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USING CONSTRAINED CUBIC SPLINE INSTEAD OF NATURAL CUBIC SPLINE TO ELIMINATE OVERSHOOT AND UNDERSHOOT IN HHT

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ABSTRACT: Hilbert-Huang Transform (HHT), proposed by N. E. Huang in 1998, is a novel algorithm for nonlinear and non-stationary signal processing. The key part of this method is decomposition the signal into finite number of Intrinsic Mode Functions (IMF) which will meet the requirements of Hilbert Transform. In this part, the algorithm uses natural cubic spline to connect all local maxima and local minima to produce upper and lower envelope of the signal. However natural cubic spline which may lead overshoot and undershoot at intermediate point. In this paper we propose to apply constrained cubic spline into first part of the HHT to eliminate overshoot in upper and lower envelope. We also propose an improvement on Improved Slope Base Method (ISBM) to limit swing at the end points. The experiments showed that our proposal gets better result than original proposal and speed up Empirical Mode Decomposition (EMD) process.
KEYWORDS: HHT; EMD; IMF; constrained cubic spline; SBM, IBSM

INTRODUCTION

The Hilbert Huang Transform consists of two phases. The first phase is decomposition a signal into finite number of IMFs. During this phase the algorithm uses natural cubic spline to connect all local maxima and minima of data to produce upper and lower envelope of data. Natural cubic spline can cause overshoot problem at intermediate points. In addition, the spline needs to cover all data points so this method needs to extend extrema at two sides of data. Each side needs at least two extrema and they must be laid out side of the original data. There were many methods dealt with extrema extension such as The Mirror Method (MM) which is proposed by Rilling et al. This method adds extrema by mirror symmetry with respect to the extrema that are closest to the edges [3]. The slope-based method (SBM) was proposed by Da¨ tig and Schlurmann [4]. This method bases on extrema at head and tail to extend two more extrema for each side. The Improved Slope Base Method (ISBM) was proposed by Fangji Wu and Liangsheng Qu [5]. This method bases on SBM method but it has an improvement to create proper extrema at head and tail. In this paper, we propose to use constrained cubic spline instead of natural cubic spline to eliminate overshoot and decrease complexity of producing spline to speed up shifting process and decrease swing at end points. We also propose an improvement on ISBM to limit swing at the end points. We implement of EMD using both natural cubic spline and constrained cubic spline combining with the improved ISBM. We also implement of EMD using ISBM and the improved ISBM that is proposed by us combine with constrained cubic spline by Java language to experiment some dataset. The running time and results were compared between each method.

THE EMPIRICAL MODE DECOMPOSITION METHOD (EMD)

The empirical mode decomposition method is necessary to deal with data from non-stationary and nonlinear processes [1]. The decomposition is based on the simple assumption that any data consists of different simple intrinsic modes of oscillations. Each intrinsic mode, linear or nonlinear, represents a simple oscillation, which will have the same number of extrema and zero-crossings. Furthermore, the oscillation will also be symmetric with respect to the "local mean." At any given time, the data may have many different coexisting modes of oscillation, one superimposing on the others. The result is the final complicated data. Each of these oscillatory modes is represented by an intrinsic mode function (IMF) with the following definition:

1. in the whole dataset, the number of extrema and the number of zero-crossings must either equal or differ at most by one.
2. at any point, the mean value of the envelope defined by the local maxima and, the envelope defined by the local minima is zero.

With the above definition for the IMF, the algorithm to decompose any data set x(t) into IFM as flowchart in figure 1 below.

In figure 1, the monotonic function is function that has maximum two extrema. There were two stoppage criterions. The first one was used in Huang et al. (1998). This stoppage criterion is determined by using a Cauchy type of convergence test. Specifically, the test requires the normalized squared difference between two successive sifting operations defined as:

$$SD_k = \frac{\sum_{t=0}^{T} |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=0}^{T} h_{k-1}(t)^2} \quad (1)$$

If this squared difference SD_k is smaller than a predetermined value, the sifting process will be stopped.

The second criterion was proposed by Huang et al. (1999, 2003). This criterion based on the agreement of the number of zero-crossings and extrema. Specifically, a S-number is pre-selected. The sifting process will stop only after S consecutive times, when the numbers of zero-crossings and extrema stay the same and are equal or differ at most by one; for the optimal sifting the range of S-numbers should be set between 4 and 8.

Summing up all IMFs and residue we obtain:

$$x(t) = \sum_{i=1}^{n} IFM_i + r_n \quad (2)$$

Thus, one achieves a decomposition of the data into n-empirical IMF modes, plus a residue, r_n.

❖ Constrained cubic spline

The constrained cubic spline was proposed in [6]. Assume we have a collection of known points (x_0, y_0), (x_1, y_1), … (x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}), … (x_n, y_n). To interpolate between these data points using constrained cubic splines, a third degree polynomial is constructed between each point. The equation to the left of point (x_i, y_i) is indicated as f_i with a y value of $f_i(x_i)$ at point x_i. Similarly, the equation to the right of point (x_i, y_i) is indicated as f_{i+1} with a y value of $f_{i+1}(x_i)$ at point x_i.

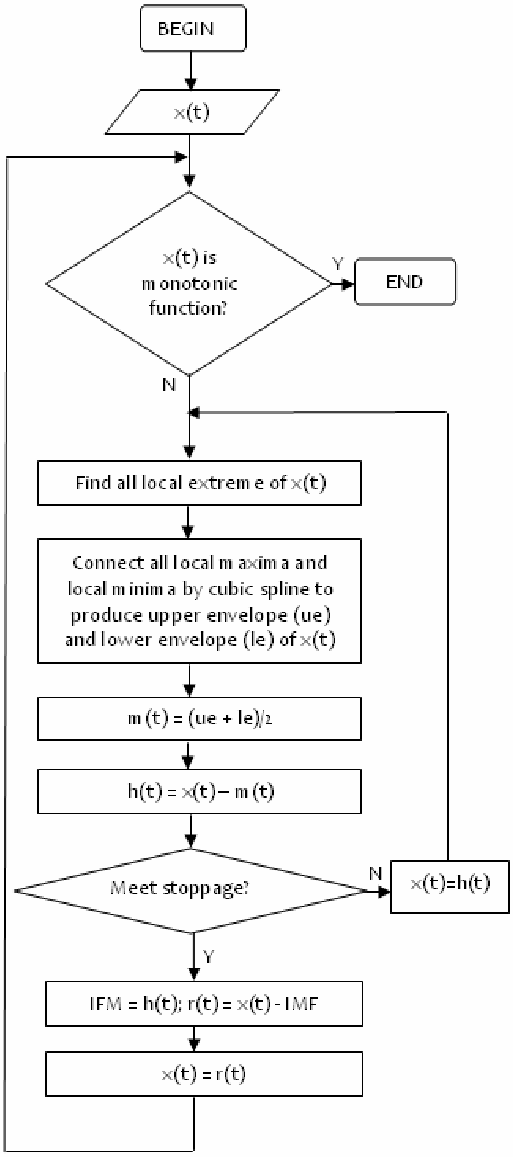

Figure 1. Flowchart of EMD algorithm

The constrained cubic spline function, f_i, is constructed based on the following criteria:

- Curves are cubic polynomials:

$$f_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \quad (3)$$

- Curves pass through all known data points:

$$f_i(x_i) = f_{i+1}(x_i) = y_i \quad (4)$$

- The first order derivative must be continuous at intermediate points:

$$f_i'(x_i) = f'_{i+1}(x_i) = f'(x_i) \quad (5)$$

- $f'(x_i)$ is calculated by following formula for intermediate points:

$$f'(x_i) = \frac{2}{\dfrac{x_{i+1} - x_i}{y_{i+1} - y_i} + \dfrac{x_i - x_{i-1}}{y_i - y_{i-1}}} \quad (6)$$

= *0 if slope changes sign at point.*

- First derivative of each end point is calculated by following formula:

$$f_1'(x_0) = \frac{3(y_1 - y_0)}{2(x_1 - x_0)} - \frac{f'(x_1)}{2} \quad (7)$$

$$f_n'(x_{n+1}) = \frac{3(y_n - y_{n-1})}{2(x_n - x_{n-1})} - \frac{f'(x_{n-1})}{2} \quad (8)$$

- Second derivative and coefficient of equation (3) are calculated by following formula:

$$f_i''(x_{i-1}) = \frac{2[f_i'(x_i)+2f_i'(x_{i-1})]}{(x_i-x_{i-1})} + \frac{6(y_i-y_{i-1})}{(x_i-x_{i-1})^2} \quad (9)$$

$$f_i''(x_i) = \frac{2[2f_i'(x_i)+f_i'(x_{i-1})]}{(x_i-x_{i-1})} - \frac{6(y_i-y_{i-1})}{(x_i-x_{i-1})^2} \quad (10)$$

$$d_i = \frac{f_i''(x_i)-f_i''(x_{i-1})}{6(x_i-x_{i-1})} \quad (11)$$

$$c_i = \frac{x_i f_i''(x_{i-1})-x_{i-1}f_i''(x_i)}{2(x_i-x_{i-1})} \quad (12)$$

$$b_i = \frac{(y_i-y_{i-1})-c_i(x_i^2-x_{i-1}^2)-d_i(x_i^3-x_{i-1}^3)}{x_i-x_{i-1}} \quad (13)$$

$$a_i = y_{i-1}-b_i x_{i-1}-c_i x_{i-1}^2-d_i x_{i-1}^3 \quad (14)$$

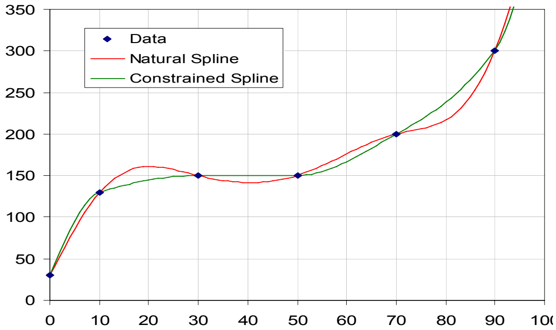

Figure 2. Constrained cubic spline and natural cubic spline for given data

So each spline was given in equation (3) can calculate directly with out solving system equations. There fore the complexity of this method is less than complexity of original method. Figure 2 above shows behavior of the constrained cubic spline which can eliminate overshoot.

❖ THE SLOPE BASE METHOD (SBM) AND THE IMPROVED SLOPE BASE METHOD (ISBM)

SBM. The SBM bases on extrema at the head end tail to extend two more extrema for each side. The algorithm of this method is illustrated by flowchart in figure 4 but it exclusives gray and blue blocks.

ISBM. The SBM method does not consider the case that the ordinate of extended maxima point at the head less than the ordinate of the first original data point and ordinate of extended minima point at the head greater than the ordinate of the first original data point. This issue also happens to extended extrema at the tail. The ISBM was proposed by Fangji Wu and Liangsheng- Qu. This method bases on SBM but it considers the issue that SBM does not. The algorithm of this method is illustrated by flowchart in figure 4 but it exclusives blue blocks. In the flowchart in figure 4 we denote:

- ☐ $x_{min}(1)$, $x_{min}(2)$, $x_{max}(1)$, $x_{max}(2)$ are corresponding abscissa of the first and second minima and maxima.
- ☐ $y_{min}(1)$, $y_{min}(2)$, $y_{max}(1)$, $y_{max}(2)$ are corresponding ordinate of the first and second minima and maxima.
- ☐ $x_{min}(n-1)$, $x_{min}(n)$, $x_{max}(n-1)$, $x_{max}(n)$ are corresponding abscissa of the before last and last minima and maxima.
- ☐ $y_{min}(n-1)$, $y_{min}(n)$, $y_{max}(n-1)$, $y_{max}(n)$ are corresponding ordinate of the before last and last minima and maxima;
- ☐ x(1) and y(1) are abscissa and ordinate of the first data point; x(n) and y(n) are abscissa and ordinate of the last data point.

❖ OUR PROPOSAL

USING CONSTRAINED CUBIC SPLINE INSTEAD OF NATURAL CUBIC SPLINE TO ELIMINATE OVERSHOOT IN EMD

As session 3 above, we have already mentioned that constrained cubic spline could eliminate overshoot and decrease complexity of calculating coefficient of polynomials in equation (3). In this paper we propose using constrained cubic spline instead of natural cubic spline in EMD process to speed up and decrease swing at end point of IMF.

IMPROVEMENT ON ISBM

The ISBM method does not consider the case that the abscissa of the extended maxima or extended minima at the head is greater than abscissa of the first data point and abscissa of the extended maxima or extended minima at the tail is less than abscissa of the last original data point. These cases may occur when the data consists of IMF which oscillations decrease toward two end sides. Figure 3 illustrates the case extended minima does not lie outside the data. It also can occur with extended maxima. If this happens it will cause strong swing at end point. We base on ISBM method and improve it to overcome problem of this method. Our improvement makes sure extended extrema lay out side the original data set. Blue blocks in figure 4 illustrate our improvement.

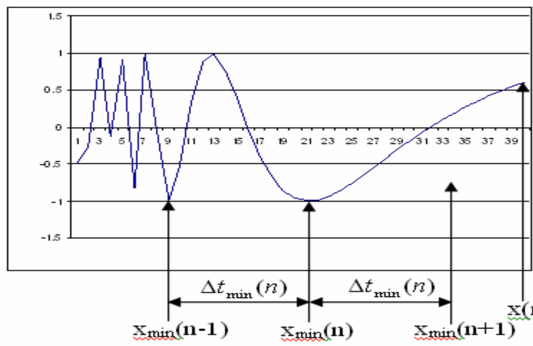

Figure3. Problem with SBM and ISBM

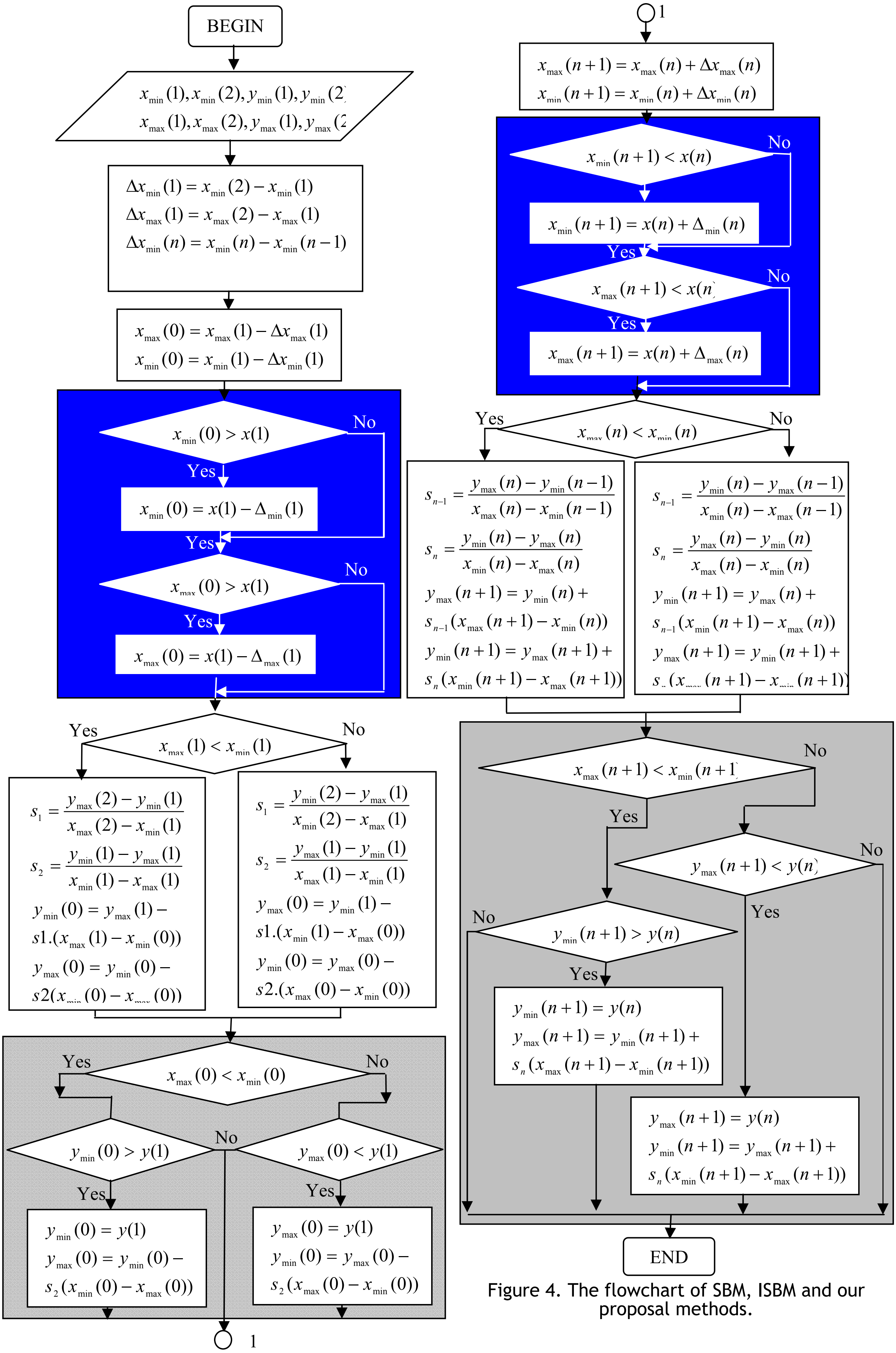

Figure 4. The flowchart of SBM, ISBM and our proposal methods.

❖ EXPERIMENTS

We used java programming language to implement of EMD with both natural cubic spline and constrained cubic spline. Extension end point uses both ISBM and our proposal methods. Using our program we experiment two datasets. The first dataset was addition of three sine signals: f (t) = sin (5t) + sin (10t) + sin (15t). The sampling period was 0.01s. Second dataset was real HVR downloaded from: http://www.signaldb.com/data/RealHRV/MIT-BIH-AD/MIT_BIH_Record_201.txt.

The hardware and software used in our experiments: Laptop IBM T42 (CPU Intel Pentium 1.7 GHz, memory 512 MB). Operating system: Windows XP service packs 2; software: JDK 6.19.

Our result shown that using constrained cubic spline speeds up shifting process because this method calculates directly coefficient of cubic spline instead of solving system equations to find coefficients of original method and decreases swing at end points. Our result also shown that apply our proposal could limit swing at the end point better than ISBM.

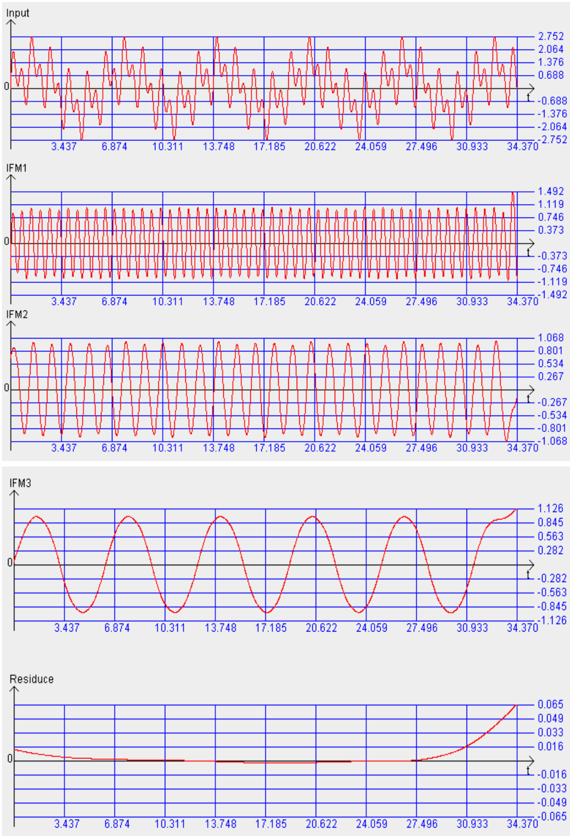

Figure 5. IMFs were extracted by shifting process using constrained cubic spline

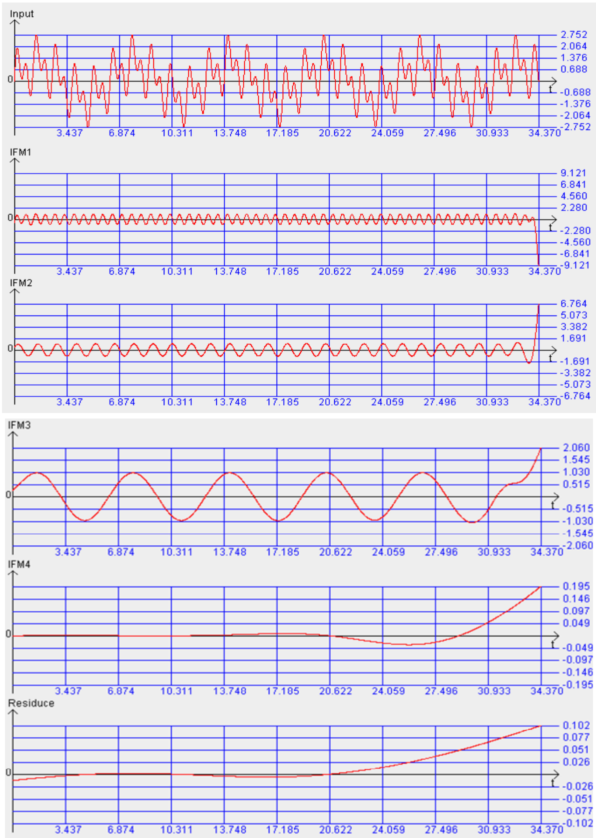

Figure 6. IMFs were extracted by shifting process using natural cubic spline

Table 1 compares running time of EMD process that uses constrained cubic spline and natural cubic spline. Figures 6 and 7 are IMFs which were extracted from dataset 1 using constrained cubic spline and natural cubic spline combining with the improved ISBM. Because of overshoot problem of natural cubic spline which influences EMD extracts redundant IFM and create large swing at the end point. Figure 8 and 9 are IFMs which were extracted from dataset 2. The ISBM method does not consider the case that extended extrema does not lay out side the original data so this method may create very large swing at the end point. We improve ISBM to make sure all extended extrema must position out side the data. The experiment showed that our improvements get a better result than ISBM. All IMFs were extracted by our proposal method are more symmetric, smoother and smaller swing at the end point than IMFs were extracted by ISBM method.

Table 1: Running time of EMD process uses natural cubic spline and constrained cubic spline

Dataset	#samples	Running time (s)		#IFMs	
		Natural cubic spline (1)	Constrained cubic spline (2)	(1)	(2)
1	3438	1.062	0.421	4	3
2	2208	2.784	1.032	9	9

❖ CONCLUSIONS

We have done an effective improvement of ISBM method and apply constrained cubic spline into shifting process. The results of our experiments showed that the swing at the end points were very small and shifting process very fast. In the future we will take some more experiments and find another effective way to deal with end point problem of EMD process.

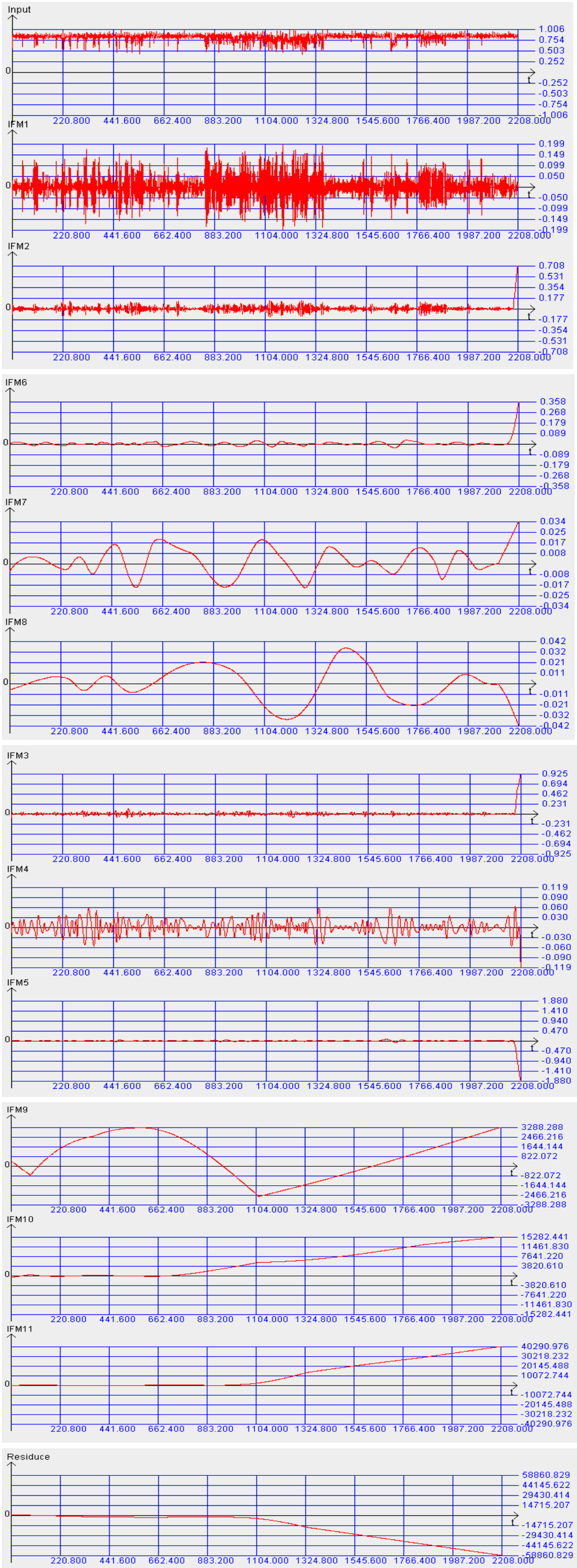

Figure 7. IMFs were extracted by shifting process used constrained cubic spline combined with ISBM

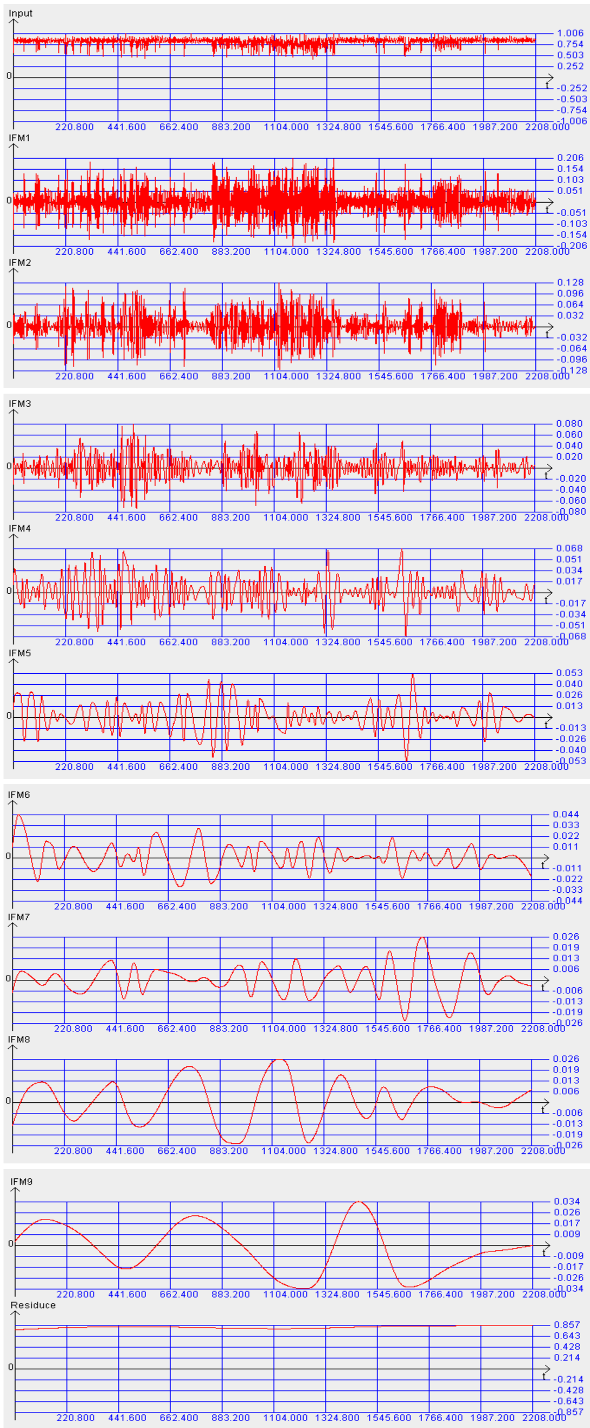

Figure 8. IMFs were extracted by shifting process used constrained cubic spline combined with our improvement on ISBM

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