ANNALS OF FACULTY ENGINEERING HUNEDOARA
- INTERNATIONAL JOURNAL OF ENGINEERING
Tome IX (Year 2011). Fascicule 3. (ISSN 1584 - 2673)

1. A.G. Vijaya KUMAR, 2. S. Vijaya KUMAR VARMA

THERMAL RADIATION AND MASS TRANSFER EFFCTS ON MHD FLOW PAST A VERTICAL OSCILLATING PLATE WITH VARIABLE TEMPERATRE AND VARIABLE MASS DIFFUSION

1. DEPARTMENT MATHEMATICS, STANLEY STEPHEN COLLEGE OF ENGINEERING & TECHNOLOGY, PANCHALINGALA, KURNOOL, INDIA
2. DEPARTMENT OF MATHEMATICS, S.V. UNIVERSITY, TIRUPATI, ANDHRA PRADESH, INDIA

ABSTRACT: An analytical study is performed to study thermal radiation and mass transfer effects on unsteady MHD flow past a vertical oscillating plate with variable temperature and also with variable mass diffusion in the presence of transverse applied magnetic field. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. At time $t > 0$, the plate temperature and concentration levels near the plate raised linearly with time t. The governing equations are solved in closed form by the Laplace transform technique. The velocity, temperature, concentration and skin- friction are studied graphically for different physical parameters like Radiation parameter, Magnetic field parameter, Schmidt parameter, Prandtl number, thermal Grashof number, mass Grashof number, Phase angle (ωt) and time t.
KEYWORDS: MHD, oscillating vertical plate, thermal radiation, variable temperature, variable mass diffusion

❖ INTRODUCTION

Study of MHD flow with heat and mass transfer play an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermo nuclear fusion and electro magnetic casting of metals. MHD finds applications in electromagnetic pumps, crystal growing, MHD couples and bearings, plasma jets and chemical synthesis. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. It the temperature of surrounding fluid is rather high, radiation effects play an important role and this situation does not exist in space technology. In such cases one has to take into account the effect of thermal radiation and mass diffusion.

Boundary layer flow on moving horizontal surfaces was studied by sakiadas [1]. The effects of transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started isothermal vertical plate was studied by Soundalgekar et al [2]. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al [3]. The dimensionless governing equations were solved using Laplace transform technique. Kumari and nath [4] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point of a two - dimensional body and over a stretching surface with an applied magnetic field. The governing equations were solved using Laplace transform technique. England and Emery [5] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar [6] have considered the radiation free convection flow of an optically thin gray- gas past a semi- infinite vertical plate. Radiation effects on mixed convection along isothermal vertical plate were studied by Hossain and Takhar [7]. In all above studies, the stationary vertical plate is considered. Raptis and Perdikis [8] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das et al [9] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique.

It is proposed to study thermal radiation and mass transfer effects on unsteady MHD flow past a vertical oscillating plate with variable temperature and also with variable mass diffusion in the presence of transverse applied magnetic field. The dimensionless governing equations involved in the presence analysis are solved using Laplace transform technique. The effect of velocity, temperature, concentration and are shown graphically and skin-friction is presented with numerical computations.

❖ MATHEMATICAL FORMULATION

Thermal radiation and mass transfer effects on unsteady MHD flow of a viscous incompressible fluid past along a vertical oscillating plate with variable temperature and also with variable mass diffusion in the presence of transverse applied magnetic field has been studied. The $x'-$ axis is taken along the plate in the vertical upward direction and the $y'-$ axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all the points. At time, $t'>0$ the plate is given an oscillatory motion in its own plane with velocity $u_0 \cos\omega' t'$. At the same time the plate temperature is raised linearly with time t and also mass is diffused from the plate linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Then by usual boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma\beta_0^2 u'}{\rho} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

With the following initial and boundary conditions

$$\begin{aligned} &t' \le 0: u' = 0,\ T' = T'_\infty,\ C' = C'_\infty,\ \text{for all } y' \\ &t' > 0:\ u' = u_0\cos(\omega' t'),\quad T' = T'_\infty + (T'_w - T'_\infty)At',\quad C' = C'_\infty + (C'_w - C'_\infty)At' \quad \text{at} \quad y' = 0 \\ &\text{and } u' = 0,\quad T' \to T'_\infty,\quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty \end{aligned} \tag{4}$$

where $A = \frac{u_0^2}{\nu}$ The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus we get

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

From equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} + 16a^*\ \sigma T'^3_\infty(T'_\infty - T') \tag{7}$$

On introducing the following non-dimensional quantities

$$\begin{aligned} &u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\nu},\ y = \frac{y' u_0}{\nu},\ \theta = \frac{T'-T'_\infty}{T'_w - T'_\infty},\ C = \frac{C'-C'_\infty}{C'_w - C'_\infty},\ a = \frac{a'\nu}{u_0^2},\quad \omega = \frac{\omega'\nu}{u_0^2} \\ &G_r = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3},\ G_m = \frac{g\beta^\bullet\nu(C'_w - C'_\infty)}{u_0^3},\ P_r = \frac{\mu C_p}{\kappa},\ S_c = \frac{\nu}{D},\ M = \frac{\sigma B_0^2\nu}{\rho u_0^2},\ R = \frac{16a^\bullet\nu^2\sigma T'^3_\infty}{k u_0^2} \end{aligned} \tag{8}$$

we get the following governing equations which are dimensionless.

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} - \frac{R}{Pr}\theta, \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{11}$$

The initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &t' \le 0: u = 0,\ \theta = 0,\ C = 0 \text{ for all } y, \\ &t > 0:\ u = \cos\omega t,\ \theta = t,\ C = t \text{ at } y = 0, \text{ and } u \to 0,\quad c \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{12}$$

The appeared physical parameters are defined in the nomenclature. The dimensionless governing equations from (9) to (11), with respect to the boundary conditions (12) are solved by usual Laplace transform technique and the solutions for hydro magnetic flow in the presence of radiation are obtained as follows.

$$\theta(y,t) = \left(\frac{t}{2} + \frac{y\,Pr}{4\sqrt{R}}\right)\exp\left(y\sqrt{R}\right)\mathrm{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{\frac{Rt}{Pr}}\right) + \left(\frac{t}{2} - \frac{y\,Pr}{4\sqrt{R}}\right)\exp\left(-y\sqrt{R}\right)\mathrm{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{\frac{Rt}{Pr}}\right) \tag{13}$$

$$C(y,t)=\left[\left(t+\frac{y^{2}Sc}{2}\right)\operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)-y\sqrt{\frac{tSc}{\pi}}\exp\left(-\frac{y^{2}Sc}{4t}\right)\right] \tag{14}$$

$$\begin{aligned}
u(y,t)=&\frac{1}{4}\left[\exp(i\omega t)\left(\begin{array}{l}\exp\left(y\sqrt{(M+i\omega)}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(M+i\omega)t}\right)\\ +\exp\left(-y\sqrt{(M+i\omega)}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(M+i\omega)t}\right)\end{array}\right)\right]+\frac{1}{4}\left[\exp(-i\omega t)\left(\begin{array}{l}\exp\left(y\sqrt{(M-i\omega)}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-i\omega)t}\right)\\ +\exp\left(-y\sqrt{(M-i\omega)}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-i\omega)t}\right)\end{array}\right)\right]\\
&-\frac{A}{2}\left[\exp\left(y\sqrt{M}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)+\exp\left(-y\sqrt{M}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\right]+B\left[\begin{array}{l}\left(\frac{t}{2}+\frac{y}{4\sqrt{M}}\right)\exp\left(y\sqrt{M}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\\ +\left(\frac{t}{2}-\frac{y}{4\sqrt{M}}\right)\exp\left(-y\sqrt{M}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\end{array}\right]\\
&+\frac{E}{2}\exp(-ct)\left[\begin{array}{l}\exp\left(y\sqrt{M-c}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(M-c)t}\right)\\ +\exp\left(-y\sqrt{M-c}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(M-c)t}\right)\end{array}\right]+\frac{G}{2}\exp(et)\left[\begin{array}{l}\exp\left(y\sqrt{M+e}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(M+e)t}\right)\\ +\exp\left(-y\sqrt{M+e}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(M+e)t}\right)\end{array}\right]\\
&+\frac{E}{2}\left[\exp\left(y\sqrt{R}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{Pr}}\right)+\exp\left(-y\sqrt{R}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{Pr}}\right)\right]\\
&-D\left[\begin{array}{l}\left(\frac{t}{2}+\frac{yPr}{4\sqrt{R}}\right)\exp\left(y\sqrt{R}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{Pr}}\right)\\ +\left(\frac{t}{2}-\frac{yPr}{4\sqrt{R}}\right)\exp\left(-y\sqrt{R}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{Pr}}\right)\end{array}\right]-\frac{E}{2}\exp(-ct)\left[\begin{array}{l}\exp\left(y\sqrt{R-cPr}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{\left(\frac{R}{Pr}-c\right)t}\right)\\ +\exp\left(-y\sqrt{R-cPr}\right)\operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{\left(\frac{R}{Pr}-c\right)t}\right)\end{array}\right]\\
&+G\operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)+F\left[\left(t+\frac{y^{2}Sc}{2}\right)\operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)-y\sqrt{\frac{tSc}{\pi}}\exp\left(\frac{-y^{2}Sc}{4t}\right)\right]-\frac{G}{2}\exp(et)\left[\begin{array}{l}\exp\left(y\sqrt{eSc}\right)\operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{et}\right)\\ +\exp\left(-y\sqrt{eSc}\right)\operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{et}\right)\end{array}\right]
\end{aligned} \tag{15}$$

where $b=\frac{Gr}{Pr-1}, C=\frac{R-M}{Pr-1}, d=\frac{Gm}{Sc-1}, e=\frac{M}{Sc-1}$

$$A=\frac{Gr(Pr-1)}{(R-M)^{2}}+\frac{Gm(Sc-1)}{M^{2}}, B=\frac{M(Gr+Gm)-RGm}{M(R-M)}, D=\frac{Gr}{(R-M)}, E=\frac{Gr(Pr-1)}{(R-M)^{2}}, F=\frac{Gm}{M}, G=Gm\left(\frac{Sc-1}{M}\right)^{2}$$

Skin friction: From velocity field, now we study skin friction (rate of change of velocity in flow with respect to y) it is given in non-dimensional form as

$$\tau=-\left[\frac{du}{dy}\right]_{y=0} \tag{16}$$

From equations (15) and (16), we get skin friction as follows.

$$\begin{aligned}
\tau=&\frac{1}{2}\left[\begin{array}{l}\exp(i\omega t)\left(\frac{\exp(-(M+i\omega))t}{\sqrt{\pi t}}+\sqrt{(M+i\omega)}\operatorname{erf}\sqrt{(M+i\omega)t}\right)\\ +\exp(-i\omega t)\left(\frac{\exp(-(M-i\omega))t}{\sqrt{\pi t}}+\sqrt{(M-i\omega)}\operatorname{erf}\sqrt{(M-i\omega)t}\right)\end{array}\right]\\
&-A\left[\frac{1}{\sqrt{\pi t}}\exp(-Mt)+\sqrt{M}\operatorname{erf}\sqrt{Mt}\right]+B\left[\sqrt{\frac{t}{\pi}}\exp(-Mt)+\left(t\sqrt{M}+\frac{1}{2\sqrt{M}}\right)\operatorname{erf}\sqrt{Mt}\right]+E\exp(-ct)\left[\frac{1}{\sqrt{\pi t}}\exp(-(M-c))t+\sqrt{M-c}\operatorname{erf}\sqrt{(M-c)t}\right]\\
&+G\exp(et)\left[\frac{1}{\sqrt{\pi t}}\exp(-(M+e))t+\sqrt{M+e}\operatorname{erf}\sqrt{(M+e)t}\right]+E\left[\sqrt{\frac{Pr}{\pi t}}\exp\left(\frac{-Rt}{Pr}\right)+\sqrt{R}\operatorname{erf}\sqrt{\frac{Rt}{Pr}}\right]\\
&-D\left[t\sqrt{R}\operatorname{erf}\left(\sqrt{\frac{Rt}{Pr}}\right)+\sqrt{\frac{tPr}{\pi}}\exp\left(-\frac{Rt}{Pr}\right)+\frac{Pr}{2\sqrt{R}}\operatorname{erf}\left(\sqrt{\frac{Rt}{Pr}}\right)\right]-E\exp(-ct)\left[\sqrt{\frac{Pr}{\pi t}}\exp\left(-\left(\frac{R}{Pr}-c\right)\right)t+\sqrt{R-cPr}\operatorname{erf}\sqrt{\left(\frac{R}{Pr}-c\right)t}\right]+G\sqrt{\frac{Sc}{\pi t}}\\
&+2F\sqrt{\frac{tSc}{\pi}}-G\exp(et)\left[\sqrt{\frac{Sc}{\pi t}}\exp(-et)+\sqrt{eSc}\operatorname{erf}\sqrt{et}\right]
\end{aligned}$$

❖ Discussion And Results

In order to know the influence of different physical parameters viz., radiation parameter, Schmidt parameter, Magnetic field parameter, thermal Grashof number, Mass Grashof number, Prandtl number, Phase angle and time on the physical flow field, computations are carried out for vertical velocity, temperature and concentration and they are presented in figures below for the cases of heating (Gr < 0, Gm < 0) and cooling (Gr > 0, Gm > 0) of the plate. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient.

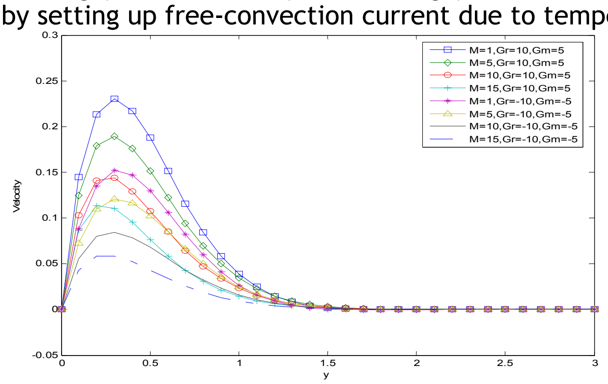

Figure 1: Velocity profiles when R=14, Sc=2.01, Pr=0.71, ωt=π/2, t=0.2

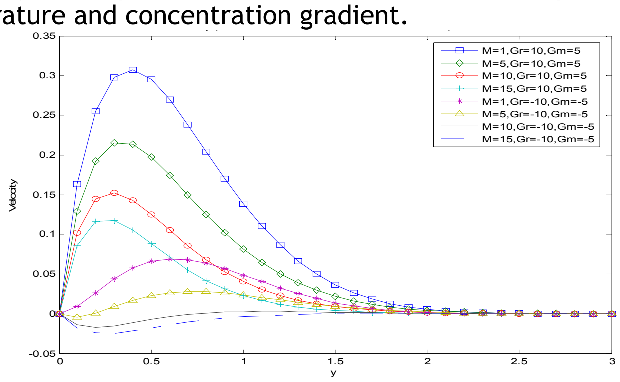

Figure 2: Velocity profiles when R=14, Sc=2.01, Pr=0.71, ωt=π/2, t=0.4

Figure (1) displays the influence of the magnetic parameter M(1, 5, 10, 15) on the velocity field in the cases of cooling and heating of the plate at time t=0.2 when R=14, Sc=2.01, Pr=0.71, ωt=π/2. It is observed that the velocity decreases with increasing of magnetic parameter for both cases of cooling and heating of the plate. It is also observed that the velocity becomes minimum moving away from the plate and finally it leads to an asymptotic value.

Figure (2) also illustrates the effect of magnetic parameter M on the velocity field in the cases of cooling and heating of the plate but it is at time t=0.4 when R=14, Sc=2.01, Pr=0.71, ωt=π/2. It is found that the velocity decreases with increase in M in both cases of cooling and heating of the plate. And it also found that the velocity becomes minimum in asymptotic direction moving far away from the plate.

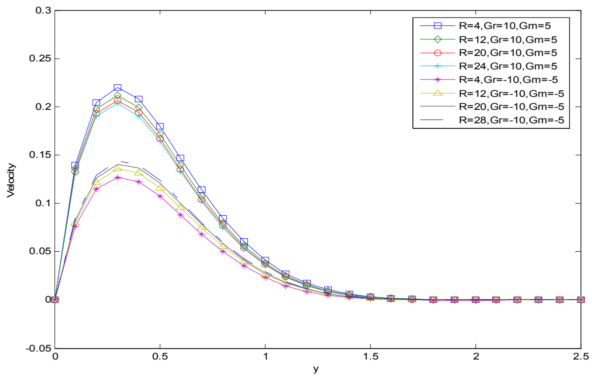

Figure 3: Velocity profiles when M=3, Sc=2.01, Pr=0.71 ωt=π/2, t=0.2

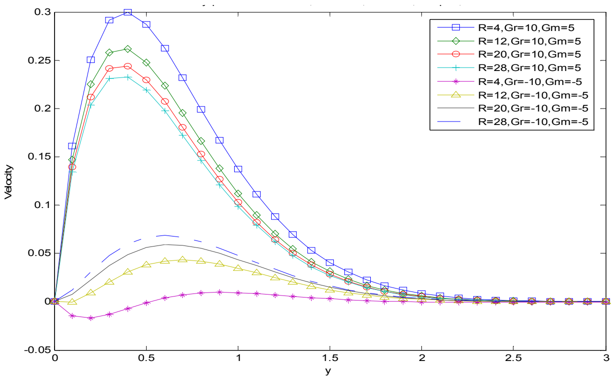

Figure 4: Velocity profiles when M=3, Sc=2.01, Pr=0.71, ωt=π/2, t=0.4

The behavior of the velocity at time t=0.2 when M=3, Sc=2.01, Pr=0.71, ωt=π/2 for different values of Radiation parameter R (4, 12, 20, 28) is exhibited through Figure (3) in the cases of cooling and heating of the plate. It is observed that the velocity decreases with increasing in Radiation parameter R in the case of cooling of the plate. But an opposite phenomenon is identified in the case of heating of the plate. And it also observed that the velocity becomes minimum moving far away from the plate and leads to an asymptotic value. Figure (4) also illustrates that the variation in velocity with increase of radiation parameter R (4, 12, 20, 28) but it is at time t = 0.4 when M=3, Sc=2.01, Pr=0.71, ωt=π/2. It is seen that the velocity decreases in the case of cooling of the plate while it increases in the case of heating of the plate. And it is also seen that the velocity becomes minimum moving away from the plate and finally it leads to an asymptotic value.

Figure (5) shows that the velocity profiles for different values of time t (0.2, 0.4, 0.6, 0.8) when R=14, M=3, Sc=2.01, Pr=0.71, ωt=π/2. It is found that the velocity increases with increase in time t in the cases of cooling of the plate. But decreases with increase in time t in the case of heating of the plate. And it is also found that moving far away from the plate the velocity tends to zero as we choose the boundary conditions to the flow of the problem. Figure (6) describes the velocity profiles for different values of phase angles ωt (0, π/6, π/4, π/3, π/2, 2π/3, 5 π/6, and π) at time t=0.2 when R=14, M=3, Sc=2.01, Pr=0.71, Gr=10, Gm=5. It is observed that increase of phase angle ωt leads to decrease of velocity. And it is observed that all the velocity profiles converge to zero moving far away from the plate.

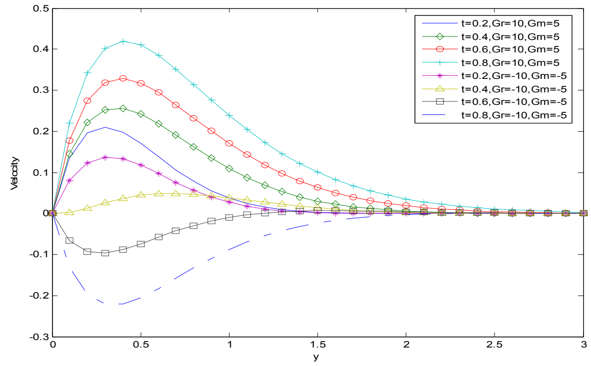

Figure 5: Velocity profiles when R=14, M=3, Sc=2.01, Pr=0.71, wt=pi/2

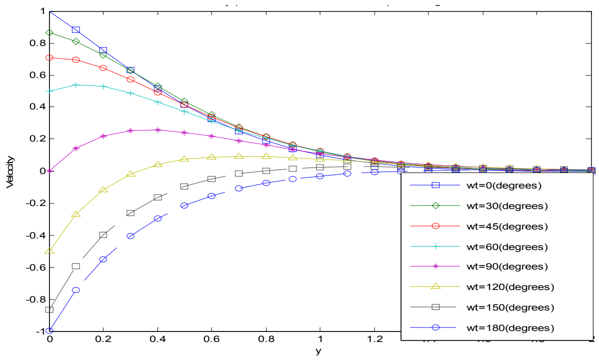

Figure 6: Velocity profiles for different values of phase angles ωt

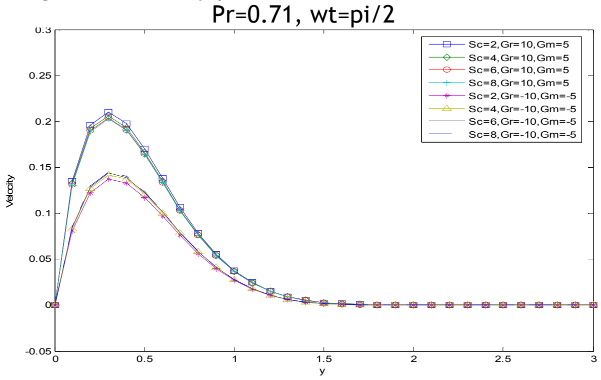

Figure 7: Velocity Profiles when R=14, =3, Pr=0.71, wt=pi/2, t=0.2

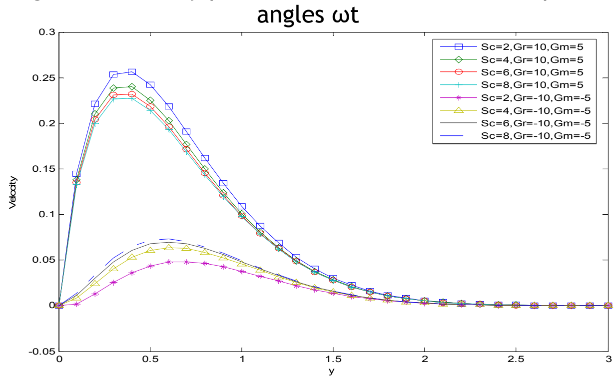

Figure 8: Velocity profiles when R=14, =3, Pr=0.71, wt=pi/2, t=0.4

The behavior of the velocity at time t=0.2 when M=3, R=14, Pr=0.71, ωt=π/2 for different values of Schmidt number Sc (2, 4, 6, 8) is shown through Figure (7) in the case of cooling and heating of the plate. It is found that the velocity decreases with increase of Schmidt number in the case of cooling of the plate but a reverse effect is noted in the case of heating of the plate and finally the velocity tends to zero as it moves far away from the plate. Figure (8) also illustrates that the variation in velocity with increase of Schmidt number Sc (2, 4, 6, 8) but it is at time t = 0.4 when M=3, R=14, Pr=0.71, ωt=π/2. It is seen that the velocity decreases in the case of cooling of the plate while it increases in the case of heating of the plate. And it is also seen that the velocity becomes minimum moving away from the plate and finally it leads to an asymptotic value.

Figure (9) represents the concentration profiles for different values of time t (0.2, 0.4, 0.6, 0.8, and 1.0) with Sc=2.01. Since the concentration is considered as time dependent, therefore this figure clearly reflects situation that the concentration increases with increase of time t. Finally, the concentration in the flow reaches to minimum value and this situation leads to an asymptotic stage. In figure (10) the concentration profiles are presented for different values Sc (2, 4, 6, and 8) at time t=0.2. It is observed that increase of Schmidt number leads to the decrease in concentration of the species. And it is also observed that the concentration will be zero as it moves far away from the plate.

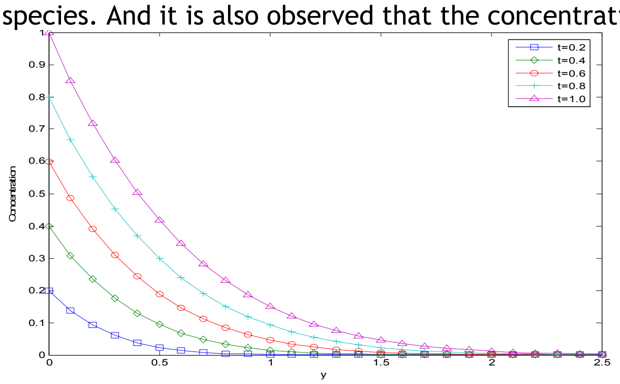

Figure 9: Concentration profiles when Sc=2.01

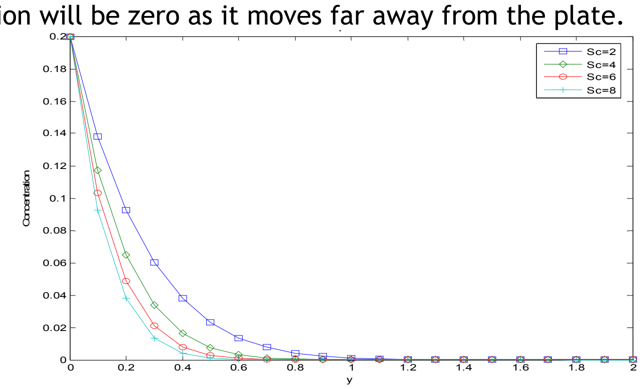

Figure 10: Concentration profiles when time t=0.2

Figure (11) also demonstrates that the variation in concentration with increase of Schmidt number Sc (2, 4, 6, 8) but it is at time t = 0.4. It is found that the concentration decreases with increase of Schmidt number the same effect is noted as we pointed out in figure (10) at time t=0.2. Figure (12) illustrates the temperature profiles for different values of radiation parameter (R) when prandtl number Pr = 0.71 at t = 0.2 and t = 0.4 respectively. It is observed that the temperature decreases with increase of radiation parameter (R) and becomes minimum away from the plate.

Figure (13) also displays temperature profiles when radiation parameter R = 5 at the time t = 0.2 and t = 0.4 respectively. It is found that the temperature for air (Pr=0.71) is greater than that of water(Pr= 7), this is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness. Figure (14) displays the temperature profiles for different values of time t (0.2, 0.4, 0.6, 0.8, 1.0) when Pr=0.71, R=14. This trend shows that the temperature increases linearly as time t increases. And it is also observed that the temperature comes to zero as it moves away from the plate.

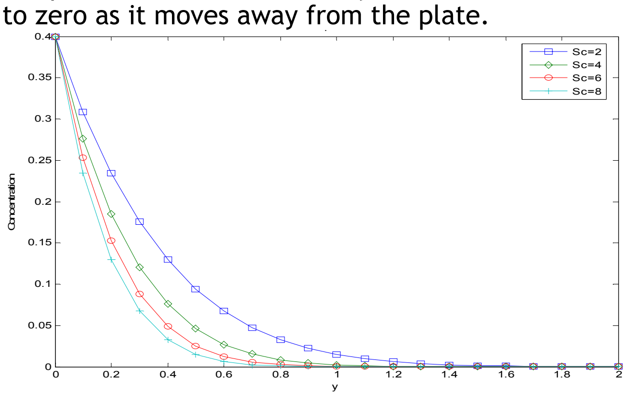

Figure 11: Concentration profiles for different Sc at time t=0.4

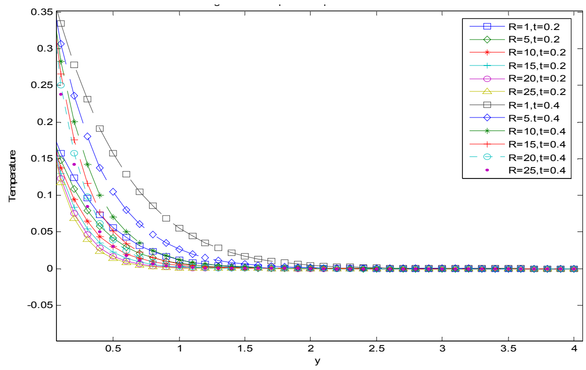

Figure 12: Temperature profiles for different R

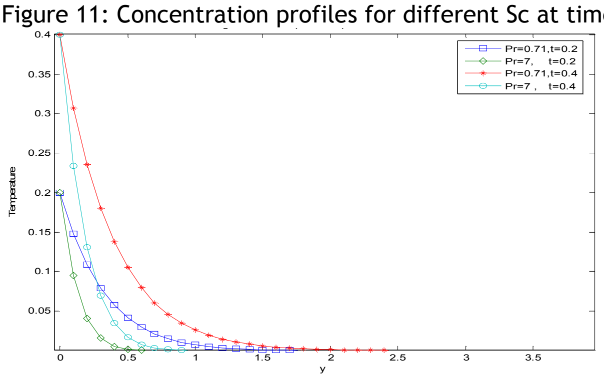

Figure 13: Temperature profiles for Pr=0.71 and Pr=7

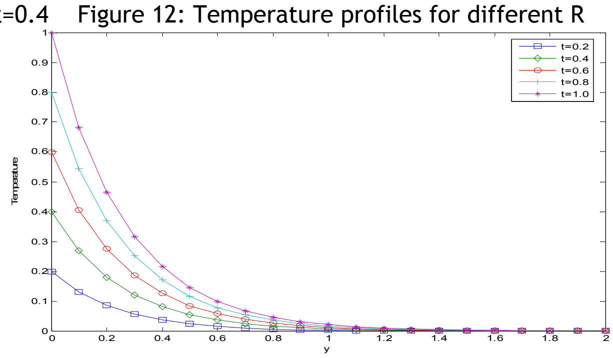

Figure 14: Temperature profiles for different values of time

Table 1: Skin-friction when Gr>0, Gm>0 with phase angle ωt=π/2

Pr	Gr	Gm	Sc	M	R	t	Skin-friction
0.71	10	5	2.01	3	4	0.2	-1.1829137534
0.71	10	5	2.01	3	8	0.2	-1.1600996418
0.71	10	5	2.00	3	14	0.2	-1.1344440038
0.71	10	5	4.00	3	14	0.2	-1.1196204227
0.1	10	5	2.01	3	4	0.2	-1.2404334550
0.71	10	5	2.01	3	4	0.4	-1.8718853319

Table 2: Skin-friction when Gr<0, Gm<0 with phase angle ωt=π/2

Pr	Gr	Gm	Sc	M	R	t	Skin-friction
0.71	-10	-5	2.01	3	4	0.2	-0.2903961386
0.71	-10	-5	2.01	3	8	0.2	-0.3132102502
0.71	-10	-5	2.00	3	14	0.2	-0.3388658882
0.71	-10	-5	4.00	3	14	0.2	-0.3638395166
0.1	-10	-5	2.01	3	4	0.2	-0.2328764370
0.71	-10	-5	2.01	3	4	0.4	0.4984535222

The skin-friction is presented in table (1) & (2) for both cooling and heating of the plate with phase angle ωt=π/2. From these tables we conclude that the skin-friction increases with increase in Radiation parameter (R), Schmidt number (Sc) and Prandtl number (Pr) in the case of cooling of the plate. But a reverse effect is identified in the case of heating of the plate. It is also found that the skin friction decreases with increase in time t in the case of cooling of the plate while it increases in the case of heating of the plate.

❖ CONCLUSIONS

A theoretical analysis is performed to study the influence of thermal radiation and mass transfer effects on hydro magnetic flow past a vertical oscillating plate with variable temperature and also with variable mass diffusion in the presence of applied transverse magnetic field. Exact solutions of equations are obtained by usual Laplace transform technique. It is found that the velocity decreases with increasing of magnetic field parameter M in both cases of cooling and heating of the plate. It is observed that with increasing of radiation parameter R the velocity decreases in the case of cooling of the plate but increases in the case of heating of the plate. It is also noticed that with increase in Sc (Schmidt number) the velocity decreases in the case of cooling of the plate and a reverse effect identified in the case of heating of the plate. It is found that the velocity increases with increase in time t in the cases of cooling of the plate but decreases in the case of heating of the plate. It is interesting to note that with increase of phase angle leads to decrease of velocity. It is also observed that the concentration increases with increase in time t while it decreases with increase in Sc. It is found that the temperature for air (Pr=0.71) is greater than that of water(Pr= 7), this is due to the fact that thermal conductivity of fluid decreases with increasing Pr, resulting a decrease in thermal boundary layer thickness. It is also observed that the temperature decreases with increase of radiation parameter (R). Finally, the skin friction increases with increase of R, Sc and Pr but decreases with time t in the case of cooling of the plate. But an opposite phenomenon is noted in the case of heating of the plate.

❖ NOMENCLATURE

a^* Absorption coefficient
a Exponential index
B_0 External magnetic field
C' Species concentration
C'_w Concentration of the plate
C'_∞ Concentration of the fluid far away from the plate.
C Dimensionless concentration
C_p Specific heat at constant pressure.
g Acceleration due to gravity.
G_r Thermal Grashof number
G_m Mass Grashof number
κ Thermal conductivity of the fluid
M Magnetic field parameter
Pr Prandtl number
q_r Radiative heat flux in the y - direction
R Radiative parameter
T' Temperature of the fluid near the plate
T'_w Temperature of the plate
T'_∞ Temperature of the fluid far away from the plate
Sc Schmidt number
t' Time
t Dimensionless time
u' Velocity of the fluid in the x' - direction
u_0 Velocity of the plate
u Dimensionless velocity
y' Co-ordinate axis normal to the plate
y Dimensionless co-ordinate axis normal to the plate
α Thermal diffusivity
β Volumetric Coefficient of thermal expansion
β^* Volumetric Coefficient of expansion with concentration
μ Coefficient of viscosity
ν Kinematic viscosity
ρ Density of the fluid
σ Electric conductivity
θ Dimensionless temperature
erf Error function
erfc Complementary error function
exp Exponential funtion

❖ REFERENCES

[1.] Sakiadis B.C., Boundary layer behavior on continuous solid surfaces:II boundary layer on a continuous solid flat surfaces AiChE journal, vol.7 , pp. 221-225, 1961.
[2.] Soundalgekar V. M., Gupta S.K. Birajdar N.S., Effects of mass transfer and free convection currents on MHD Stokes problem for a vertical plate, Nuclear engg Des Vol. pp. 339-346, 1979.
[3.] Soundalgekar V. M., Patil M.R. and Jahagirdar M.D., MHD stokes problem for a vertical plate with variable temperature, Nuclear Engg Des., Vol.64 pp.39-42, 1981.
[4.] Kumari . M and Nath.G., Devolpement of two dimensional boundary layer with n applied magnetic field due to an impulsive motion. Indian journal of pure and applied mathematics. Vol 30.pp.695-708, 1999.
[5.] England W.G. Emery A.F., Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas. Journal of Heat transfer, Vol 91, pp. 37-44, 1969.
[6.] Soundalgekar V. M. and Takhar H.S. Radiation effects on free convection flow past a semi-infinite vertical plate, modeling, measurement and control, Vol B.51, PP.31-40, 1993.
[7.] Hossain . M.A and Takhar H.S. Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and mass transfer, Vol 31. Pp.243-248. 1996.
[8.] Raptis A.Perdikis C., Radiation and free convection flow past a moving plate. Int. j. of .app. mech and engg. Vol.4, pp. 817-821, 1999.
[9.] Das U.N., Deka R.K. and Soundalgekar V. M. Radiation effects on flow past an impulsively started vertical infinite plate. J. Theo. Mech. Vol 1, pp.111-115, 1996.

ANNALS OF FACULTY ENGINEERING HUNEDOARA
– INTERNATIONAL JOURNAL OF ENGINEERING

Faculty of Engineering Hunedoara,
5, Revolutiei, 331128, Hunedoara,
ROMANIA
http://annals.fih.upt.ro