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1. M. MURALIDHARAN, 2. R. MUTHUCUMARASWAMY

HYDROMAGNETIC EFFECTS ON FLOW PAST AN ACCELERATED ISOTHERMAL VERTICAL PLATE IN THE PRESENCE OF THERMAL RADIATION

1. MATHEMATICS DEPARTMENT, MOHAMED SATHAK A. J. COLLEGE ENGINEERING, SIRUSERI, CHENNAI, INDIA
2. APPLIED MATHEMATICS DEPARTMENT, SRI VENKATESWARA COLLEGE ENGINEERING, PENNALUR, SRIPERUMBUDUR, INDIA

ABSTRACT: Thermal radiation effects on unsteady flow past an uniformly accelerated infinite isothermal vertical plate with uniform mass diffusion, under the action of transversely applied magnetic field has been presented. The plate temperature is raised to T_ω and the concentration level near the plate is also raised to C'_ω. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profiles, temperature and concentration are studied for different physical parameters like magnetic field parameter, radiation parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter.
KEYWORDS: accelerated, isothermal, radiation, vertical plate, heat transfer, mass diffusion, magnetic field

INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery [6] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [7]. The governing equations were solved analytically. Das *et al* [8] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Gupta *et al* [2] studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [3] extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis al [4]. MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh [5]. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar [7]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh[6]. Basant Kumar Jha and Ravindra Prasad [1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

Hence, it is proposed to study hydromagnetic effects on flow past an uniformly accelerated infinite isothermal vertical plate with heat and mass transfer in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

MATHEMATICAL FORMULATION

The unsteady flow of a viscous incompressible fluid past an uniformly accelerated isothermal vertical infinite plate in the presence of magnetic field has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is accelerated with a velocity u= $\frac{u_0^3}{\nu} t'$ in its own plane and the temperature from the plate is raised to T_ω and the concentration level near the plate are also raised to C'_ω. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all } y, t' \le 0 \\ t' > 0: & u = \dfrac{u_0^{\,3}}{\nu}t', & T = T_\omega, & C' = C'_\omega & \text{at } y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^{\,2}}{\nu}, \quad Y = \frac{y u_0}{\nu},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^{\,3}}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \tag{5}$$
$$R = \frac{16a^*\sigma T_\infty^{\,3}}{k}\left(\frac{\nu^2}{u_0^{\,2}}\right), Gc = \frac{g\nu\beta^*(C'_w - C'_\infty)}{u_0^{\,3}}, M = \frac{\sigma B_0^{\,2}\nu}{\rho u_0^{\,2}}, Pr = \frac{\mu C_P}{k}, \ Sc = \frac{\nu}{D}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^{\,4} - T^4) \tag{6}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^{\,3}T - 3T_\infty^{\,4} \tag{7}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_P\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^{\,3}(T_\infty - T) \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} - \frac{R}{Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0 & \text{for all } Y, t \le 0 \\ t > 0: & U = t, & \theta = 1, & C = 1 & \text{at } Y = 0 \\ & U \to 0, & \theta \to 0, & C \to 0 & \text{as } Y \to \infty \end{array} \tag{12}$$

The dimensionless governing equations (9) to (11), subject to the initial and boundary conditions (12) are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \mathrm{erfc}(\eta\sqrt{Pr}) \tag{13}$$

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\,\mathrm{erfc}\left(\eta\sqrt{Pr} + \sqrt{at}\right) + \exp(-2\eta\sqrt{Rt})\,\mathrm{erfc}\left(\eta\sqrt{Pr} - \sqrt{at}\right)\right] \tag{14}$$

$$U = \left(\frac{t}{2} + d + e\right)\left[\exp(2\eta\sqrt{Mt})\mathrm{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\mathrm{erfc}(\eta - \sqrt{Mt})\right]$$
$$-\frac{\eta\sqrt{t}}{2\sqrt{M}}\left[\exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt}) - \exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt})\right]$$

$$- d\exp(bt)\left[\exp\left(-2\eta\sqrt{(M+b)t}\right).\mathrm{erfc}\left(\eta-\sqrt{(M+b)t}\right)+\exp\left(2\eta\sqrt{(M+b)t}\right).\mathrm{erfc}\left(\eta+\sqrt{(M+b)t}\right)\right]$$
$$- e\exp(ct)\left[\exp\left(-2\eta\sqrt{(M+c)t}\right)\mathrm{erfc}\left(\eta-\sqrt{(M+c)t}\right)+\exp\left(2\eta\sqrt{(M+c)t}\right)\mathrm{erfc}\left(\eta+\sqrt{(M+c)t}\right)\right]$$
$$-2d\,erfc(\eta\sqrt{\Pr}) - 2e\,\mathrm{erfc}\left(\eta\sqrt{Sc}\right) - d\left[\exp\left(2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{pr}-\sqrt{at}\right)\right]$$
$$+ d\exp(bt)\left[\exp\left(-2\eta\sqrt{\Pr(a+b)t}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}-\sqrt{(a+b)t}\right)+\exp\left(2\eta\sqrt{\Pr(a+b)t}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}+\sqrt{(a+b)t}\right)\right]$$
$$+ e\exp(ct)\left[\exp\left(-2\eta\sqrt{ct\,Sc}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{ct}\right)+\exp\left(2\eta\sqrt{ctSc}\right)\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{ct}\right)\right] \quad (15)$$

where, $a=\frac{R}{\Pr}$, $b=\frac{M-R}{\Pr-1}$, $c=\frac{M}{Sc-1}$, $d=\frac{Gr}{2b(1-\Pr)}$, $e=\frac{Gc}{2c(1-Sc)}$ and $\eta = Y/2\sqrt{t}$.

❖ RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, M and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 2.01 which corresponds to water-vapor. The value of Prandtl number Pr is chosen such that they represent water (Pr = 7.0).The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 illustrates the effect of the concentration profiles for different values of the Schmidt number $(Sc = 0.16, 0.3, 0.6, 2.01)$ at $t = 0.2$. The effect of the Schmidt number is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the velocity increases with decreasing Schmidt number.

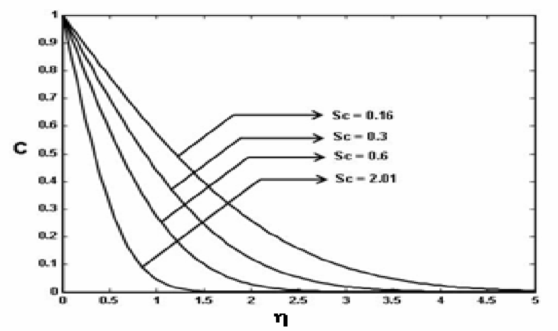

Figure 1. Concentration profiles for different Sc

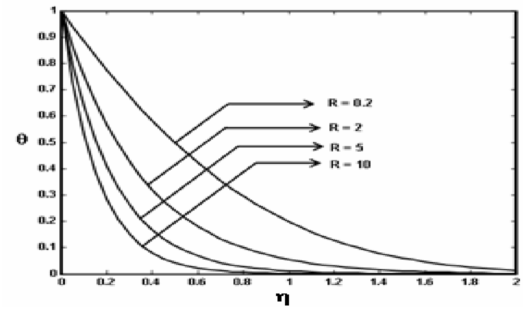

Figure 2. Temperature profiles for different R

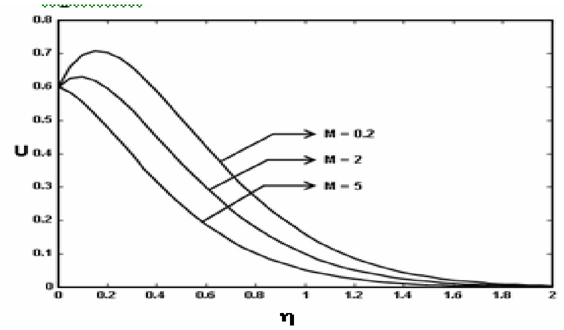

Figure 3. Velocity profiles for different M

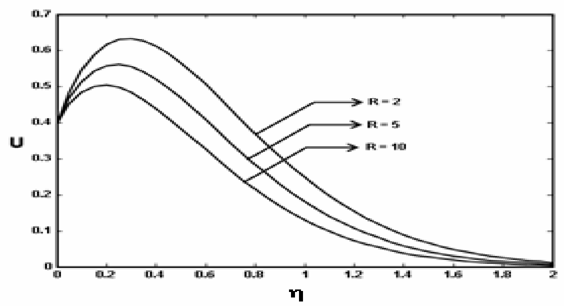

Figure 4. Velocity profiles for different R

The temperature profiles are calculated for different values of thermal radiation parameter$(R = 0.2, 2, 5, 10)$ at time $t = 1$ and these are shown in figure 2. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

Figure 3 illustrates the effects of the Magnetic field parameter on the velocity when $(M = 0.2, 2, 5)$, R=10, Gr=5, Gc=2 and $t = 0.6$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

Figure 4 demonstrates the effects of the magnetic field parameter on the velocity when $(R = 2, 5, 10)$,M=0.2, Gr=5, Gc=2 and $t = 0.4$. It is observed that the velocity increases with decreasing magnetic field parameter.

Figure 5 shows the effects of different thermal Grashof number $(Gr = 2, 5)$, mass Grashof number $(Gc = 2, 5)$, R=2 and $M = 0.2$ on the velocity at time $t = 0.4$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The velocity profiles for different time (t= 0.2, 0.3, 0.4), R=2, Gr=5, Gc=2 and $M = 0.2$ are studied and presented in figure 6. It is observed that the velocity increases with increasing values of the time t.

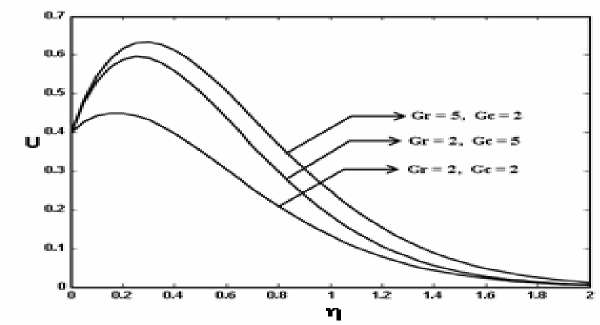

Figure 5. Velocity profiles for different Gr, Gc

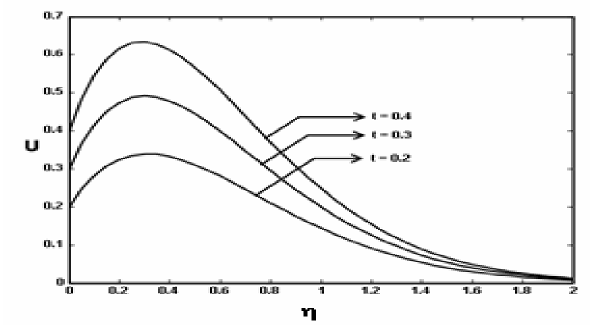

Figure 6. Velocity profiles for different t

❖ CONCLUSIONS

The theoretical solution of hydromagnetic flow past an uniformly accelerated infinite isothermal vertical plate in the presence of thermal radiation have been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the magnetic field parameter.

❖ NOMENCLATURE, GREEK SYMBOLS

a^* absorption coefficient
C' species concentration in the fluid
C dimensionless concentration
C_w wall concentration
C_∞ concentration in the fluid far away from the plate
C_p specific heat at constant pressure
D mass diffusion coefficient
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity
k thermal conductivity of the fluid
Pr Prandtl number
Sc Schmidt number
q_r radiative heat flux in the y-direction
R radiation parameter
T temperature of the fluid near the plate
T_w temperature of the plate
T_∞ temperature of the fluid far away from the plate
t' time
t dimensionless time
u velocity of the fluid in the x-direction
u_0 velocity of the plate
U dimensionless velocity component in x-direction
x spatial coordinate along the plate
y' spatial coordinate normal to the plate
β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration
μ cofficient of viscosity
υ kinematic viscosity
ρ density of the fluid
σ Stefan-Boltzmann constant
θ dimensionless temperature
η similarity parameter
erfc complementary error function

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