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RADIATIVE FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

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ABSTRACT: Thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and species concentration level near the plate is made to rise C_w . The dimensionless governing equations are tackled by the Laplace transform method, when the plate is exponentially accelerated with a velocity $u=u_0 \exp(at')$ in its own plane. The effects of concentration, temperature and velocity fields are studied for different parameters like thermal radiation parameter, Schmidt number, 'a' and time are studied. It is observed that the velocity increases with increasing a or t. But the trend is just reversed with respect to the thermal radiation parameter R.
KEYWORDS: accelerated, vertical plate, radiation, exponential, heat transfer, mass diffusion

❖ INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic Engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

Cess [3] have analyzed thermal radiation effects on heated vertical plate using singular perturbation technique. England and Emery[5] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [7]. Raptis and Perdikis [8] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das [4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [9]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [6]. Basant Kumar Jha (et al.)[2] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Basant Kumar Jha and Ravindra Prasad [1] have studied hydromagnetic effects on flow past an exponentially accelerated infinite vertical plate.

The objective of the present investigation is to study thermal radiation effects on exponentially accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

❖ ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of thermal radiation is studied. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time t'≤0, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time t'>0, the plate is exponentially accelerated with a velocity $u=u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised linearly with time t and the mass is diffused from the plate to the fluid uniformly. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_\rho \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned} &u = 0, \qquad T = T_\infty, C' = C'_\infty \quad for\ all\ y, t' \le 0 \\ &t' > 0 : u = u_0 \exp(a't'), T = T_\infty + (T_w - T_\infty)At', \quad C' = C'_w\ at\ y = 0 \\ &u \to 0 \qquad T \to T_\infty, C' \to C'_\infty\ as\ y \to \infty \\ &where\ A = \frac{u_0^2}{v} \end{aligned} \tag{4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3(T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} &U = \frac{u}{u_0}, t = \frac{t'u_0^2}{v}, Y = \frac{yu_0}{v}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ &Gr = \frac{g\beta v(T_w - T_\infty)}{u_0^3}, C = \frac{C - C_\infty}{C_w - C_\infty}, Gc = \frac{vg\beta^*(C_w - C_\infty)}{u_0^3}, \\ &\Pr = \frac{\mu C_p}{k},\ a = \frac{a'v}{u_0^2},\ Sc = \frac{v}{D} \end{aligned} \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{aligned} &u = 0, \qquad \theta = 0, \quad C = 0, \quad for\ all\ Y, t \le 0 \\ &t > 0 : u = u_0 \exp(at), \ \theta = t, \quad C = 1, \quad at\ \ Y = 0 \\ &U = 0, \qquad \theta \to 0, \quad C \to 0\ \ as\ Y \to \infty \end{aligned} \tag{12}$$

The dimensionless governing equations (6) to (8) with the initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = \frac{t}{2}\left[\exp(\,2\eta\sqrt{Rt}\,)erfc\,(\eta\sqrt{\Pr} + \sqrt{at}\,) + \exp(\,-2\eta\sqrt{Rt}\,)erfc\,(\eta\sqrt{\Pr} - \sqrt{at}\,)\right]$$
$$-\frac{\eta\Pr\sqrt{t}}{2\sqrt{R}}\left[\exp(\,-2\eta\sqrt{Rt}\,)erfc\,(\eta\sqrt{\Pr} - \sqrt{at}\,) - \exp(\,2\eta\sqrt{Rt}\,)erfc\,(\eta\sqrt{\Pr} + \sqrt{at}\,)\right] \tag{13}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{14}$$

$$U = \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})erfc(\eta - \sqrt{at})\right]$$
$$-d\exp(ct)\left[\exp(2\eta\sqrt{ct})erfc(\eta + \sqrt{ct}) + \exp(-2\eta\sqrt{ct})erfc(\eta - \sqrt{ct})\right]$$
$$+2cdt\left[(1+2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] + 2d\,erfc(\eta)$$
$$-e\left[(1+2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2) - (1+2\eta^2 Sc)erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc)\right]$$
$$-d(1+bt)\left[\exp(2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr} - \sqrt{at})\right]$$
$$+\frac{bd\eta\Pr\sqrt{t}}{\sqrt{R}}\left[\exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr} - \sqrt{at}) - \exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr} + \sqrt{at})\right]$$
$$+d\exp(ct)\begin{bmatrix}\exp(-2\eta\sqrt{\Pr(b+c)t})erfc(\eta\sqrt{\Pr} - \sqrt{(b+c)t}) \\ +\exp(2\eta\sqrt{\Pr(b+c)t})erfc(\eta\sqrt{\Pr} - \sqrt{(b+c)t})\end{bmatrix} \tag{15}$$

$$where, b = \frac{R}{\Pr}, c = \frac{R}{1-\Pr},\ d = \frac{Gr}{2c^2(1-\Pr)},\ e = \frac{Gct}{(1-Sc)}\ and\ \eta = \frac{Y}{2\sqrt{t}}$$

❖ RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters a, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the value of Prandtl number Pr are chosen such that they represent air (Pr=0.71). The numerical values of the velocity are computed for different physical parameters like a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 represents the effect of concentration profiles at time t=0.2 for different Schmidt number (Sc=0.16,0.3,0.6,2.01). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

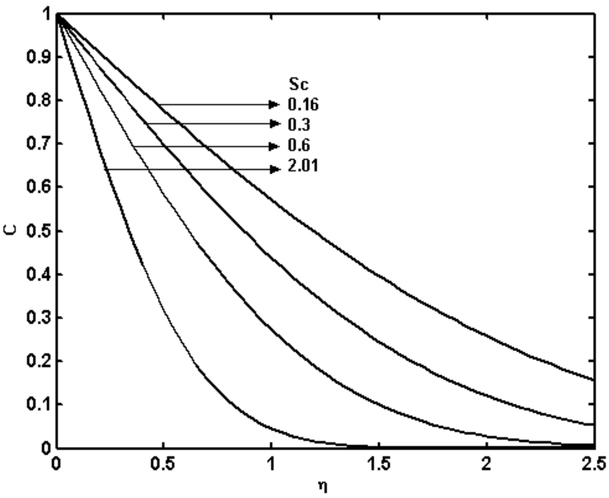

Figure 1. Concentration Profiles for different Sc

The temperature profiles are calculated for different values of thermal radiation parameter(R=0.2, 2, 5) and time (t=0.2) are shown in figure 2 for air (Pr=0.71). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. The trend is just reversed with respect to time t.

The velocity profiles for different (a=0.2, 0.6, 1), Gr=Gc=5, R=2 and t=0.2 are studied and presented in figure 3. It is observed that the velocity increases with increasing values of a. The effect of velocity for different values of the radiation parameter (R=0.2, 0.4, 20), a=0.2, Gr=5=Gc and t=0.2 are shown in figure 4.The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

The velocity profiles for different values of time (t=0.3, 0.4, 0.6, 0.8), Gr=Gc=5 and a=0.2 are shown in figure 5. It is clear that the velocity increases with increasing values of the time t.

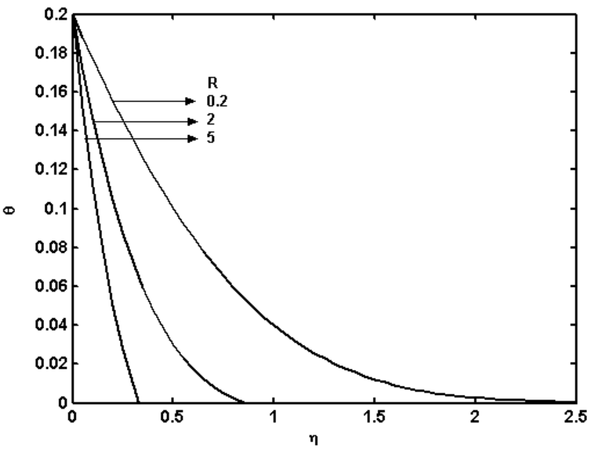

Figure 2. Temperature Profiles for different R

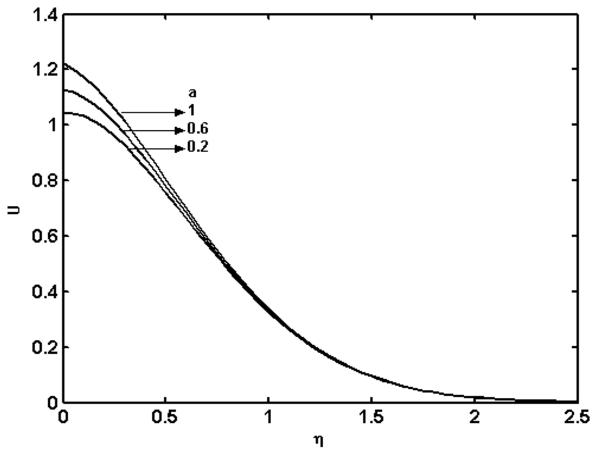

Figure 3. Velocity Profiles for different a

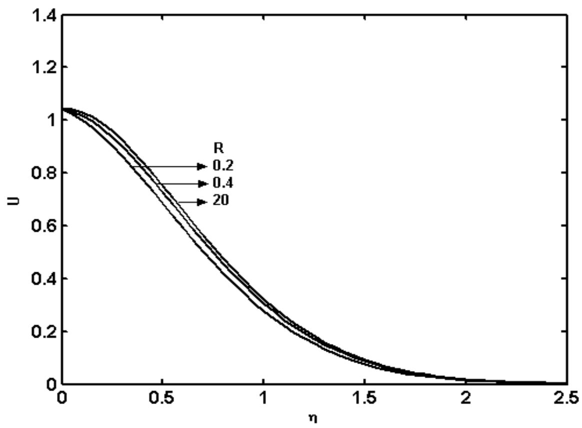

Figure 4. Velocity Profiles for different R

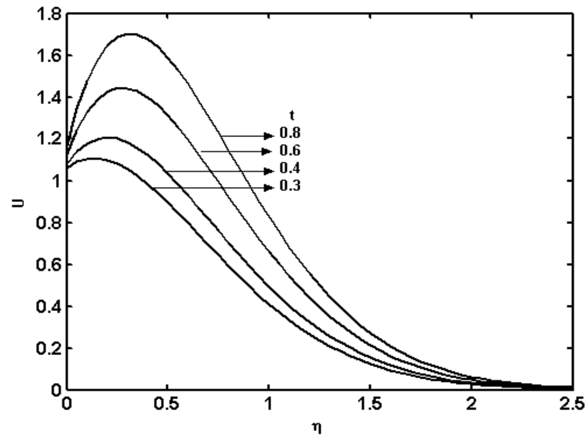

Figure 5. Velocity Profiles for different t

❖ CONCLUSION

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal radiation parameter, a, Schmidt number and t are studied graphically. It is observed that the velocity increases with increasing values of t and a. But the trend is just reversed with respect to the thermal radiation parameter R.

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