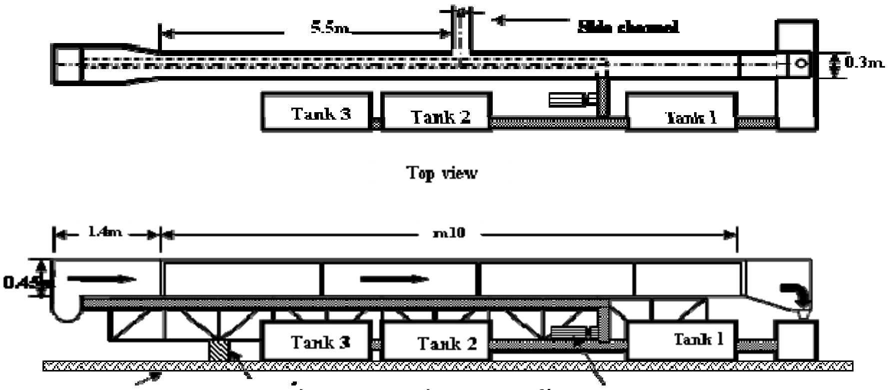


Figure 2. Laboratory flume

The side weirs were made from wood (Mwafaq & Ahmed, 2011) at (15*15*1) cm dimension located at the entrance of side channel, fig.(2), the lip of weir crest at upstream were cutted to three different heights (14,13,12)cm as well as 15cm crest height then four different angles (four cases, case 1, when horizontal crest, case 2, when crest upstream cutted at 14 cm, case 3, when crest upstream cutted at 13 cm, and case 4, when crest upstream cutted at 12 cm) with five different discharge for every case were tested. The range of various parameters is given in table (1).

Table 1. Range of parameters measured for different four cases

Crest height u/s (cm)	Crest height d/s (cm)	P (cm)	Q_1 (l/s)	Q_2 (l/s)	Q_3 (l/s)	F_r	X (cm)
15,14,13,12	15	15	9.38-18.314	8.938-15.114	0.22-2.85	0.157-0.266	15

RESULTS AND DISCUSSIONS

The observed data for all four cases runs were analyzed and eq. (4 & 6) are solved numerically by fourth order Runge Kutta method as shown (Peter, [14])

$$y_{(i+1)} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{9}$$

where

$$k_1 = f(t_i, y_i) \tag{10}$$

$$k_2 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right) \tag{11}$$

$$k_3 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right) \tag{12}$$

$$k_4 = f(t_i + h, y_i + k_3 h) \tag{13}$$

h= intermediate value, y_i = initial value, and $y_{(i+1)}$ = end value.

Eq. (4 & 6) were solved using Matlab language V.R2010a with the values of coefficient of discharge C_d from eq. (8) were required and initial condition as shown

$$\text{At } x=0: \; y=y_1 \tag{14}$$

$$Q=Q_1 \tag{15}$$

in which y_1 = upstream depth; Q_1 = upstream discharge

The solution gave the computed values of water depth and discharge at various x values a long the side weir until (x=b) the water depth and discharge down stream become y_2 and Q_2 respectively.

The computed discharge (Q_3) over the side weir is given by

$$Q_3=Q_1-Q_2 \tag{16}$$

Model parameters (C_1,C_2,C_3 and C_4) at eq. (8) were estimated by SPSS V.17 to adjust model parameters and excitations until the fit between model outputs (Q_{1m}) and laboratory observation (Q_{1o}) are optimized in the weighted least squares sense. The computed side weir discharge (Q_{3m}) is then compared with the observed side weir discharge (Q_{3o}) to yield the average percentage error (E%) as

$$E = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{Q_{3o} - Q_{3m}}{Q_{3o}}\right| \tag{17}$$

in which n = the total number of runs

Table (2) shows the parameters that are estimated by SPSS for every run at the end for all runs. The average percentage error (E%) is a function of the constants in the elementary discharge coefficient equations.

Table 2. Parameters estimated by SPSS using eq (8) in different runs

no	C_1	C_2	C_3	C_4	Q_1	Q_2	Q_3	Error Q_1, %	Error Q_3, %
1	2.17	0.1	0.1	-0.06	9.45-17.11	8.938-15.11	0.24-1.93	0.456	1.968
2	2.045	0.1	0.1	-0.05	9.388-17.66	8.94-15.2	0.22-2.44	0.225	1.296
3	1.544	0.1	0.1	-0.03	9.88-17.75	8.939-15.21	0.54-2.22	0.464	0.495
4	2.426	0.1	0.1	-0.042	9.83-18.314	8.94-15.22	0.55-2.96	1.02	0.892

Fig.3 show the variation of water depth of flow passing a long the longitudinal side weir, from fig. observed clearly increasing of water depth at x direction towards end of side weir and case 4, given the grater water depth height.

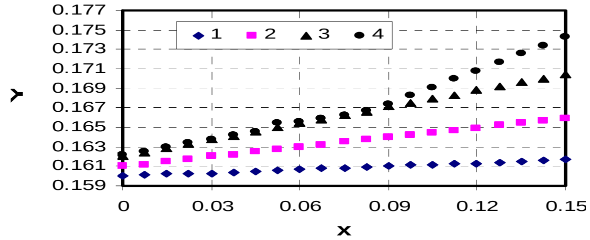


Figure 3. Water depth variation a long side weir

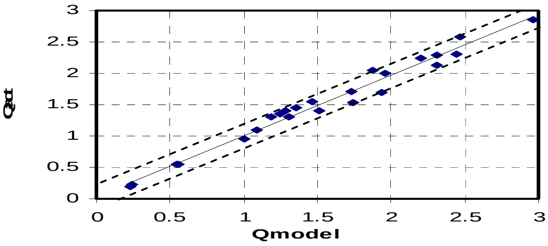


Figure 4. Compared between observed and computed discharge

Fig.4 show a comparison between observed and computed discharges. It shown that the majority of depth points fall in the error not exceed ± 8% and with average error not exceed ± 2% which indicates the validity of elementary discharge coefficient.

Fig. 5 and 6 shows the variation between (y-p)/p and coefficient of discharge across and longitudinal side weir respectively, from fig. observed clearly increasing of C_d with ((y-p)/p) increases, so case 4, have the greatest value of C_d with respect to other cases.

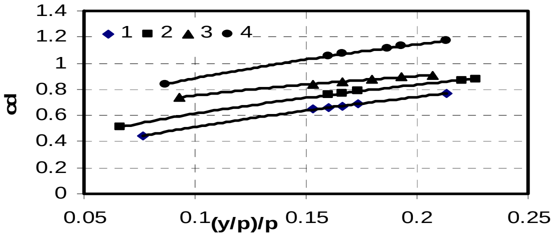


Figure 5. Variation of ((y-p)/p) and coefficient of discharge

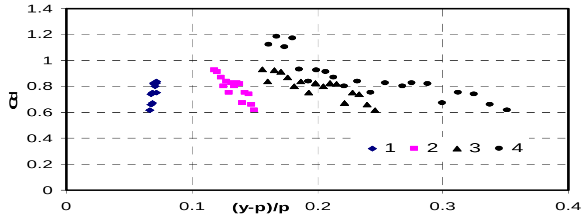


Figure 6. Variation of longitudinal ((y-p)/p) and coefficient of discharge

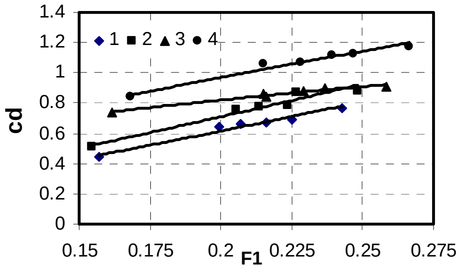


Figure 7. Variation of Froude number and coefficient of discharge

Fig. 7 show the variation of Froude number upstream main channel F_1 respect to C_d, it is shown that increasing of C_d values when Froude number increases for all cases so, case 4, have the greatest value of C_d at the same value of F_1.

CONCLUSION

The elementary discharge coefficient model of inclined side weir was developed in this study, an equation of calculated C_d was developed and shows the possibility of increasing discharge passing over side weir at same depth of water at 40% when crest lip inclined compared with horizontal side weir.

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