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NEW METHODOLOGY CALCULATIONS OF RADIAL STIFFNESS NODAL POINTS SPINDLE MACHINE TOOL

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ABSTRACT: Spindle - bearings system of the machine tools play a major role in the fulfilling the required working accuracy and productivity. The number of spindles supported on angular contact ball bearings is increasing proportionally with increasing demands on the machine tool quality. It is caused by the fact that these spindle bearings in various combinations can to reach sufficient radial and axial stiffness and revolving frequency of the spindle-bearing system. The complex analysis stiffness of nodal points is difficult and complicate. In this paper is introduced as well simplified mathematical apparatus for evaluation of radial stiffness bearing knot.
KEYWORDS: Spindle, Bearing, Machine Tools, Stiffness

INTRODUCTION

The number of spindles supported on angular contact ball bearings is increasing proportionally with increasing demands on the machine tool quality. It is caused by the fact that these bearings can be arranged in various combinations to create bearing arrangements which can be enabling to eliminate radial and also axial loads. The possibility of variation of the number of bearings, preload value, bearings dimensions and contact angle of bearings used in bearing arrangements create wide spectrum of combination to reach sufficient radial stiffness and revolving frequency of the spindle-bearing system, (Fig.1). The sufficient stiffness and revolving frequency of headstock are necessary criteria for reaching demanding manufacturing precision and machine tool productivity.

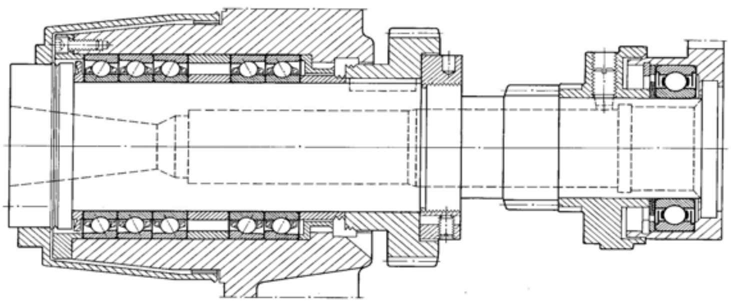

Figure 1: Horizontal machining centre, Thyssen-Hüller Hille GmbH , Germany. Work nodal – 3x71914 ACGB/P4 - 2x71914 ACGB/P4, Opposite side– 6011-2Z

THE CALCULATION OF RADIAL STIFFNESS OF NODAL POINTS

Assumptions of solution

According to the Hertz assumptions [2], there is dependence between the load "P" and deformation "δ" at the contact point of the ball with the plane, given by the relationship

$$P = k_{\delta}.\delta^{3/2} \tag{1}$$

1. the bearings in the nodal points are of the same type and dimensions, with exact geometric dimensions
2. the value of the contact angle is equal for all directionally-arranged bearings in the nodal point, which causes equal distribution of strain on these bearings
3. radial load is equally distributed onto all bearings of the nodal point

Stiffness of nodal points with directionally-arranged bearings

The calculation of the stiffness of a nodal point is based on the stiffness of the bearing itself [4], which is defined as

$$C_{r1} = \frac{d\ F_{r1}}{d\ \delta_{ro}} \tag{2}$$

As radial displacement δ_{r0} is a function of contact deformation δ_0 of the ball with the highest load [3], the equation for calculating stiffness of radial beveled bearings will have the form of

$$C_{r1} = \frac{d\ F_{r1}}{d\ \delta_0}.\frac{d\ \delta_0}{d\ \delta_{ro}} \tag{3}$$

When calculating stiffness, the distribution of load among the rollers must be determined, and the dependence between the load on the top ball and external load must be found. The distribution of load in the bearing can be derived from the static condition of balance [4]

$$F_{r1} = \frac{F_{r1}}{i} = \sum_{j=0}^{z} P_j.\cos(\alpha_j).\cos(j.\gamma) \tag{4}$$

where $\gamma = \dfrac{360}{z}$ is the spacing angle of balls.

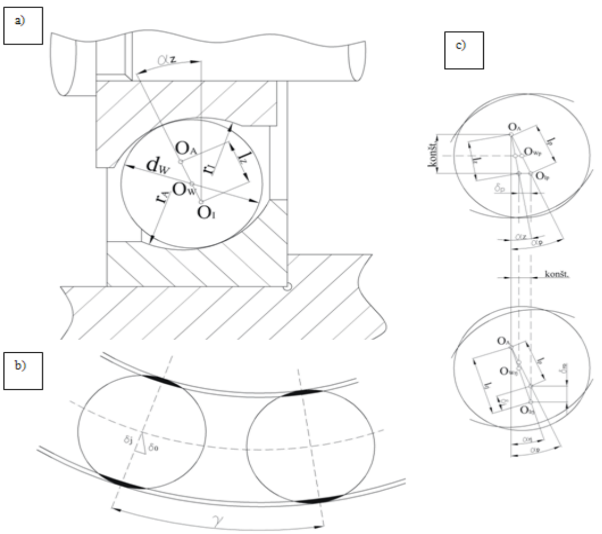

Figure 2: Built-in bearing scheme, a – unloaded, b – pre-stressed, c – radial loaded

The values of contact deformations δ_j and angles α_j differ from each other around the circumference of the bearing and can be expressed as follows, (Fig.2):

$$\delta_j = l_{rj} - l_p = \sqrt{\left[l_z.\sin(\alpha_z) + \delta_p\right]^2 + \left[l_z.\cos(\alpha_z) + \delta_{ro}.\cos(j.\gamma)\right]^2} - l_p \tag{5}$$

$$\cos(\alpha_j) = \frac{l_z.\cos(\alpha_z) + \delta_{ro}.\cos(j.\gamma)}{\sqrt{\left[l_z.\sin(\alpha_z) + \delta_p\right]^2 + \left[l_z.\cos(\alpha_z) + \delta_{ro}.\cos(j.\gamma)\right]^2}} \tag{6}$$

By loading the pre-stressed bearing by radial force is distance between center of balls O_AO_{ip} constant, (Fig.1 b, c).

$$l_p.\sin(\alpha_p) = l_{rj}.\sin(\alpha_{rj}) = konst. \tag{7}$$

The dependence between the deformation of the j-th ball and the top ball can be determined by the relation

$$\delta_j = \delta_0.\cos(j.\gamma) \tag{8}$$

By derivation of the equation (4) we get

$$\frac{d\ F_{r1}}{d\ \delta_0} = i.\sum_{j=0}^{z}\left[\frac{d\ P_j}{d\ \delta_j}.\cos(\alpha_j) - P_j.\sin(\alpha_j).\frac{d\ \alpha_j}{d\ \delta_j}\right].\frac{d\ \alpha_j}{d\ \delta_0}.\cos(j.\gamma) \tag{9}$$

The unknown derivatives in equation (9) can be calculated by derivation/simplification of the relations (1), (7), (8).

$$\frac{dP_j}{d\delta_j} = \frac{3}{2} k_\delta^{2/3} P_j^{1/3} \tag{10}$$

$$\frac{d\alpha_j}{d\delta_j} = -\frac{tg(\alpha_j)}{l_{rj}} \tag{11}$$

$$\frac{d\delta_j}{d\delta_0} = \cos(j.\gamma) \tag{12}$$

The dependence of the contact deformation and radial displacement, Fig. 1, can be determined from the relation

$$\frac{d\ \delta_0}{d\ \delta_{ro}} = \left(\frac{d\ \delta_j}{d\ \delta_0}\right)^{-1}.\frac{d\ \delta_j}{d\ \delta_{ro}} \tag{13}$$

where $\dfrac{d\ \delta_j}{d\ \delta_{ro}}$ is calculated from the equation (5) by inserting equations (14) and (12) into equation (13)

$$\frac{d\delta_j}{d\delta_{ro}} = \frac{1}{2}.\frac{2(l_z \cos\alpha_z + \delta_{ro}\cos(j.\gamma))\cos(j.\gamma)}{\sqrt{(l_z \cos\alpha_z + \delta_{ro}\cos(j.\gamma))^2 + (l_z \sin\alpha_z + \delta_p)^2}} = \cos\alpha_j \cos(j.\gamma) \tag{14}$$

$$\frac{d\ \delta_0}{d\ \delta_{ro}} = \frac{1}{\cos(j.\gamma)}.\cos(\alpha_j).\cos(j.\gamma) = \cos(\alpha_j) \tag{15}$$

After inserting equations (15) and (9) into equation (3) we will get the resulting relation for the stiffness of a pre-stressed nodal point with directionally-arranged bearings.

$$C_r = i.\sum_{j=0}^{z}\left[\frac{3}{2}.k_\delta^{2/3}.P_j^{1/3}.\cos^2(\alpha_j)+P_j.\frac{\sin^2(\alpha_j)}{l_{rj}}\right].\cos^2(j.\gamma) \qquad (16)$$

The stiffness of nodal point with bearings arranged according to the shape

When calculating the nodal point with bearings arranged according to the shape we divide the nodal point into part "1" and part "2" (Fig.1), with the *same* orientation of contact angles -nodes as directionally-arranged bearings, and the stiffness of the parts will be calculated as follows:

$$C_{r1} = i_1.\sum_{j=0}^{z}\left[\frac{3}{2}.k_\delta^{2/3}.P_j^{1/3}.\cos^2(\alpha_{1j})+P_j.\frac{\sin^2(\alpha_{1j})}{l_{r1j}}\right].\cos^2(j.\gamma) \qquad (17a)$$

$$C_{r2} = i_2.\sum_{j=0}^{z}\left[\frac{3}{2}.k_\delta^{2/3}.P_j^{1/3}.\cos^2(\alpha_{2j})+P_j.\frac{\sin^2(\alpha_{2j})}{l_{r2j}}\right].\cos^2(j.\gamma) \qquad (17b)$$

for Fig. 1: numbers of balls: $i_1 = 3$, $i_2 = 2$, *contact angles* $a_1 = a_2 = 25^\circ$

By their subsequent addition we determine the total stiffness of the nodal point with

$$C_r = C_{r1} + C_{r2} \qquad (18)$$

In order to optimize the stiffness and load-bearing capacity for determined technological conditions, the manufacturers of machine tools have come out with a new, non-traditional solution of nodal points. By diminishing the contact angle of the bearing in Part 2, the axial stiffness of the nodal point is partially decreased, but at the same time the value of the radial stiffness and boundary axial load is increased.

Approximate calculation of stiffness

When evaluating the overall stiffness of a spindle, the designer must take into account the approximate calculation of the stiffness of the nodal points. If all the rollers are loaded, and their number is more than 2 per bearing [4], the following equation can be applied:

$$\sum_{j=0}^{z}\cos^2(j.\gamma)=\frac{z}{2} \qquad (19)$$

If the bearing angle is loaded only in an axial direction by the pre-stressing force, then the load on the rollers is constant around the whole circumference and can be expressed, for the particular parts of the nodal point [8],, in the form

$$P_{1j} = \frac{F_p}{i_1.z.\sin(\alpha_{p1})};\; P_{2j} = \frac{F_p}{i_2.z.\sin(\alpha_{p2})} \qquad (20)$$

The magnitude of the contact angles of spindle bearings is not greater than 26 degrees. In that case the value of the second expression in equations (17a) and (17b) is negligible.

Taking into consideration these assumptions, we will get the relationship for the approximate calculation of the radial stiffness of a bearing angle with directionally placed bearings.

$$C_r = \frac{3.10^{-3}}{4}.z^{2/3}.k_\delta^{2/3}.i^{2/3}.F_p^{1/3}.\frac{\cos^2(\alpha)}{\sin^{1/3}(\alpha)} \qquad (21)$$

and with bearings arranged according to shape:

$$C_r = \frac{3.10^{-3}}{4}.z^{2/3}.k_\delta^{2/3}.i_1^{2/3}.F_p^{1/3}.\frac{\cos^2(\alpha_1)}{\sin^{1/3}(\alpha_1)}.\left[1+\frac{i_2^{2/3}.\cos^2(\alpha_2).\sin^{1/3}(\alpha_1)}{i_1^{2/3}.\cos^2(\alpha_1).\sin^{1/3}(\alpha_2)}\right] \qquad (22)$$

where the approximate value of the deformation constant is $k_\delta = \sqrt{1{,}25.d_w}$, d_w – diameter of balls.

The pre-stressing value "F_p" can be calculated according to the standard STN 02 46 15. Some foreign manufacturers (fy SKF, FAG, SNFA ...) publish this value in their catalogues. The number of balls "z" and diameters of balls "d_w" of some types of bearings are quoted in literature, e.g. [7].

❖ VERIFICATION OF MEASURED AND CALCULATED VALUES

The results obtained according to this mathematical model were compared with the values measured by means of the experimental device shown in Fig 3. This device was used to measure the deformation characteristics of nodal points with different combinations of arrangement, pre-stressing values, contact angles, loads and revolution frequencies. This new equation (22) for middle stiffness of the bearing arrangement "C_r" calculation was experimental verified, [1], [5], [6].

At Fig. 3 we have been compared experimental measure stiffness, exactly theoretical and middle calculated radial stiffness of the bearing arrangement B7216 AATB P4 O UL. Results are very good. At zero frequencies the values of radial stiffness are experimental higher than theoretical values, Fig.3.

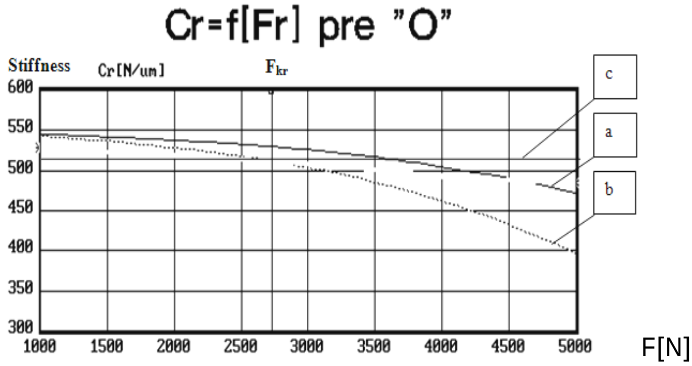

Figure 3. Radial stiffness of the bearing arrangement B7216 AATB P4 O UL, a - experimental, b – exactly theoretical, c – middle theoretical

The dependence of stiffness on loading has a digressive (decreasing) course. The decrease is nearly linear, until a certain critical force “F_{kr}”, at this point the least-loaded roller becomes unloaded. The deformation characteristic of the nodal point is influenced by the type of flange. The direction and gradation of the stiffness change under the given operation conditions depend on their construction. At Fig. 4 and Fig. 5 is experimental stand for messing radial and axial stiffness.

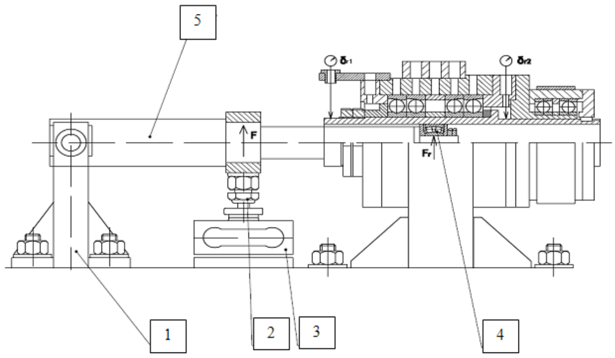

Figure 4 Cut of experimental stand, 1- holder, 2- to retighten screw, 3- dynamometer, 4- force bearing, 5- force arm

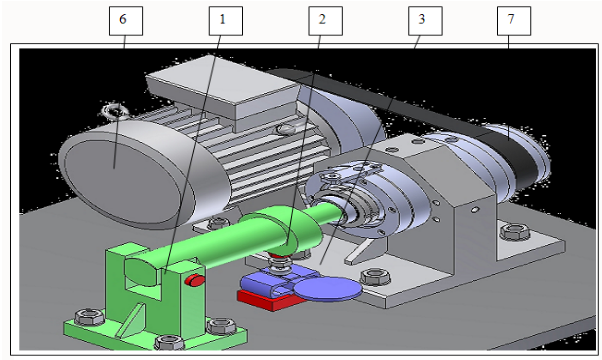

Figure 5.3D model of experimental stand, 1- holder, 2- to retighten screw, 3- dynamometer, 6- drive, 7- Poly V belt

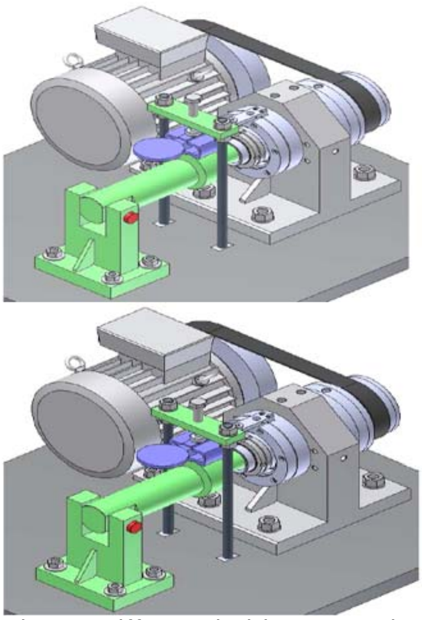

Figure 6.Different decisions measuring radial stiffness bearing knot

❖ RESULTS / SUMMARY

In this paper are presented the regulations for selection and calculation of the radial stiffness of nodal points composed of radial ball bearings with beveled contact. The resulting radial stiffness of spindle nodes is a function of various factors. Its calculation, considering the operation conditions of the node, is quite complicated, and cannot be accomplished without the use of computer technologies.

The research results show that the change of the radial stiffness of pre-stressed nodes is relatively small at zero revolution frequencies, in dependence on loading, and can be mineralized.

In this field the results of the precise and the approximate mathematical model are practically equal. From the preceding it follows that in a preliminary design of mounting, a simplified mathematical model for calculating the stiffness of nodal points can be used, as derived in this article.

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❖ REFERENCES

[1.] Šooš, Ľ.: Statika ložiskových uzlov vretien obrábacích strojov. [Kandidátska dizertačná práca]. STU Bratislava. 1992, 140 s.

[2.] Harris, T.A.: Rolling Bearing Analysis. New York - London - Sydney, 1966, 481s.

[3.] Balmont, V.B. - Russkich,S.P.: Rasčet radialnoj žestkosti radialno - upornogo podšipnika. Trudy instituta. M., Specinformcentr VNIPPa, 69, 1978, č.1, s..93 - 107.

[4.] Kovalev, M.P.- Narodeckiij,M.Z.: Rasčet vysokotočnych šarikopodšipnikov. 2 vyd. Moskva, Mašinostroenie 1980. 279s.

[5.] Šooš, Ľ.: Quality of design engineering: case of machine tools headstock. In: Quality Festival 2008 : 2nd International quality conference. - Kragujevac, May 13-15, 2008. - Kragujevac : University in Kragujevac, 2008. - ISBN 978-86-86663-25-2.

[6.] Lavorčík, L.- Šooš, Ľ.- Zoň, J.: Applied Software Technology of Designing a Bearing Housing Fitted with Rolling Bearing Arrangement. In..: “ICED 91”. Zurich, August, 1991, s.1228 -1233.