ANNALS OF FACULTY ENGINEERING HUNEDOARA
- INTERNATIONAL JOURNAL OF ENGINEERING
Tome IX (Year 2011). Fascicule 3. (ISSN 1584 - 2673)

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CAD REPRESENTATIONS OF 3D SHAPES WITH SUPERELLIPSOIDS AND CONVEX POLYHEDRONS

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ABSTRACT: The purpose of this paper is to present the results of a CAD study for construction of complex 3D shapes with superellipsoids and convex polyhedrons based on computational geometry. To obtain the relevant geometric informations concerning the shape and profile for different 3D complex objects, the Madsie Freestyle 1.5.3 application for computation is used. The results of this study are to be used as the basis for complex geometric constructions and optimized CAD structures used in engineering and sculpture design.
KEYWORDS: engineering design, sculpture design, superellipsoid, convex polyhedron, implicit surface

INTRODUCTION

The increasingly important role of computer aided design and mathematical definition of 3D shapes has important applications for geometric modeling, visualization and animation in sculpture design [1, 2, 3]. Interactive technological tools for geometric modeling and graphical rendering have tremendous potential to address a broad range of objectives to explore and analyze possible new artistic shapes in virtual form [4, 5, 6, 7, 8].

Free-form surfaces are non-analytic surfaces and to create and manipulate these surfaces many powerful surface modeling tools have been developed.

SUPERELLIPSOID AND CONVEX POLYHEDRON

Superquadrics were introduced by Barr A. H. in 1981 by generalizing the superellipsoids and also he introduced the notation common in the computer vision literature [9]. He demonstrated that superquadric models, in particular for CAD design, have compactly represented continuum useful forms with rounded edges that can easily be rendered and shaded and further deformed by parametric deformations [10].

Superquadrics are a class of surfaces with a natural parametric and implicit description. The superquadrics are: the superellipsoid, the superhyperboloid of one and two sheets, and the supertoroid [11].

In last decade, 3D objects modeling using superellipsoids has been extensively applied for their simple and flexible shape description and efficient CAD representation [10, 11, 12].

a) SUPERELLIPSOID. A superellipsoid, as an ellipsoid's extension, is the result of the spherical product of two 2D models (two superellipses) [12]. A superellipse, analogous to a circle, is expressed as [10]:

$$\left(\frac{x}{a}\right)^{2/\varepsilon}+\left(\frac{y}{b}\right)^{2/\varepsilon}=1,\ a>0, b>0. \quad (1)$$

where exponentiation with ε is a signed power function such that:

$$\cos^{\varepsilon}\theta = sign(\cos\theta)\left|\cos\theta\right|^{\varepsilon}. \quad (2)$$

Superellipsoids can be expressed by a spherical product of a pair of such superellipses [10]:

$$r(\eta,\omega)=s_1(\eta)\otimes s_2(\omega)=\begin{bmatrix}\cos^{\varepsilon 1}\eta\\ a_3\sin^{\varepsilon 1}\eta\end{bmatrix}\otimes\begin{bmatrix}a_1\cos^{\varepsilon 2}\omega\\ a_2\sin^{\varepsilon 2}\omega\end{bmatrix}=\begin{bmatrix}a_1\cos^{\varepsilon 1}\eta\cos^{\varepsilon 2}\omega\\ a_2\cos^{\varepsilon 1}\eta\sin^{\varepsilon 2}\omega\\ a_3\sin^{\varepsilon 1}\eta\end{bmatrix},\ -\frac{\pi}{2}\le\eta\le\frac{\pi}{2};\ -\pi\le\omega\le\pi. \quad (3)$$

The a_1, a_2, a_3 parameters are scaling factors along the three coordinate axes. ε_1 and ε_2 are derived from the exponents of the two original superellipses.

This flexibility achieved by rising each trigonometric term to an exponent is of particular interest to us. In simple terms, these exponents control the relative roundness and squareness in both the horizontal and vertical directions.

The shape of the superellipsoid cross section parallel to the [*xoy*] plane is determined by ε_2, while the shape of the superellipsoid cross section in a plane perpendicular to the [*xoy*] plane and containing *z* axis is determined by ε_1.

A superellipsoid is defined as the solution of the general form of the implicit equation [11]:

$$\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2}+\left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1}+\left(\frac{z}{a_3}\right)^{2/\varepsilon_1}=1. \qquad (4)$$

All points with coordinates (x, y, z) that correspond to the above equation lie on the surface of the superellipsoid. This is a compact model defined by only five parameters that permits to handle different shapes.

The exponent functions are continuous to ensure that the superellipsoid model deforms continuously and thus has a smooth surface. This form provides an information on the position of a 3D point related to the superellipsoid surface, that is important for interior/exterior determination [9, 12]. We have an inside-outside function F (x, y, z):

$F(x, y, z) = 1$ when the point lies on the surface;
$F(x, y, z) < 1$ when the point is inside the superellipsoid;
$F(x, y, z) > 1$ when the point is outside.

The existence of the inside-outside functions means that superquadrics can be manipulated by means of solid boolean operations, such as union, intersection, and subtraction [9].

Superquadrics are an easy class of objects to use because they have well defined normal and tangent vectors. Normal vectors are used in intensity calculations during rendering. Both the normal and tangent vectors are used to calculate the curvature of the surface [13].

b) CONVEX POLYHEDRON. A convex polyhedron [14] is a figure composed of finitely many planar polygons so that:

- it is possible to pass from one polygon to another through polygons having common sides or segments of sides;
- the entire figure lies on one side of the plane of each constituent polygon.

It is the second condition that defines convexity; the first means that a polyhedron does not split into parts meeting only at vertices or even disjoint from each other.

A convex solid polyhedron is defined as a body bounded by finitely many planar polygons so that it lies on one side of the plane of each of the polygons.

The image boundary of a convex solid is a convex polygon.

GRAPHICAL REPRESENTATIONS

The Madsie Freestyle 1.5.3 application for computation was used to generate the 3D objects with superellipsoids and convex polyhedrons [15] and the graphical representations are given in Table 1. The parameters for superellipsoid are:

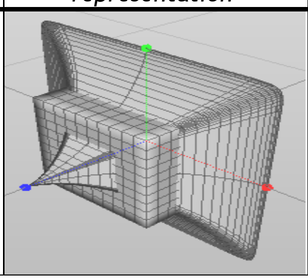
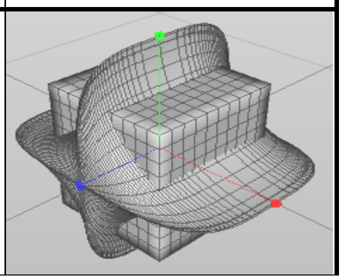
Radius X (r_1) - the radius of the ellipsoid along the X-axis;
Radius Y (r_2) - the radius of the ellipsoid along the Y-axis;
Radius Z (r_3) - the radius of the ellipsoid along the Z-axis;
Stacks (n_1) - the number of segments along the Z-axis;
Slices (n_2) - the number of radial segments around the ellipsoid;
Stack Exponent (e_1) - the shape of the ellipsoid;
Slice Exponent (e_2) - the shape of the ellipsoid.

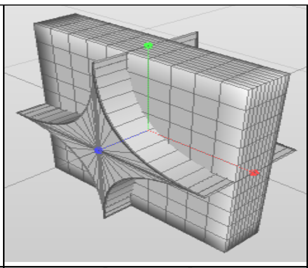
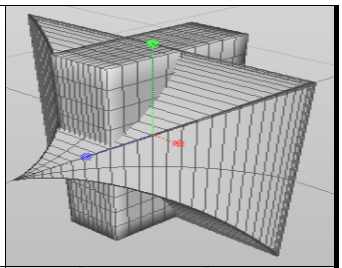
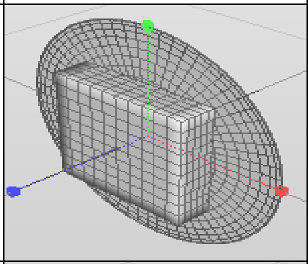
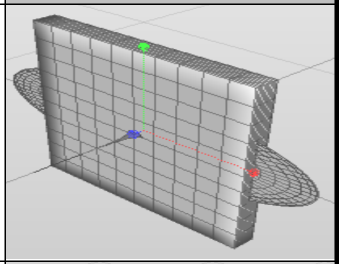
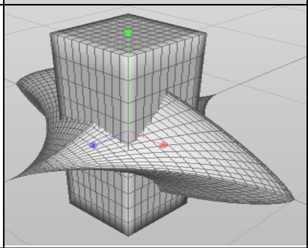
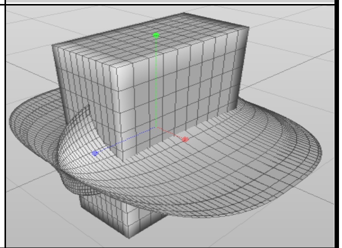
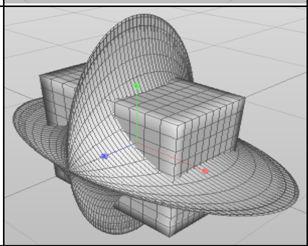
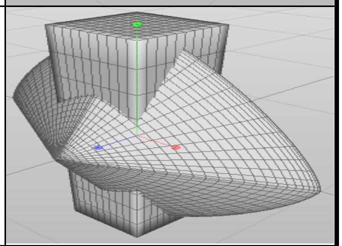
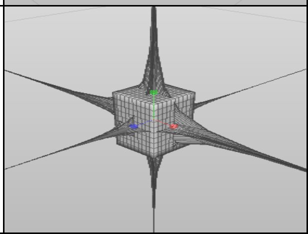
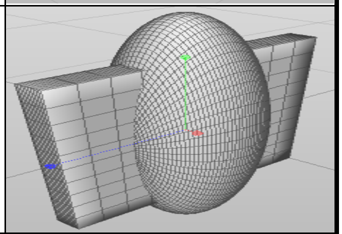
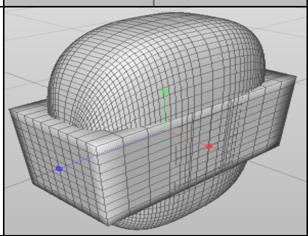
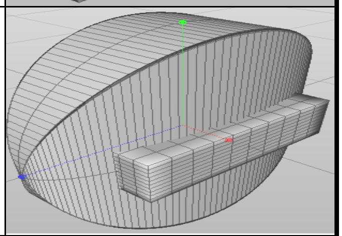
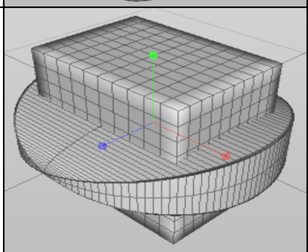
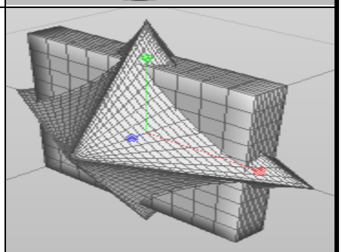
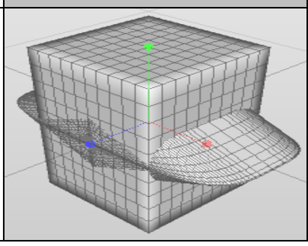
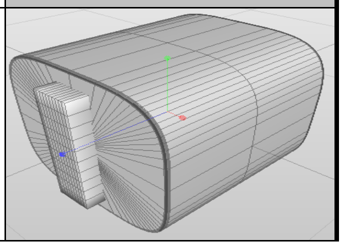
The parameters for convex polyhedron are:

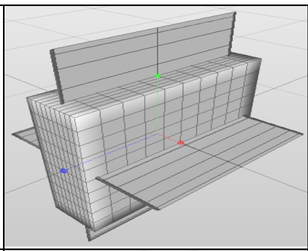
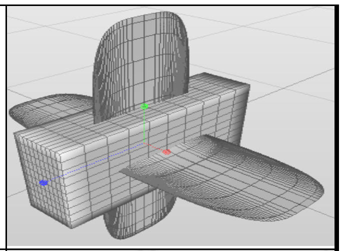
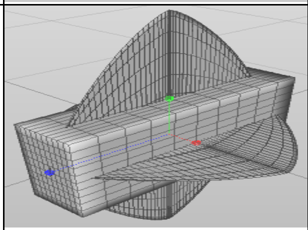
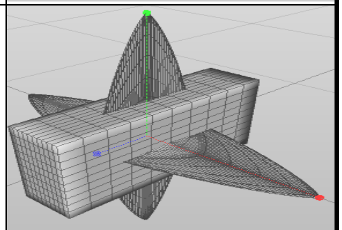
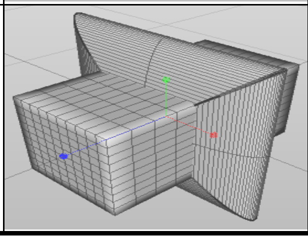
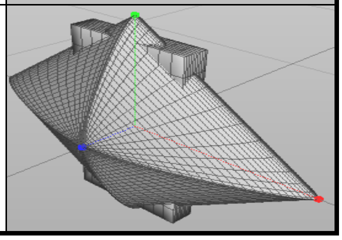
Size X (r_4) - the size of convex polyhedron along the X-axis;
Size Y (r_5) - the size of convex polyhedron along the Y-axis;
Size Z (r_6) - the size of convex polyhedron along the Z-axis;
Segments X (n_3) - the number of segments along the X-axis;
Segments Y (n_4) - the number of segments along the Y-axis;
Segments Z (n_5) - the number of segments along the Z-axis.

Because there are some numerical issues in computation with both very small and very large values of the superellipsoid exponents, in this study, for safety, they are chosen in the range of 0.01 to about 8. Also, in this study it is considered: $n_1 = n_2 = 64$, $n_3 = n_4 = n_5 = 10$.

Table 1. Graphical representations of 3D complex objects

No.	*Values of parameters*	*Axonometric representation*	*No.*	*Values of parameters*	*Axonometric representation*
1	*Superellipsoid:* $r_1 = 1$, $r_2 = 1$ $r_3 = 1$ $e_1 = 0.1$, $e_2 = 3$ *Polyhedron:* $r_4 = 0.5$, $r_5 = 0.5$, $r_6 = 0.5$		*2*	*Superellipsoid:* $r_1 = 1$, $r_2 = 1$ $r_3 = 1$ $e_1 = 1$, $e_2 = 0.5$ *Polyhedron:* $r_4 = 0.5$, $r_5 = 0.5$, $r_6 = 0.5$	

3	Superellipsoid: $r_1 = 1$, $r_2 = 1$ $r_3 = 0.5$ $e_1 = 4$, $e_2 = 0.01$ Polyhedron: $r_4 = 1$, $r_5 = 1$, $r_6 = 0.2$		4	Superellipsoid: $r_1 = 1$, $r_2 = 1$ $r_3 = 1$ $e_1 = 0.01$, $e_2 = 3$ Polyhedron: $r_4 = 0.2$, $r_5 = 1$, $r_6 = 0.5$	
5	Superellipsoid: $r_1 = 1$, $r_2 = 1$ $r_3 = 1$ $e_1 = 1$, $e_2 = 8$ Polyhedron: $r_4 = 0.5$, $r_5 = 0.5$, $r_6 = 0.25$		6	Superellipsoid: $r_1 = 1.5$, $r_2 = 0.5$, $r_3 = 1$ $e_1 = 1$, $e_2 = 8$ Polyhedron: $r_4 = 1$, $r_5 = 1$, $r_6 = 0.1$	
7	Superellipsoid: $r_1 = 2$, $r_2 = 1$ $r_3 = 1.5$ $e_1 = 1.5$, $e_2 = 3$ Polyhedron: $r_4 = 0.5$, $r_5 = 1.5$, $r_6 = 0.5$		8	Superellipsoid: $r_1 = 2$, $r_2 = 1$ $r_3 = 1.5$, $e_1 = 4.5$, $e_2 = 0.7$ Polyhedron: $r_4 = 0.5$, $r_5 = 1.6$, $r_6 = 1$	
9	Superellipsoid: $r_1 = 2$, $r_2 = 2$ $r_3 = 1$ $e_1 = 4$, $e_2 = 1$ Polyhedron: $r_4 = 1$, $r_5 = 1$, $r_6 = 0.5$		10	Superellipsoid: $r_1 = 2$, $r_2 = 2$ $r_3 = 1$ $e_1 = 1.5$, $e_2 = 2$ Polyhedron: $r_4 = 0.5$, $r_5 = 2$, $r_6 = 0.5$	
11	Superellipsoid: $r_1 = 2$, $r_2 = 2$ $r_3 = 1$ $e_1 = 8$, $e_2 = 8$ Polyhedron: $r_4 = 0.1$, $r_5 = 0.2$, $r_6 = 0.1$		12	Superellipsoid: $r_1 = 0.5$, $r_2 = 1.5$, $r_3 = 0.5$, $e_1 = 1$, $e_2 = 1$ Polyhedron: $r_4 = 0.1$, $r_5 = 1$, $r_6 = 1$	
13	Superellipsoid: $r_1 = 0.5$, $r_2 = 1.5$, $r_3 = 1$ $e_1 = 0.5$, $e_2 = 0.7$ Polyhedron: $r_4 = 0.5$, $r_5 = 0.6$, $r_6 = 1.1$		14	Superellipsoid: $r_1 = 0.5$, $r_2 = 1.5$, $r_3 = 1.5$, $e_1 = 0.01$, $e_2 = 1$ Polyhedron: $r_4 = 0.7$, $r_5 = 0.3$, $r_6 = 1$	
15	Superellipsoid: $r_1 = 1$, $r_2 = 0.25$ $r_3 = 1$ $e_1 = 0.01$, $e_2 = 1$ Polyhedron: $r_4 = 0.7$, $r_5 = 0.7$, $r_6 = 0.5$		16	Superellipsoid: $r_1 = 1$, $r_2 = 1$ $r_3 = 0.5$ $e_1 = 2.5$, $e_2 = 2$ Polyhedron: $r_4 = 0.7$, $r_5 = 0.7$, $r_6 = 0.1$	
17	Superellipsoid: $r_1 = 1$, $r_2 = 0.5$ $r_3 = 0.5$, $e_1 = 2$, $e_2 = 0.5$ Polyhedron: $r_4 = 0.5$, $r_5 = 0.8$, $r_6 = 0.5$		18	Superellipsoid: $r_1 = 1$, $r_2 = 1$ $r_3 = 1$, $e_1 = 0.5$ $e_2 = 0.01$ Polyhedron: $r_4 = 0.2$, $r_5 = 0.8$, $r_6 = 1.2$	

19	*Superellipsoid:* $r_1 = 1$, $r_2 = 1.5$ $r_3 = 1$, $e_1 = 8$, $e_2 = 0.01$ *Polyhedron:* $r_4 = 0.3$, $r_5 = 0.9$, $r_6 = 1.1$		20	*Superellipsoid:* $r_1 = 1.5$, $r_2 = 1.5$, $r_3 = 0.5$, $e_1 = 8$, $e_2 = 0.5$ *Polyhedron:* $r_4 = 0.25$, $r_5 = 0.5$, $r_6 = 1$	
21	*Superellipsoid:* $r_1 = 1$, $r_2 = 1.5$ $r_3 = 1$, $e_1 = 8$, $e_2 = 1.5$ *Polyhedron:* $r_4 = 0.25$, $r_5 = 0.5$, $r_6 = 1$		22	*Superellipsoid:* $r_1 = 1.5$, $r_2 = 1.5$, $r_3 = 0.5$, $e_1 = 8$, $e_2 = 1.5$ *Polyhedron:* $r_4 = 0.25$, $r_5 = 0.5$, $r_6 = 1$	
23	*Superellipsoid:* $r_1 = 1$, $r_2 = 1$, $r_3 = 0.25$, $e_1 = 0.01$, $e_2 = 1$ *Polyhedron:* $r_4 = 0.5$, $r_5 = 0.5$, $r_6 = 1$		24	*Superellipsoid:* $r_1 = 1.5$, $r_2 = 1.2$, $r_3 = 0.5$, $e_1 = 2$, $e_2 = 1.5$ *Polyhedron:* $r_4 = 0.5$, $r_5 = 1$, $r_6 = 0.1$	

❖ CONCLUSIONS

Surface modeling techniques are important in different visual computing fields such as engineering design, sculpture design, interactive graphics, animation and virtual environments.

This paper presents a CAD method for generation of complex shapes with superellipsoids and convex polyhedrons.

The Madsie Freestyle 1.5.3 application for computation helps in obtaining conclusions referring to shape of complex 3D objects. The ease of generating new 3D shapes allows the user to incorporate the resulting solids into other models from engineering and sculpture design.

❖ ACKNOWLEDGMENTS

The author wish to thank to Mr. Mads Andersen for consultation, permission to use documentary material and The Madsie Freestyle 1.5.3 application from http://www.madsie.com.

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