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EMPIRICAL MATHEMATICAL MODELS OF THE DEPENDENCE OF THE SPECIFIC CUTTING FORCE ON THICKNESS OF CUT IN TURNING

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ABSTRACT: Based on the analysis of the theoretical models for determining the specific cutting force, boundary conditions of its dependence on the thickness of cut are offered. Hypothetical graphic relations of the specific cutting force versus the thickness of cut with its change in a large rate for brittle and ductile workpiece material are offered in addition with possible hypothetical mathematic models for approximation of these relations. Experimental research of the influence of the thickness of cut on the specific cutting force in turning of different workpiece materials is done. By mathematical processing of the experimental data, the suggested hypothetical mathematical models are gained and analyzed and as the best new empirical mathematical model of the dependence of the specific cutting force on the thickness of cut is recommended this one which is with the best adequacy and accuracy. The deviations between the values of the specific cutting force received from calculations by the mathematic models based on reference data and the values received from the suggested new model have been explored. It turns out that in some cases for some workpiece materials these deviations reach 40-50% and more. In consequence, when calculating the cutting forces in turning by such methods significant errors can be received.
KEYWORDS: specific cutting force, thickness of cut, hypothetical model, empirical model, accuracy of models

❖ INTRODUCTION

The formulae for calculating the cutting forces for various cutting operations most often are based on the Kintzle mathematical model [1], derived from experimental research. This model of the main cutting force in turning can be presented as:

$$F_c = k_{c1.1}.b.h^{1-m_c}\ , \tag{1}$$

where $k_{c1.1}$ is the basic value of the specific cutting force at the nominal cross-sectional area of the cut $A = a.b = 1.1 = 1\,\text{mm}^2$, b - the width of cut, h - the thickness of cut, m_c - the exponent

The mathematical model of the specific cutting force can be expressed using a power function

$$k_c = k_{c1.1}.h^{-m_c}\ . \tag{2}$$

The values for $k_{c1.1}$ and m_c depend on the type and the mechanical properties of the workpiece material. These values for various materials and at various cutting conditions are published in specialized and reference literature [2,3,4,5]. Correction coefficients are established for calculation of the cutting forces at different conditions and for different cutting operations, These coefficients take into account the influence of the rake angle γ_0, the cutting speed v_c, the tool wear, the type of cutting operations, etc.

Fleischer et al [6] propose to apply formula (1) for micromachining ($h = 1 \div 100$ µm) of steel AISI 10045, AISI 02 and armco-iron by establishing a correction coefficient that takes into account the influence of the cutting edge radius.

In the reference book [7], formulas for calculation of the cutting forces in turning of various work materials are given when v_c=*const* and constant other cutting conditions:

$$F_c = C_{Fc} \cdot a_p^{x_{Fc}} \cdot f^{y_{Fc}} \tag{3}$$

Having in mind that the depth of the cut a_p influences proportionally $(x_{F_c} = 1)$ and that the depth *h* is proportional to the feed *f*, the specific cutting force is calculated using model (2):

$$k_c = \frac{F_c}{a_p . f} = \frac{C'_{F_c}}{h^{1-y_{F_c}}} = k_{c1.1}.h^{-m_c}\ , \tag{4}$$

where $C'_{F_c} = k_{c1.1}$, $m_c = 1 - y_{F_c}$.

The model (2) of the specific cutting force is often used in different studies of the cutting forces. For example, Ahmadi et al [8] use it for the tangential and radial specific cutting forces (called 'cutting force coefficients' in this article) in flank milling of aluminium alloy 60-61-T6. The same model for these specific forces is used by Diez Cifuentes et al [9] in milling of aluminium alloy AL-7040, as well as by Arit et al [10] in ultra-precision end-milling of brittle material (glass).

The extrapolation of the values of k_c in turning, calculated using formula (2) at small thickness of cut, can lead to considerable errors.

Results of conducted researches that may be seen in [3] show that with decreasing of the thickness of the cut the intensity of its influence on the specific cutting force grows which is expressed with the growth of the power index m_c. So it is suggested that the range of thickness of cut from 1 to 1000 μm is divided into three sub-ranges where $k_{c1.1}$ and m_c are determined for each one of them.

Other empirical mathematical models for approximation of the $k_c = f(h)$ dependence are also suggested.

The dependencies of the tangential k_t and the radial k_r specific cutting forces on the feed per tooth f_t, the cutting speed v_c, the radial depth of cut a_e, and the axial depth of cut a_p when end milling of aluminium alloys are approximated by Wang and Chang [11] using a linear function. When $v_c = const$, $a_e = const$, and $a_p = const$ the equation looks like:

$$k_i = a_{0i} - a_{1i}.f_t, \quad i \in \{t,r\} \tag{5}$$

The approximation of this case is accurate enough because during the experiments the factors do not vary much. In this model the thickness of cut is indirectly presented. During milling it is variable but the average thickness under given circumstances are proportional to the feed.

When researching the main cutting force in face milling of steel AISI 5020 Anderson et at [12] use a mathematical model of the specific cutting force C_r (called "cutting resistance" in this article) that looks like:

$$C_r = \frac{C_{r_1}}{h} + C_{r_2}, \tag{6}$$

where C_{r_1} and C_{r_2} are constants derived from the cutting force data.

Based on experimental studies of the cutting forces in orthogonal turning of steel AISI 1045, Conzalo et al. [13] derive this model of the main cutting force:

$$F_c / a_p = a_0 + a_1 . f \tag{7}$$

Since in this case $f = h$, and using formula (7), one can derive a model like (6) for the specific cutting force $k_c = F_c / (a_p.h)$.

Such a model for the main specific force an also be derived from the formulas by Ding et al [14] for the cutting forces in end milling of hard steel AISI H3 at $v_c = const$, $a_p = const$ and $a_e = const$.

The dependence of the thrust force and torque on the feed f (mm/rev), drill diameter d (mm) and spindle speed *n* (rpm) in drilling of coir composite using a HSS drill are researched by Jayabal et al [15]. The dependence of the torque $M = f(f,d,n)$ is approximated by a second-degree polynomial which when $d = const$ and $d = const$ is:

$$M = b_0 + b_1 . f + b_2 . f^2 \tag{8}$$

If the torque is expressed by the specific cutting force

$$M = \frac{k_c . f . d^2}{2} = b_0 + b_1 . f + b_2 . f^2 .$$

Then the model of the specific force is

$$k_c = \frac{a_0}{f} + a_1 + a_2 . f, \tag{9}$$

where, when taking into consideration the data from [15], $a_0 > 0$, $a_1 > 0$, $a_2 > 0$.

The dependencies, proposed by Chang et al [16], of the normal k_n, radial k_r and tangential k_t specific cutting forces in ball end milling of aluminium alloys on the feed per tooth f_t and the hardness of material h_d are approximated by a third-degree polynomial. When $h_d = const$ the model is:

$$k_{ci} = C'_{1i} - C'_{2i} . f_t + C'_{3i} . f_t^2 - C'_{4i} . f_t^3 \;; \quad i \in \{u,r,t\} \tag{10}$$

The relationship between re-scaled uncut chip thickness t_{cn} and log-scaled normal specific cutting force k_n in end milling of aluminium alloy is approximated by Yun et al [17] using a Boltzman function

$$\ln(k_n) = \frac{A_1 - A_2}{1 + e^{(t_{cn} - x_0)/dx}} + A_2, \tag{11}$$

where the coefficients $A_1 > A_2 > 0$; $x_0 < 0$; $dx > 0$.

The abovementioned establishes that when choosing a mathematical model for approximating the experimental dependence of the specific cutting force an empirical approach should be applied. In consideration is taken the physical nature of the dependence of a certain parameter on the factors that influence it, and not the basic requirement when choosing a mathematical model for adequate presentation of the experimental dependency. This empirical approach allows for the calculation of the specific cutting force with a satisfactory accuracy having a limited variation of the influencing factors. Otherwise the results may have substantial errors.

The current research aims to achieve more accurate calculation of the cutting forces using new empirical mathematical models for the specific cutting force based on the physical nature of the influence of the thickness of cut. Accomplishing this goal faces a variety of problems that need solving: analysing the theoretical dependencies of the specific cutting force and based on the mechanical nature of the process of cutting substantiating the hypothetical graphical dependencies of the specific cutting force on the thickness of cut in a wide variation range of its values; suggesting various mathematical models for a possible approximation of this dependence for various work materials; an experimental research of the dependence of the specific cutting force on the thickness of cut for various work materials; deriving and researching of the suggested empirical models of this dependence; research of the deviation of specific cutting force values by comparing the results of the suggested new mathematical models based on reference data compared to determine those values using the new mathematical models which are proposed and referred to as the best according to physical and statistical criteria.

❖ HYPOTHETICAL MATHEMATICAL MODELS

It is necessary to analyze the results of the theoretical studies, conducted for estimating cutting force in order to choose the kind of the empirical mathematical models that determine the dependence between the specific cutting force and the thickness of cut based on the physical nature of its influence in a wide range of variation. The main goal of these researches is to analytically calculate the cutting forces by using mechanical properties of the machined material and some parameters of the process of cutting, and accepting different hypotheses about the deformation of the cut during the chip formation process.

The resulting theoretical models are very different from one another. Some theoretical formulae for determining the main specific cutting force are shown in Table 1. They are derived form the formulae for determining the main specific cutting force $\left(k_c = F_c/(h.b)\right)$ suggested by different researchers of a 2D model – free orthogonal cutting with one shear plane.

It must be noted that while determining k_c using models 1, 2, 3 the forces on the major flank of the tool have not been taken into consideration. They are ignored due to their insignificant value, which is allowed at large thickness of cut, clearance angles and an unworn tool with ductile materials.

Table 1. Some theoretical mathematical models for calculation of the specific cutting force

№	Theoretical mathematical model	Author/Source
1.	$k_c = \dfrac{\tau_\phi .\cos(\rho_\gamma + \gamma_o)}{\sin\phi.\cos(\phi+\rho_\gamma-\gamma_o)}$	H. Ernst, M. E. Merchant 1941 [18]
2.	$k_c = \tau_\phi .(\cot g\phi + tg\,C_z)$	N. N. Zorev, 1956 [19]
3.	$k_c = 0{,}185.HV\dfrac{\varepsilon}{1-\sin\rho_\gamma/\left(\lambda.\cos(\rho_\gamma-\gamma_o)\right)}$	A. M. Rozenberg, A. N. Eremin, 1956 [20]
4.	$k_c = k_{c\gamma} + k_{c\alpha}$ $k_c = R_e\,\lambda^n(\cos\gamma_o + \mu_\gamma.\sin\gamma_o)$ $k_{c\alpha} = \mu_\alpha\,F_{\alpha N}\,/(b.h)$	V. D. Kuznezov, V. A. Krivouhov 1954 [20]

Symbols: τ_ϕ - shear strength on the shear plane; ρ_γ - angle of friction between chip and face; γ_0 - rake angle; ϕ - shear angle; C_z - constant angle for work materials; HV - Vickers hardness of chip; ε - shear strain; R_e - yield strength in compression; λ - chip length compression ratio; $F_{\alpha N}$ - major flank perpendicular force; n - exponent; μ_α - coefficient of friction between major flank and transient surface.

The theoretical formulae used to calculate the specific cutting force is inaccurate and too complicated for practical application visualize the mechanics of process of cutting.

With wide variations of the thickness of cut the friction force on the major flank is not to be ignored and the specific cutting force is viewed as a composite force (see Model 4, Table 1):

$$k_c = k_{c\gamma} + k_{c\alpha}\,, \tag{12}$$

where $k_{c\gamma}$ - the component which is formed by the normal force and the friction force on the race;

$k_{c\alpha}$ - the component formed by the friction on the major flank .

The variable intensity of the influence of the thickness of cut can be explained and the limits of its variation can be determined using the results from the theoretical researches that take into consideration the physical nature of formation of the specific force.

With smaller thickness of thickness of cut ductile cutting plastic materials the component of the specific cutting force $k_{c\gamma}$ increases according to the theoretical models 1-4 (Table 1). This is due to the increase of the chip length compression ratio λ and thus the conventional shear angle ϕ decreases. The component of the specific cutting force $k_{c\alpha}$ increases significantly faster which is due to the smaller cross-sectional area. The friction force on the major flank has a determined value, which does not depend on the thickness [19] (model 4 – Table 1). The overall specific force increases with ever growing intensity and when h tends to zero the specific force should be an indefinite large value (fig. 1a):

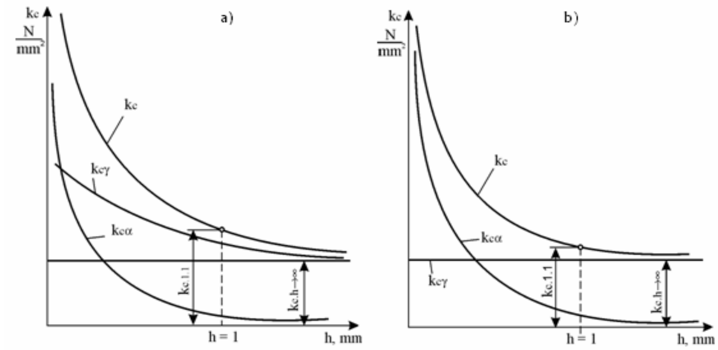

Figure 1. Hypothetical graphical dependencies of the specific cutting forces $k_{c\gamma}$, $k_{c\alpha}$ and k_c on the thickness of cut: a) Ductile materials; b) Brittle materials

With $h \to 0$, $k_c \to \infty$ (13)

With an increase of h the component $k_{c\alpha}$ decreases and at large thicknesses it has insignificantly small values. The component of the specific force $k_{c\gamma}$ according to the same theoretical models decreases because λ decreases ϕ increases, and the friction angle ρ_γ decreases. At thicknesses of given work and tool materials the chip length compression ratio is a constant value $(\lambda \to const \geq 1)$ and thus $\phi \to const$. This leads to the assumption that when the thickness has significantly high values then the specific cutting force is a constant value (fig.1a).

$$h \to \infty, k_c \to const = k_{c,h\to\infty}. \tag{14}$$

When cutting brittle materials the component $k_{c\alpha}$ changes similarly to the change of the thickness. When cutting ductile materials the change of $k_{c\gamma}$ is based on the varying degrees of plastic deformation of the cut and is mainly due to the secondary plastic deformation due to the friction on the rake. When cutting brittle materials the plastic deformation is poor $(\lambda \approx 1)$. Thus it can be said that for an ideally brittle material (fig. 1b):

$$k_{c\gamma} \approx const = k_{c,h\to\infty}. \tag{15}$$

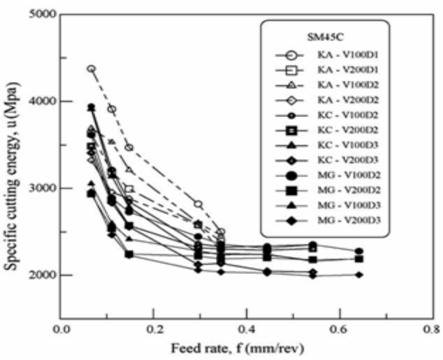

Figure 2. Specific cutting energy for SM45C [21]; KA, KC and MC – types chip formers of insert TNMG 160408; V – cutting speed; D – depth of cut.

A new parameter $k_{c,h\to\infty}$ is used when studying the specific cutting force. It can be called a limit specific cutting force and is a characteristic property of a given work material. It depends mostly on the rake angle when an unworn tool is used. Such an assumption is confirmed by some experimental studies. For example when turning steel SM45C Lee et al. [21] have obtained dependencies of the specific cutting force (called 'specific cutting energy' in this article) on the feed, shown on Fig.2. The empirical mathematical models that approximate the dependence of the specific cutting force on the thickness of cut in wide range of variation must comply with the limit conditions (13) and (14).

The mathematical model (2) complies with condition (13) but when $h \to \infty$ it does not comply with condition (14), i.e. $k_c \to 0$.

Furthermore it can't express the variable intensity of h in a logarithmic coordinate system because $d\,ln\,k_c / d\,ln\,h = -m_c = const$ which does not conform to experimental data.

Mathematical model (5) does not comply with both conditions - when $f_t \to 0, k_i = a_{0i} = const$ and when $f_t \to \infty, k_i < 0$. The model (6) satisfies both conditions: when $h \to 0$, $C_r \to \infty$ and $h \to \infty$, $C_r = C_{r_2}$. It is necessary to verify this model for various work materials. The model (9) complies condition (13), but does not comply with condition (14), and model (10) complies only with condition (14). It is established that when $h_{cn} \to 0$ $\ln k_n = (A - A_2)/(1 + e^{-x_0/dx}) = const$, model (11) does not comply with condition (13). When $h_{cn} \to 0$ $\ln k_n = A_2$, i.e. condition (14) is fulfilled.

Table 2. Hypothetical Mathematical Models

№	Mathematical models	Conditions	$k_{c1.1}$	$k_{c,h\to\infty}$
1.	$k_{c1} = a_0 + \frac{a_1}{h} + \frac{a_2}{h^2}$	$a_0 > 0$; $a_1 > 0$; $a_2 > 0$	$a_0 + a_1 + a_2$	a_0
2.	$k_{c2} = a_0 + \frac{a_1}{h}$	$a_0 > 0$; $a_1 > 0$	$a_0 + a_1$	a_0
3.	$k_{c3} = a_0 + \frac{a_1}{h^2}$	$a_0 > 0$; $a_1 > 0$	$a_0 + a_1$	a_0
4.	$k_{c4} = a_0\, e^{a_1 h^{a_2}}$	$a_0 > 0$; $a_1 > 0$; $a_2 < 0$	$a_0\, e^{a_1}$	a_0
5.	$k_{c5} = a_0 + \frac{a_1}{h^{a_2}}$	$a_0 > 0$; $a_1 > 0$; $a_2 > 0$	$a_0 + a_1$	a_0

The mathematical models that approximate hypothetical graphical dependencies of the specific cutting force on the thickness of cut (fig. 1) and that comply with conditions (13), (14) and (15) are chosen from [22]. Some of them are modified based on structure (table 2).

A mathematical model has to be selected through mathematical processing of experimental data for obtaining the coefficients of these models and statistical analysis of the results. This model shall approximate the dependence of the specific cutting force on the thickness of cut with the best adequacy and accuracy.

❖ RESEARCH METHODOLOGY - DETERMINING THE SPECIFIC CUTTING FORCE

For each test the specific cutting force is calculated using the following formula:

$$k_c = F_c / A_{eff}, \quad N/mm^2, \tag{16}$$

where F_c, N, is the main cutting force, determined experimentally; A_{eff} is the effective cross-sectional area of cut, mm^2.

If when determining k_c during turning the nominal cross-sectional area of cut is $A = a.f$, and not the actual $A_{eff} < A$, with comparatively large feeds f, small depths of cut a_p, and corner radiuses r_ε some significant mistakes can be made. Therefore it is necessary to determine precisely the effective cross-sectional area of cut. An approach to determine the width b and its average thickness h_m has to be adopted when researching the dependence $k_c = f(h)$.

❖ RESEARCH METHODOLOGY - DETERMINING THE DIMENSIONS OF CUT

Depending on the cutting conditions – the depth of cut a_p, the feed f and the geometry of the tool – the tool cutting edge angle κ_r and minor cutting edge angle κ_r' and the corner radius at the tip r_ε there are four possible schemes of cutting during turning. Here will be reviewed the determining of the elements of the cross-sectional area of cut in two of these schemes – the (fig. 3 a) first scheme is obtained when complying with the following conditions:

$$a_p > r_\varepsilon(1 - \cos\kappa_\gamma), \quad f \le 2r_\varepsilon \sin\kappa_r', \tag{17}$$

and the second scheme (fig. 3 b) complying with

$$a_p > r_\varepsilon(1 - \cos\kappa_\gamma), \quad f > 2r_\varepsilon \sin\kappa_r'. \tag{18}$$

These schemes the most frequently met in practice and are thus used in the experimental researches performed.

When cutting is done using curved cutting edges, several approaches for determining the width of cut are recommended according to ISO 3002/3 - 1984/E: $b_m = \overline{AC}$ or according to [2] $b_m = \overset{\frown}{AB} + \overline{BC}$ (fig. 3). For the current study accepts:

$$b_m = \overline{AC} = \sqrt{\overline{AG}^2 + \overline{CG}^2} \tag{19}$$

where $AG = a_p$;

$CG = [a_p - r_\varepsilon(1 - \cos\kappa_r)]\cot g\kappa_r + r_\varepsilon \sin\kappa_r$

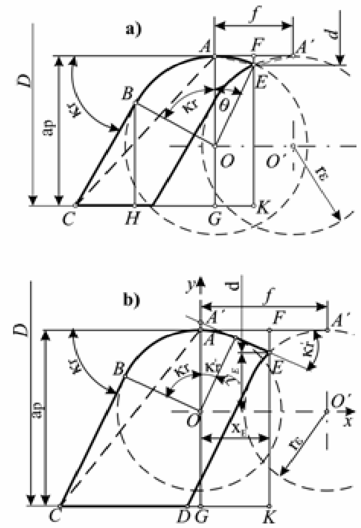

Figure 3. Schemes of cut. a) with straight and curved areas of the major cutting edge and with curved area of the minor cutting edge; b) with straight and curved areas of the major cutting edge and the minor cutting edge.

The effective cross-sectional area of cut in both schemes is determined when the area of the tip $A_{AA'E}$ is subtracted from the nominal cross-sectional area of cut $A_n = a_p . f$. The area $A_{AA'E}$ is determined using formulae that are not quoted here.

$$A_{eff} = A_n - A_{AA'E}, \text{ mm}^2 \tag{20}$$

When during cutting there are curved cutting edges, the thickness of cut in the different points of the cutting edge is variable. Therefore, it is accepted to determine the average thickness of cut [2]:

$$h_m = A_{eff} / b_m, \text{ mm}. \tag{21}$$

The exact value of the actual depth of cut is needed to determine the dimensions of the cross-section A and b_m. That is why the diameter of the work surface D is measured with a micrometer when beginning the test. The diameter d of the machined surface is measured after the test is done. When taking into account the roughness of the surface and ignoring its accidental component, the actual depth of cut is determined using the following formulae:

$$a_p = \frac{D - d}{2} + \frac{f^2}{8r_\varepsilon}, \text{ mm (fig. 3, a) it } f \ge 0{,}2\ mm/rev \tag{22}$$

$$a_p = \frac{D - d}{2} + r_\varepsilon - y_E, \text{ mm (fig. 3, b)}, \tag{23}$$

where y_E is calculated by the given formula.

❖ RESEARCH CONDITIONS

The experimental researches are done using the work materials: AISI carbon steel W1-1.0C, CuSn7P0,7 bronze, aluminium alloys AlCu4,5Mn0,5Mg1,6 and grey iron GG15.

The experiments are done on a lathe model SU500 with tooling: Insert SNMG120412-MR, GC4225 grade; tool holder PSBNR2525M12 (SANDVIK Coromant), cutting edge angle $\kappa_r = 75^o$; minor cutting edge angle $\kappa_r^{'} = 15^o$; corner radius $r_\varepsilon = 1.2\,mm$.

A three-component dynamometric system with sensitiveness of 0.15 μm of the inductive transducers – IWT of VEB RFT – Germany and a universal measuring device N2301 IEMI, have been used to measure the cutting forces. Table 3 show the cutting conditions.

Table 3. Cutting conditions

№	Work Material	Brinnel hardness HB	Cutting speed $v_c, m/\min$	Depth of cut (nominal) a_p, mm	Feed $f, mm/rev$	Thickness of cut h_m, mm	Number of tests n
1.	Steel AISI W1-1.0C	180	90	2.00	0.018 ... 1.24	0.014 ... 0.93	12
2.	Grey iron GG 15	156	55	2.00	0.018 ... 1.24	0.014 ... 0.93	12
3.	Bronze CuSn7P0.7	93	80	2.00	0.018 ... 1.74	0.014 ... 0.575	10
4.	Aluminium alloy AlCu4.5Mn0.5Mg1.6	107	100	2.00	0.018 ... 1.74	0.014 ... 0.575	10

❖ DETERMINING THE COEFFICIENTS OF THE MATHEMATICAL MODELS

The determination of the mathematical models' coefficients that approximate the dependence of the specific cutting force and the statistical analysis is done through processing of the experimental data using a computer program created especially for the purpose [23].

The coefficients of mathematical models 1, 2 and 3 (Table 2) are determined through the least squares method (LS Method).

Mathematical model 4 is liberalised through taking a double logarithm and model 5 – through taking a single one. The initial value of the coefficient $a_o < k_{ci}$ and iteration are chosen. On each step through LS Method the coefficients a_o, a_1, a_2 and the sum of the squares of the errors $S_{\min}$ are determined. Valid values of the coefficients a_1 and a_2 are those for which $S_{\min}$ has the smallest value. The significance of the model coefficient is estimated by the Student's t - criterion and the uniformity of the dispersions – by the Cochran's G – criterion.

The Fisher criterion, the correlation coefficient R and absolute value of the maximal relative error $\left|\Delta k_{c\max}\right|\%$ are used to estimate the adequacy, the accuracy and the efficiency of the mathematical models.

The Fisher criterion is used in two variants:

a) $F = F_1 = s_a^2 / s_b^2 \le F_{\alpha,\nu_1,\nu_2}$ (24)

b) For better predictability properties of the model, the F - criterion should be several times (more than 4 times) greater than the critical value F_{α,ν_1,ν_2}, where α is the significance level, and ν_1 and ν_2 are the degrees of freedom [24].

$$F = F_2 = s_{\bar{k}_{cn}}^2 / s_a^2 \ge 4 F_{\alpha,\nu_1,\nu_2} \quad (25)$$

The dispersion of reproducibility s_a^2, residual dispersion s_b^2 and the dispersion in relation to the mean value of the specific cutting force $s_{\bar{k}_c}^2$ are calculated through known formulae.

All tests are repeated three times.

The actual depth of cut, the effective cross-sectional area of cut, the width and the average thickness as well as the specific cutting force for each test were calculated by using the specially created for that purpose computer program.

❖ RESULTS AND ANALYSES

The established dependencies of the specific cutting force on the thickness of cut for each material and each test are shown in fig. 4. In the area of small thicknesses ($h_m \le 0.1$ mm), the specific cutting force decreases intensively when the thickness increases for all researched work materials. With greater thicknesses of cut the specific cutting force decreases less intensively and is close to a constant value. For the ductile materials – bronze and aluminium alloy - the decrease starts when the thickness $h_m \ge 0.20$ mm and for steel - $h_m \ge 0.60$ mm. For a brittle material cast iron, the specific cutting force varies insignificantly for thickness of $h_m = 0.20$ mm is almost constant.

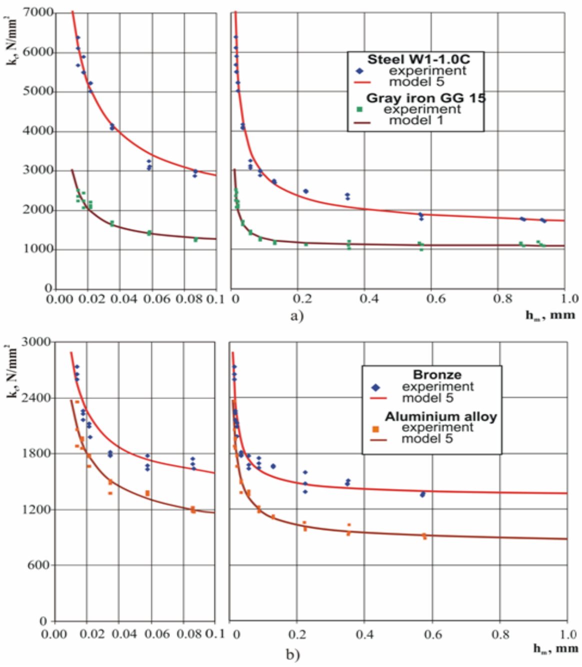

Figure 4. Variation of the specific cutting force on the thickness of cut. a) steel (AISI W1-1.0C), grey iron (GG15) 6) bronze (CuSn7P0.7), aluminium alloy (AlCn4.5Mn0.5Mg1.6)

The hypothetical mathematical models given in Table 2 were analyzed in order to approximate the experimental dependencies of the specific cutting force on the thickness of cut. The resulting coefficients for these models of the experimental date were mathematically processed and are given in Table 4. Choosing a mathematical model which best approximates the experimental data is rather complex. The statistical criteria should be applied when the conditions given in Table 2 are fulfilled, as well as condition

$$a_o < \bar{k}_{cn} \ , \qquad (26)$$

where $\bar{k}_{cn}$ is the average value of the specific cutting force at maximum thickness of cut (n^{-th} trial).

Table 4. Coefficients of the mathematical models, the statistical criteria and the conditions

Work Material	Mathematical Model	a_o	a_1	a_2	F_2	F_{cr}	R	Conditions	Adequacy	$\bar{k}_{cn}$	$\bar{k}_{cn}$ <a$_0$	$\lvert\Delta k_{c\max}\rvert$ %
Steel W1-1.0C	1	1832	95.83	-0.518	89.7	9.2	0.994	-	+	173 6	-	-
	2	2036	61.52	-	30.4	9.04	0.984	+	+		-	-
	3	2494	0.836	-	5.14	9.04	0.966	+	-		-	
	4	963.7	0.596	-0.266	124	9.2	0.966	+	+		+	9.8
	5	1353	405.2	0.578	117	9.2	0.915	+	+		+	9.3
Grey iron GG 15	1	1038	26.40	-0.101	81.1	9.20	0.994	-	+	1138	+	-
	2	1077	19.72	-	49.1	9.04	0.990	+	+		+	4.8
	3	1220	0.273	-	6.94	9.04	0.935	+	-		-	-
	4	1082	0,016	-0.915	17.1	9.20	0.974	+	+		+	12.7
	5	1087	10.35	1.139	23.9	9.20	0.981	+	+		+	7.2
Bronze CuSn7P0.7	1	1472	9.64	0.082	25.4	10.1	0.981	+	+	1357	-	-
	2	1428	15.3	-	20.6	9.80	0.977	+	+		-	-
	3	1556	0.213	-	15.4	9.80	0.969	+	+		-	-
	4	1302	0.051	-0.603	19.8	10.1	0.973	+	+		+	6.3
	5	1321	48.2	0.757	17.0	10.1	0.969	+	+		+	6.3
Aluminium alloy AlCu4.5Mn 0.5 Mg1.6	1	920.0	22.8	-0.093	101.1	10.1	0.995	-	+	910	-	-
	2	969.2	16.4	-	50.0	9.80	0.990	+	+		-	-
	3	1118	0.216	-	6.99	9.80	0.935	+	-		-	-
	4	718.3	0.193	0.637	219	10.1	0.998	+	+		+	4.4
	5	797.3	84.2	-0.400	188	10.1	0.997	+	+		+	4.5

When evaluating the model's adequacy to the Fisher criterion F_1 (variant 'a'), most of the models are adequate but only for critical values of the criterion at different levels of significance. The second variant 'b' using Fisher's criterion has been used so that there is a general evaluation of the adequacy. Its critical value is chosen for a level of significance $\alpha = 0.05$.

The values of the F_2 criterion, its critical value $F_{cr} = 4F_{\alpha,\nu_1,\nu_2}$, the correlation coefficient *R* and the maximal relative error $\lvert\Delta k_{c\max}\rvert$% are given in Table 4 and are used for the analyzed mathematical models of the various work materials. In the „Conditions", "Adequacy" and „Condition (26)" columns "plus" or "minus" specifies if the conditions have been met or not.

For work material- steel AISI W1 – 1.0C, mathematical models 4 and 5 comply with all conditions and statistical criteria. They are almost equivalent, but model 5 has a smaller maximum relative error.

For grey iron GG15 model 1 does not comply with the condition $a_2 < 0$ and model 3 is inadequate. According to the statistical criteria and the relative fault model 2 is the best one followed by models 5 and 4. It is accepted that the friction does not depend on the thickness of cut and that the cut of brittle materials is turned into a chip with insignificant plastic deformations. Then the component of the specific cutting force $k_{c\gamma}$ does not depend on the thickness and the coefficient a_0 expresses the limit specific force $a_o = k_{c\gamma} = k_{ch\to\infty}$ while the coefficient a_1 expresses the friction on a width of cut unit. The best model applied with brittle materials is model 2 followed by model 5 with a coefficient a_2 = 1.139 which is close to a_2 = 1 of model 2.

For the work material- bronze CuSn7P0.7, models 4 and 5, which are almost equivalent, complies with the conditions and the statistical criteria. The same is established for the machined material- aluminium alloy AlCu4.5Mg 1.6 Mn0.5.

This research shows that mathematical model 2, corresponding to model (6) [12], is suitable for brittle materials - in this case – grey iron. However, further additional research is needed to support this finding.

Mathematical models 4 and 5 prove to be most suitable for ductile materials. Mathematical model 5 can be recommended since it expresses the physical nature of the formulation of the specific cutting force in a better way, and model 2 is a particular case of model 5. Therefore, the experimental mathematical model, that approximates the dependence of the specific cutting force on the thickness of cut in turning can, be represented in general form

$$k_c = k_{c.h\to\infty} + k'_{c1.1}.h_m^{-m_c} \ , \qquad (27)$$

where $k_{c.h\to\infty} = a_0$ is a component of $k'_{c1.1}$ in $h \to \infty$, $k'_{c1.1} = a_1$ - component of $k_{c1.1}$ in $h = 1$ mm, $m_c = a_2$ - exponent.

The values of $k_{c.h\to\infty} = a_0$, $k'_{c1.1}$ and m_c for the researched machined materials are given in Table5.

Table 5. *Values of* $k_{c.h\to\infty}$, $k'_{c1.1}$ *and* m_c

№	Work Material	$k_{c.h\to\infty}$ N/mm^2	$k'_{c1.1}$ N/mm^2	m_c	$k_{c.1.1}$, N/mm^2
1	Steel AISI W1-1.0C	1353	405	0.578	1758
2	Grey iron GG15	1077	19.7	1.000	1097
3	Bronze CuSn7P0.7	1321	48.2	0.767	1369
4	Aluminium alloy AlCu4.5Mn0.5Mg1.6	797	84.2	0.637	882

❖ Accuracy Of The Empirical Mathematical Models

This research aims at estimating the accuracy of some accepted empirical models of the specific cutting force on the thickness of according to reference data given in. The models have been marked as A[4], (SANDVIK Coromant), B [2] and C [7]. The suggested new models of these dependencies are accepted as the best ones based on this estimation.

The specific cutting force, which depends on the thickness of cut, for all materials, is calculated by using the following formula (model A):

$$k_c = k_{c.0,4}\left(\frac{0{,}4}{h_m}\right)^{m_c}, \tag{28}$$

where $k_{c.0,4}$ is the basic value of the specific cutting force with thickness of cut $h_m = 0{,}4$ mm, given in tables;

The expression in the brackets is a correctional coefficient:

$$k_{ch} = \left(0{,}4/h_m\right)^{m_c}, \tag{29}$$

Then $k_c = k_{c.0,4}.k_{ch}$. (30)

In (model B) the specific cutting force data is given in a table. The values of $k_{c.0,4}$ for various materials can be taken directly as well as the power m_c.

If $k_{c.0,4}$ is accepted as a basic value calculated using formula (4) – model C, the specific cutting force is also calculated using formula (30) and the correctional coefficient - using formula (29).

The values m_c of models A, B and C are given in Table 6.

Table 6. *Values of* m_c *of mathematical models A, B and C*

№	Work Material	A[4]	B[2]	C[7]
1	Steel AISI W1-1.0C	0.15	0.18	0.27
2	Grey iron GG15	0.28	0.21	0.25
3	Bronze CuSn7P0.7	0.25	0.17	0.34
4	Al-alloy AlCu4.5Mn0.5Mg1.6	0.25	0.25	0.30

The specific cutting force according to the best suggested model (27) can also be calculated using a formula of type (30):

$$k_{co} = k_{co.0.4}.k_{coh}, \tag{31}$$

where $k_{c0.0.4}$ is determined as model (27) when $h_m = 0.4$ mm and $k_{coh} = k_{co}/k_{co.0.4}$.

So that the basic value of the specific cutting force is eliminated because it differs for materials' mechanical characteristics, it is accepted that it will have a constant value for all researched models ($k_{c.0.4} = k_{co.0.4}$). Thus the accuracy of the respective model is estimated regardless of the specific basic value of the cutting force.

The difference between the empirical models according to reference data and the suggested new ones describing the dependence of specific cutting force on the thickness of cut is determined using the following formula:

$$\Delta k_{ch} = \frac{k_c - k_{co}}{k_{co}}\cdot 100\,\% = \frac{k_{ch} - k_{coh}}{k_{coh}}\cdot 100\,\%. \tag{32}$$

The calculations are done for scope thicknesses of the cut $h_m = 0.05 \div 1.0$ mm that approximately corresponds to the most widely used feeds during turning $f = 0.07 \div 1.3$ mm/r.

The dependence of the deviations Δk_{ch} of the various models on the thickness of cut for the different materials is shown in fig. 5.

For carbon steel the smallest deviations are achieved using the C model (-9.30÷3.17 %), which are larger in the B model (-18.5÷0.0 %) and the largest ones using the A model (-7.0÷1.70 %). Generally the deviations are relatively small and are comparable to the relative error of the suggested new model.

For bronze there are significant deviations. The smallest deviations are the ones for the B model (-11.0 ÷ 12.9), larger are - in the A model (-17.2 ÷ 33.3) and the largest - in the C (- 24.4 ÷ 61.1 %).

For the workpiece material aluminium alloy the models A and B have the smallest deviations (-13.9 ÷ 17.2) and C - has the largest (-18.2 ÷ 29.9%).

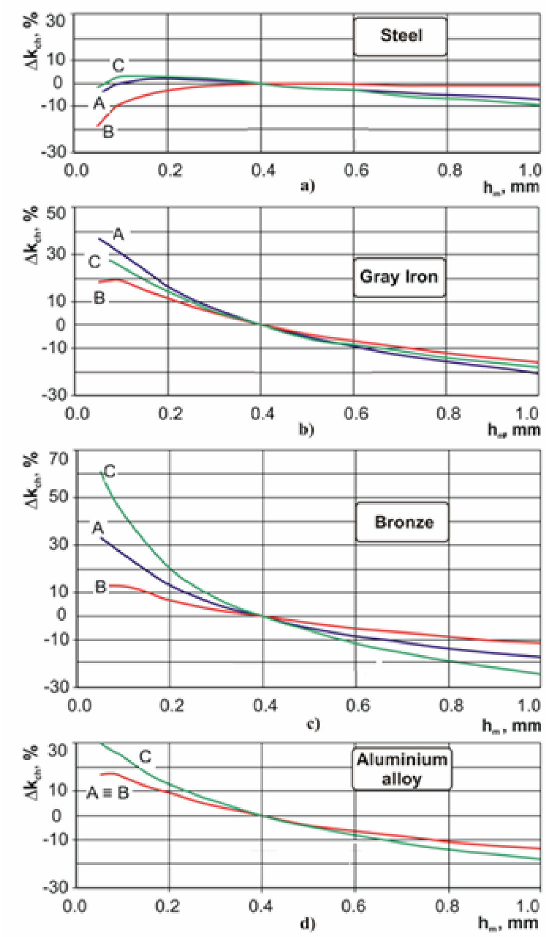

Figure. 4. Dependencies of the deviations of the mathematical models according to reference data on the thickness of cut: a)AISI W1-1.0C steel; b) bronze CuSn7P0.7 ; c) aluminium alloy AlCu4.5Mn0.5Mg1,6 ; d) grey iron GG15

Sizable deviations also occur with the workpiece material gray iron. The C model has the smallest deviations (-17.9 ÷ 28.3%) and the A model has the largest ones (-20.9 ÷ 36.6%).

The deviations of the suggested models for the specific cutting force according to reference data for bronze, aluminium alloy and cast iron are too large and may lead to significant errors if used to calculate the cutting forces.

❖ CONCLUSIONS

The following conclusions can be made from the analysis that has been done:

a) Limit conditions have been defined based on the theoretical dependencies of the specific cutting force and the physical nature of the cutting process mechanics. The mathematical models used to approximate the dependence of the specific cutting force on the thickness of cut with a wide range variation must comply with these limit conditions.

b) Hypothetical graphical dependencies $k_c = f(h)$ for ductile and brittle work materials and hypothetical models for the approximation of these dependencies that comply with the limit conditions defined have been suggested.

c) A new parameter of the specific cutting force has been suggested - limit specific cutting force that would occur for very large thicknesses of cut $(h_m \to \infty)$, characteristic for a certain work material.

d) New mathematical models have been reached and suggested through experimental studies and mathematical processing of the experimental data by using statistical and physical criteria that better approximate the dependence of the specific cutting force on the thickness of cut with a wide range variation for machining various materials during turning.

e) Despite the limited volume of experimental studies the suggested hypothesis that with very large thicknesses $(h_m \to \infty)$ the specific cutting force is a constant value for certain cutting conditions can be affirmed.

f) The experimental tests confirm the thesis that for brittle materials the decrease of the specific cutting force is mostly due to the decrease of its component which is a result of the major flank friction force.

Based on the study of the accuracy of the empirical mathematical models according to reference data compared to the new recommended models. It has been established that for some models and some work materials the deviations are in some cases insignificant but for others the deviations may reach up to 40%-50%. As a result when calculating the cutting forces by using such models some significant errors may occur.

❖ REFERENCES

[1.] O. Kinzle. Die Bestimmung von Kräffen und Leistungen an spanenden Werkzeuden und Werkzeugmaschinen. VDI – Z 94 (1952).11/12, 299-305 (in German).

[2.] W. Degner, H. Lutze, E. Smeikal, Spanende Formung : Teorie, Berechnung, Richtwerte, Münhen, Wien, C. H. Verlag, 1993.

[3.] G. Spur, T. Steferle. Handbuch der Fertigungstechnek. Vol. 1, Moskwa, Mashinostroenie, 1985 (in Russian).

[4.] SANDVIK Coromant. TechnicalCuide. 2010.

[5.] W. König, K. Essel. Spezufische Schittkraftwerte die Zerspanung metallischer Werkstofe. Verlag Stahleisen, Düsseldorf, 1973 (in German).

[6.] J. Fleischer, V. Schulze and J. Kotchenreuter. Extension of cutting force formulae for microcutting. CIRP J. Manuf. science Technol.Vol. 2, iss. 1. (2009), 75-80.

[7.] Reference book of the technologist of machining. Vol. 2. Edited by S. Pashov. Tehnika, Sofia, 1990 (in Bulgarian).

[8.] K. Ahmadi and F. Ismail. Machining chatter in flank milling. International Journal of Machine Tools and Manufacture, 50 (2010), 75-85.

[9.] E. Diez Cifuentes, N. Perez Garcia, M. Guzman Villeansenor and A. Vitan Idoipe. Dynamic analysis of runout correction in milling. International Journal of Machine Tools and Manufacture, 50 (2010), 709-717.

[10.] M. Arif., M. Rahman and E.-Y. San. Analytical model to determinate the critical feed per edge for ductile-brittle transition in milling process of brittle materials. International Journal of Machine Tools and Manufacture, 51 (2011), 170-181.
[11.] M.-Y. Wang, and H.-Y. Chang. A simulation shape error for end milling AL6061-T6. Int. J. Adv. Manuf. Technol. vol. 22, 10, 2003, 689-696.
[12.] C. Anderson, M. Anderson and J.-E. Stahl. Experimental studies of cutting force variation in face milling. International Journal of Machine Tools and Manufacture 51, (2011), 67-76.
[13.] O. Gonzalo, J. Beristain, H. Jaurege and C. Sanz. A method for the identification of the specific force coefficients for mechanistic milling simulation. International Journal of Machine Tools and Manufacture, 50 (2010), 765-744.
[14.] T. Ding, S. Zhang and Y. Wang. Empirical models and optimal cutting parameters for cutting forces and surface roughness in hard milling of AISI H13 steel, Int. J. Adv. Manuf. Technol., vol. 51 (2010), 45-55.
[15.] S. Jayabal, U. Natarajan. Optimization of thrust force, torque, and tool wear in drilling of coir-reinforced composites using Nelder-Mead and genetic algorithm methods. Int. J. Adv. Manuf. Technol., vol. 51 (2010), 371-381.
[16.] S. - T. Shiang, C.-M.Tsai, A-C. Lee. Analysis of cutting forces in ball-end milling. Journal of Materials Processing Technology. Vol.47, (1995), 231-249.
[17.] W.-S. Yun, D.-W. Cho. Accurate 3-D cuttung force prediction using cutting condition independent coefficients in end milling. International Journal of Machine Tools and Manufacture, 41, (2001), 463-478.
[18.] G. Boothroyd, W. A. Knight. Fundamentals of Machining and Machine Tools. Third Edition, CRC Press, Taylor & Francis Group. Boca Raton, 2006, 573.
[19.] Development of the Metal Cutting Science, edited by N.N. Zorev, Moscow, Mashinostroenie, 1967, 416 (in Russian).
[20.] A. M. Vulf. The cutting of metals. Moscow, Mashgiz, 1968 (in Russian).
[21.] Y.-M. Lee, S.-H. Yang, S.-I. Chang. Assessment of chip-breaking characteristics using new chip-breaking index. J. Mat. Proc. Tech. 173 (2006), 166-171.
[22.] A. Mitkov, P. Nenov, D. Minkov and T. Todorov. Approximation of a function with one variable. VTU "A.Kanchev", Rousse, 1982 (in Bulgarian).
[23.] I. Kolev. Mathematical models of specific cutting force in turning – a methodology for determining of coefficients. University of Ruse, Procedings. Vol. 49, book 2, (2010), 114-118, (in Bulgarian).
[24.] N. Draper, H. Smith, Applied Regression Analysis. Moscow, Statistica, 1973, 391 (in Russian).
[25.] A. Mitkov, D. Minkov. Mathematical Methods of engineering research. V.T.U. "A. Kanchev", Rousse, 1985 (in Bulgarian).

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