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# RANDOM SEARCH METHOD USING CHAOTIC FORCING FOR LOCATION OF GLOBAL OPTIMUM OF MULTIMODAL FUNCTIONS

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ABSTRACT: Chaotic forcing signal, presented as a dynamic system, is introduced in the optimization algorithm of random search with back steps to locate the global optimum of a function. Searching algorithm using n-sphere-based chaotic forcing overcomes the potential barriers between local minima/maxima. When the magnitude of the forcing signal decays slowly, the random search with back steps algorithm is able to approach the global optimum of the multimodal potential function. Under chaos conditions an additional stop criterion is considered. The good performance of the proposed optimization algorithm is confirmed by tests with Shekel functions.
KEYWORDS: global optimum, random search with back steps, chaos

## ❖ INTRODUCTION

Multimodality is a phenomenon, which occurs very often in the engineering, economic and other optimization tasks using complex high-order objective (cost) functions. Many algorithms are developed to locate the global optimum [3, 10], and the convergence of some of them is mathematically proved. The random search methods are widely used because of their simplicity, the small number of calculations, the wide area for investigation from a given starting point, the minimum restrictions (continuity of the target function is not required), etc. The random search from a multitude of starting points, being a trivial method for finding a global extremum, is not very effective with a big number of variables and complex objective functions with a very big number of extrema. Intelligent methods for global search such as genetic algorithms [8] and immune networks [2] have been used recently. But there is not an effective computational algorithm, which could guarantee finding the global optimum in a rational number of iterations.

It has been reported in [7] that it is possible to locate the global optimum of convex functions by solving the problem in a dynamic setting. A dynamic chaotic intervention has been applied to a gradient descent algorithm. Chaotic forcing of appropriate magnitudes leads to the occurrence of crisis bifurcation(s) in the multimodal system. Thus the system sequentially overcomes the local energy barriers and stabilizes in the state in which its energy is the lowest (the global minimum). The same method has been applied to a global optimum search under equality constraints in [11]. Random search method combined with a chaotic intervention for 2D optimization has been presented in [12].

The purpose of the paper is to propose a new approach for a global n-dimensional optimization of multimodal objective functions. The random search with back steps algorithm is modified by adding an n-sphere-based chaotic forcing signal into the recurrent rule. An additional criterion for stopping under chaos conditions has been introduced. The good performance of the proposed algorithm is confirmed by tests with 2D and 4D variants of Shekel's function [9].

## ❖ GRADIENT DESCENT WITH CHAOTIC FORCING

The chaotic forcing of the negative gradient is considered here in the way it is proposed in [7] and used in [11, 12]. Consider the problem of locating the global optimum (minimum) $x = x^*$ of a continuous convex multimodal function $Q(x)$, where $x = (x_1, x_2, ..., x_n)^T$, $n \geq 1$. The derivatives vanish at the extrema of the function, i.e.

$$\frac{\partial Q(x)}{\partial x_i} = 0, \quad i = 1,2,...,n. \tag{1}$$

The solutions depend on the choice of the initial guess and often correspond to a local optimum. Let the real-valued function $Q(x)$ is differentiable in respect to $x$. The *fastest* decrease of $Q(x)$ can be obtained if one goes into the direction of the negative gradient of $Q(x)$, i.e.

$$\frac{dx_i}{dt} = -\frac{\partial Q}{\partial x_i}, \quad i = 1,2,...,n. \tag{2}$$

Depending on the choice of the initial conditions $x_i(t=0)=x_i(0)$, $i=1,2,...,n$, the equilibrium point attained by the system (2) will correspond to one of the minima of the objective function $Q(x)$ and one of them will be the global minimum. Even for relatively simple problems the initial conditions which guarantee an approach to the desired equilibrium point (the global optimum) are often difficult to estimate.

Let the dynamics of (2) is subjected to chaotic forcing. The system obtained is [7, 11]

$$\frac{dx_i}{dt}=-\frac{\partial Q}{\partial x_i}+k_1C_ie^{-k_2t},\quad i=1,2,...,n, \tag{3}$$

where $C_i$ is the chaotic forcing term, $k_1>0$ is its magnitude, and $k_2>0$ controls the rate of decay of the forcing. The Lorenz equation with parameters $\sigma=10$, $\rho=60$, and $\beta=8/3$ has been used [7, 11, 12] to generate the chaotic signal ($C_i=z_2$, $i=1,2,...,n$):

$$\begin{aligned}\dot{z}_1&=\sigma(z_1-z_2)\\ \dot{z}_2&=\rho z_1-z_2-z_1z_3\,.\\ \dot{z}_3&=-\beta z_3+z_1z_2\end{aligned} \tag{4}$$

This system is one of the first chaotic systems, which have been studied. It was discovered by Edward Lorenz in 1963 when he tried to describe the convection in the Earth's atmosphere.

Chaotic resonance can be associated with the transitions induced by chaotic forcing of nonlinear systems [1, 6]. The presence of deterministic chaos plays the role of a random perturbation. A multi-stable system can show intra-well dynamics or inter-well dynamics according to the magnitude of the chaotic forcing signal. In several systems [7], chaotic resonance has been shown to occur near the onset of chaos and near crisis bifurcation points. A crisis bifurcation point refers to the bifurcations after which a system undergoes a transition from intra-well to inter-well dynamics [4, 7]. Such transitions can be easily obtained by increasing the magnitude of the forcing. Once the system starts to show inter-well dynamic behavior, the magnitude of the forcing signal is reduced in a gradual manner. It has been shown in [7] that for a large set of initial conditions the system (2) stabilizes in the state which corresponds to the global minimum of the function $Q(x)$.

## ❖ CHAOTIC INTERVENTION IN ALGORITHM OF RANDOM SEARCH WITH BACK STEPS - RANDOM SEARCH WITH BACK STEPS

The main idea behind the random search method with back steps is the consequent replacing of one starting point $x_0$ by another $x_1$, in case that the objective function in this point has a better value, i.e. $Q(x_1)>Q(x_0)$ (the optimum is a maximum). Reaching the new point is accomplished by a step in a random direction along the direction of a random vector, for which all the possible directions in space are equally probable. If the step is successful, the new point with the better result for the objective function is assumed to be the starting one and a new successful direction is searched for. If the step is not successful, the opposite direction is also checked, since the probability for it to turn out to be successful is considerable. If both directions prove not to be successful, a new random successful direction is searched for. The optimum point is considered to be found when in *M* possible random directions a better result for the objective function cannot be found, where *M* is calculated, following the empiric formulae [10]

$$M=2^n+4\text{, if } n\le 3\ ,\ M=2n+4\text{, if } n>3\ . \tag{5}$$

The step in a random direction is accomplished by the formula [10]

$$x_i^{(1)}=x_i^{(0)}\pm h_i\xi_i,\quad i=1,2,...,n, \tag{6}$$

where $h_i$ is a parameter (a scale coefficient) of the step along the $i$-th controlling variable, and $\xi_i$, $i=1,2,...,n$ are the components of a normalized random vector, obtained in the following way:

1) Random numbers $\beta_i$, $i=1,2,...,n$, uniformly distributed in the interval $(0\div 1)$, are generated;
2) These numbers are transformed into a new sequence $\alpha_i$, $i=1,2,...,n$, uniformly distributed in the interval $(-1\div 1)$

$$\alpha_i=2(\beta_i-0.5),\quad i=1,2,...,n \tag{7}$$

3) The components of the normalized random vector $\xi_i$ are calculated by the formula

$$\xi_i=\frac{\alpha_i}{\sqrt{\sum_{j=1}^{n}\alpha_j^2}},\quad i=1,2,...,n. \tag{8}$$

The sign "-" in the formula (6) is related to a step in the direction, opposite to the random vector. With a normalized random vector (with length of 1), the parameters of the step $h_i$ represent the maximum possible change of the corresponding controlling variable of the current step. They define the accuracy of localization of the extremum point along the separate controlling variables. Variable parameters of the step $h_i$ can be used. The search starts with comparatively high values, enabling high speed of movement towards the extremum area, and the gradual reduction to predetermined minimum values guarantees the desired accuracy.

## CHAOTIC FORCING OF THE RANDOM SEARCH

The equation (3) can be presented in the following discrete form:

$$x_i^{(k+1)} = x_i^{(k)} - \frac{\partial Q}{\partial x_i}\Big|_{x_i^{(k)}} T_0 + k_1 T_0 z_2^{(k)} e^{-k_2 k T_0}, \quad i = 1,2,\ldots,n, \tag{9}$$

where $T_0$ and $k$ are correspondingly a step and a moment of the discretization, and $z_2^{(k)}$ is obtained from the Lorenz equation (4), given in a discrete form:

$$\begin{aligned} z_1^{(k+1)} &= z_1^{(k)} + \sigma(z_1^{(k)} - z_2^{(k)})T_0 \\ z_2^{(k+1)} &= z_2^{(k)} + (\rho z_1^{(k)} - z_2^{(k)} - z_1^{(k)} z_3^{(k)})T_0 \\ z_3^{(k+1)} &= z_3^{(k)} + (-\beta z_3^{(k)} + z_1^{(k)} z_2^{(k)})T_0 \end{aligned} \tag{10}$$

The dependence (6), by which the new point in the random search method is calculated, is modified in accordance with (9) as it follows [12]:

$$x_i^{(k+1)} = x_i^{(k)} \pm (h_i \xi_i^{(k)} + k_1 z_2^{(k)} e^{-k_2 k}), \quad i = 1,2,\ldots,n, \tag{11}$$

where $k_1 \leftarrow k_1 T_0$ and $k_2 \leftarrow k_2 T_0$. The chaotic component changes both its direction (slightly) and the magnitude of the random vector (considerably), and at $k \to \infty$ this component tends to zero.

Here, n-sphere-based modification of the chaotic forcing component in (11) is proposed in order to extend the angular scope of chaotic forcing. The rough inspiration comes from well-known hyper spherical coordinates' transformation

$$\begin{aligned} x_1 &= r\cos(\phi_1) \\ x_2 &= r\sin(\phi_1)\cos(\phi_2) \\ x_3 &= r\sin(\phi_1)\sin(\phi_2)\cos(\phi_3) \\ &\ldots \\ x_{n-1} &= r\sin(\phi_1)\ldots\sin(\phi_{n-2})\cos(\phi_{n-1}) \\ x_n &= r\sin(\phi_1)\ldots\sin(\phi_{n-2})\sin(\phi_{n-1}). \end{aligned} \tag{12}$$

The radial coordinate $r$ is chosen to be $r = k_1 z_2(t) e^{-k_2 t}$, and the angular coordinates are chosen $\phi_1 = \phi_2 = \ldots = \phi_{n-1} = z_2(t)t$. Thus, the search rule (11) obtains the form:

$$\begin{aligned} x_1^{(k+1)} &= x_1^{(k)} \pm (h_1 \xi_1^{(k)} + k_1 z_2^{(k)} e^{-k_2 k} \cos(z_2^{(k)} k T_0)) \\ x_2^{(k+1)} &= x_2^{(k)} \pm (h_2 \xi_2^{(k)} + k_1 z_2^{(k)} e^{-k_2 k} \sin(z_2^{(k)} k T_0)\cos(z_2^{(k)} k T_0)) \\ x_3^{(k+1)} &= x_3^{(k)} \pm (h_3 \xi_3^{(k)} + k_1 z_2^{(k)} e^{-k_2 k} \sin(z_2^{(k)} k T_0)\sin(z_2^{(k)} k T_0)\cos(z_2^{(k)} k T_0)) \\ &\ldots \\ x_n^{(k+1)} &= x_n^{(k)} \pm (h_n \xi_n^{(k)} + k_1 z_2^{(k)} e^{-k_2 k} \underbrace{\sin(z_2^{(k)} k T_0)\ldots\sin(z_2^{(k)} k T_0)}_{n-1}). \end{aligned} \tag{13}$$

This modification expands the space area of the variables $x_i$, accessible for impact on the side of the chaotic intervention. Thus the forcing component describes parts of a hyper sphere with chaotically changing radial and angular coordinates.

Additional criterion for stopping is introduced, according to which after $L$ successful iterations the average deviation $\Delta Q^*$ of the $L$-th sequential optima from their averaged value $\overline{Q}$ for this interval and the corresponding mean square deviation $\Delta x^*$ of the control variables are compared with predetermined boundary values - $\delta_Q$ and $\delta_x$.

The deviations are calculated by the formulae:

$$\Delta Q^* = \frac{1}{L}\sum_{r=1}^{L} \left| Q^{(r)} - \overline{Q} \right|, \tag{14}$$

$$\Delta x^* = \frac{1}{L}\sum_{r=1}^{L} \sqrt{\sum_{i=1}^{n} (x_i^{(r)} - \overline{x}_i)^2}, \tag{15}$$

where the average values of $Q$ and $x$ for the last $L$ successful iterations are $\overline{Q} = \frac{1}{L}\sum_{r=1}^{L} Q^{(r)}$ and $\overline{x}_i = \frac{1}{L}\sum_{r=1}^{L} x_i^{(r)}, \quad i = 1,2,\ldots,n$. After $M$ unsuccessful directions the random search without chaotic intervention would cease, eventually, at a local minimum. Following the modified method, however, the time counter is nulled $k = 0$ (or $t = 0$), and the chaotic component starts with a minimum value of decay, so the search continues. If at this moment the following conditions are fulfilled,

$$\Delta Q^* < \delta_Q \ \& \ \Delta x^* < \delta_x, \tag{16}$$

the search stops at the moment of reaching the predetermined minimum size of the step $h_i < \delta_{h_i}$, $i = 1,2,...,n$, and, in case it is not reached, the step halves and the search continues, though without the chaotic forcing component (by the random search method with back step). As a last measure for stopping, achievement of a certain number of iterations could be predetermined.

## ❖ ALGORITHM OF THE PROPOSED METHOD

STEP 1. INITIALIZATION. The following information is given: the number of control variables $n$; the step of discretization $T_0$; the initial size of the scaling coefficient along the $i$-th controlling variable $h_i = h_i^0$, $i = 1,2,...,n$ together with its minimum size $\delta_{h_i}$, $i = 1,2,...,n$, defining the accuracy of localization of the optimum; the starting point $x^{(0)} = (x_1^{(0)}, x_2^{(0)},..., x_n^{(0)})$; the initial precondition $z^{(0)} = (z_1^{(0)}, z_2^{(0)}, z_3^{(0)})$ for solving the Lorenz equation (10); the final number of iterations $N_{max}$. The objective function $Q_0 = Q(x^{(0)})$ and the constant $M$ are calculated here. The following components are nulled: the counter of the discrete time $k$, the counter of the iterations $N$, and the counter of the unsuccessful directions $l$; Two arrays are reserved $Q_{M-best}$ and $x_{M-best}$ with dimensions of $1 \times L$ and $n \times L$ correspondingly, which will contain the last L best values of $Q$ and $\mathbf{x}$ (in search for a maximum $Q_{M-best}$ is initialized by an $L$ number of very small values).

STEP 2. A normalized random vector $\xi = (\xi_1, \xi_2,..., \xi_n)$ is generated following (8). It is checked whether to introduce a chaotic signal into the random search, i.e. if the following condition is fulfilled $h_i = h_i^0$, $i = 1,2,...,n$, a step is realised from the starting point following the formulae (13) and in accordance with (10), using the sign "+". Otherwise the step is done following the formula (6) with a "+" sign. The counters of discrete time ($k$) and the iterations ($N$) are increased by 1.

STEP 3. The objective function $Q_1 = Q(x^{(1)})$ is calculated. If $Q_1 > Q_0$, the algorithm continues with step 4, while in the opposite case it continues with step 5.

STEP 4. The coordinates of point $x^{(1)}$ are saved as a new starting point $x^{(0)}$; $Q_0$ assumes the value $Q_1$; the arrays $Q_{M-best}$ and $x_{M-best}$ are actualized by deleting the oldest value and introducing the new optimum ($Q_1$ and $x^{(1)}$) in its place; the counter of unsuccessful directions $l$ is nulled and the algorithm continues with step 2.

STEP 5. It is checked whether to introduce a chaotic signal in the back step. If the condition $h_i = h_i^0$, $i = 1,2,...,n$ is fulfilled, then a back step is undertaken, following (13) and in accordance with (10), using the sign "-". In the opposite case the step is realized following the formula (6) with the sign "-". The counters of discrete time ($k$) and iterations ($N$) are increased with 1.

STEP 6.The objective function $Q_1 = Q(x^{(1)})$ is calculated. If $Q_1 > Q_0$, the algorithm continues with step 4, and in the opposite case it continues with step 7.

STEP 7. One unsuccessful direction is counted, i.,e, the counter $l$ increases with 1. If $l < M$ the algorithm continues with step 2, while in the opposite case the counter of unsuccessful directions is nulled $l = 0$, as well as the counter of discrete time $k = 0$, and the process continues with step 8.

STEP 8. The criterion for stopping is checked - if a predetermined maximum number of iterations is reached. If the condition $N \le N_{max}$ is fulfilled, the process is continued with the next step 9, and in the opposite case - step 10 follows.

STEP 9. The deviation $\Delta Q^*$ of the last $L$ optima from their mean value is calculated following (14), together with calculating the mean square deviation $\Delta x^*$ of their corresponding control variables following (15). The criterion for stopping is checked under chaotic conditions, i.e., if the condition (16) is fulfilled, then the process continues with step 10, otherwise - with step 2.

STEP 10. The scaling coefficient $h_i$ is halved. If the condition $h_i < \delta_{h_i}$, $i = 1,2,...,n$ is fulfilled, the algorithm continues with step 11, otherwise - with step 2.

STEP 11. End of the algorithm. The search is ceased, and solution of the task is the point $x^{(0)}$ with a result for the objective function in it - $Q_0$.

## ❖ TEST FUNCTIONS

As an illustration of the proposed modified random search method with a forcing chaotic signal in the role of a test example a Shekel function is used [9, 5]:

$$Q(x) = \sum_{i=1}^{m} \frac{1}{c_i + \sum_{j=1}^{n} (x_j - a_{ji})^2}, \tag{17}$$

which is $n$-dimensional and has $m$ maxima. Two variants of the Shekel function are used - 2D ($n = 2$) and 4D ($n = 4$). In both cases the function has $m = 10$ maxima, and the search domain is $0 \le x_i \le 10$. For the 4D variant the following parameters are used in (17) [5]:

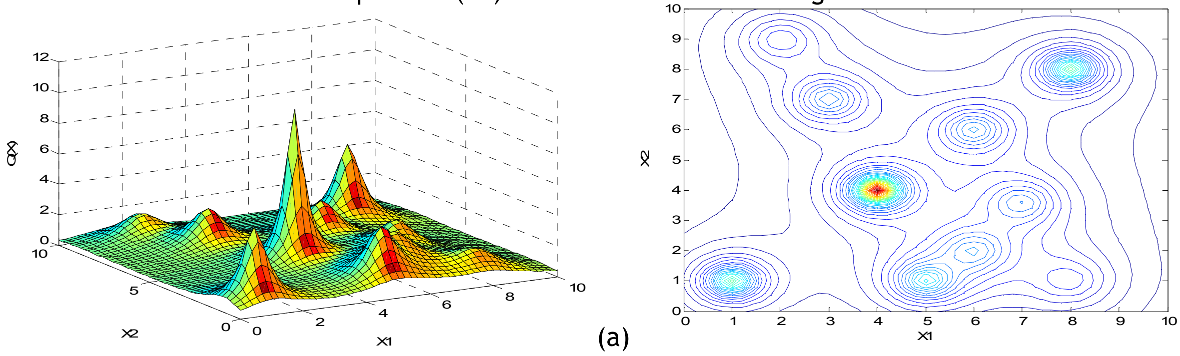
$$C^{(4D)} = \frac{1}{10}[1, 2, 2, 4, 4, 6, 3, 7, 5, 5]^T,$$

$$A^{(4D)} = \begin{bmatrix} 4.0 & 1.0 & 8.0 & 6.0 & 3.0 & 2.0 & 5.0 & 8.0 & 6.0 & 7.0 \\ 4.0 & 1.0 & 8.0 & 6.0 & 7.0 & 9.0 & 5.0 & 1.0 & 2.0 & 3.6 \\ 4.0 & 1.0 & 8.0 & 6.0 & 3.0 & 2.0 & 3.0 & 8.0 & 6.0 & 7.0 \\ 4.0 & 1.0 & 8.0 & 6.0 & 7.0 & 9.0 & 3.0 & 1.0 & 2.0 & 3.6 \end{bmatrix}.$$

For the 2D variant $C^{(2D)} = C^{(4D)}$, and $A^{(2D)}$ contains only the first two rows of $A^{(4D)}$. The maxima are correspondingly

$$4D: \quad x^* = (4, 4, 4, 4), \quad Q(x^*) = 10.5363; \text{ and } 2D: \quad x^* = (4, 4), \quad Q(x^*) = 10.6922.$$

The surface and contour plots of (17) for $n = 2$ are shown in Figure 1.



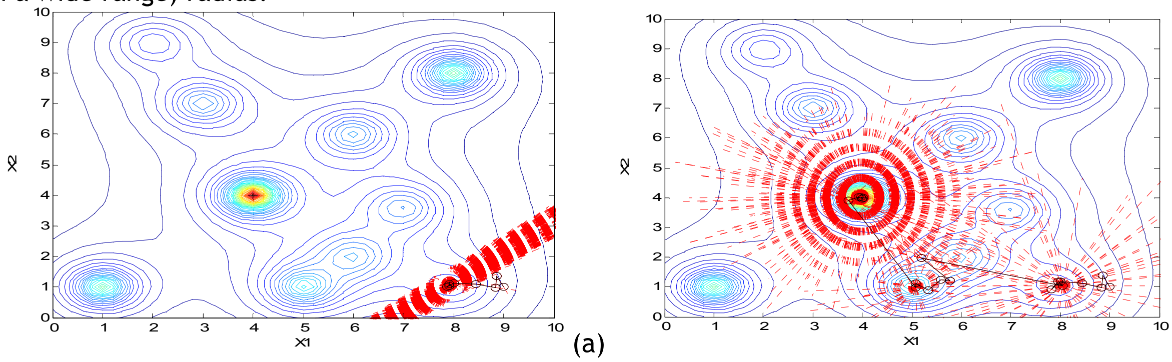
(a) (b)

Figure 1. The surface and contour plots of (17) for $n = 2$

## ❖ RESULTS AND DISCUSSIONS

In order to demonstrate the workability, the advantages and drawbacks of the proposed algorithm for global search of an extremum, a multitude of simulations was realized in MATLAB environment with the above described multimodal function (17). The following parameters were chosen and used in the proposed search algorithm: step of discretization $T_0 = 0.01\,s$; initial and minimum value of the scaling coefficients along the two controlling variables $h_1^0 = h_2^0 = h^0 = 0.4$ and $\delta_{h_1} = \delta_{h_2} = 0.05$ correspondingly; coefficients of the chaotic forcing component $k_1 = 0.1 \div 0.3$ and $k_2 = 0.1 \div 0.01$ (for the 2D variant); initial preconditions for the Lorenz equation $z_1^0 = z_2^0 = z_3^0 = 0.1$; minimum deviations for the additional criterion for stopping under chaotic conditions $\delta_Q = 0.5$ and $\delta_x = 0.5h^0 = 0.2$ (for the 2D variant).

For finding the global maximum of the Shekel (2D) function experiments were conducted with the two variants of the modified by chaotic forcing random search method, presented in the formulae (11) and (13). The first variant contains the same chaotic component as the one, used in [7], while the second one contains an improved version of it, proposed in the present paper. Figure 2 shows the results from the optimization of (17), obtained in the two ways for $n = 2$. It can be seen that the area of influence of the chaotic component in the first case is strongly restricted, what depends on various factors (the Lorenz system, generating it, the coefficients $k_1$ and $k_2$) but mostly on the way of introducing it into the law of random search (6). It participates in the same way in the equations for actualization of all controlling variables. The area of initial conditions, leading to a global extremum is expanded with respect to the classical random search with back steps though not enough to guarantee high success of the global search. In the second case search is carried out around the current point in the form of a circular motion with a chaotic frequency of going round and with a chaotically changing (in a wide range) radius.



(a) (b)

Figure 2. Global search by the random search method with chaotic forcing:
(a) the search rule (11), and (b) the modified search rule (13).

The proposed method is sensitive towards the choice of parameters to be used. The decrease in $L$, $N_{max}$ and $k_1$, as well as the increase in $k_2$, can worsen the success of the global search. The parameter $k_1$ influences the magnitude of the amplitude of the chaotic component: the small values lead to convergence toward the closest to the starting point maximum, while the excessively high values could cause omission of the global maximum and oscillations at the local maxima around it. The high values of $k_2$ lead to a premature decay of the chaotic component, while the small values lead to increasing the duration of the process of searching for the optimum.

Together with the development of the iterations Figure 3 shows the actualization of the values of the controlling variables $x$ and of the objective function $Q$, corresponding to the experiment shown in Figure 2(b). Since the successful iterations are denoted by the sign "o", it gives a visual idea of the number of calculations of the objective function in the chaotically generated unsuccessful points.

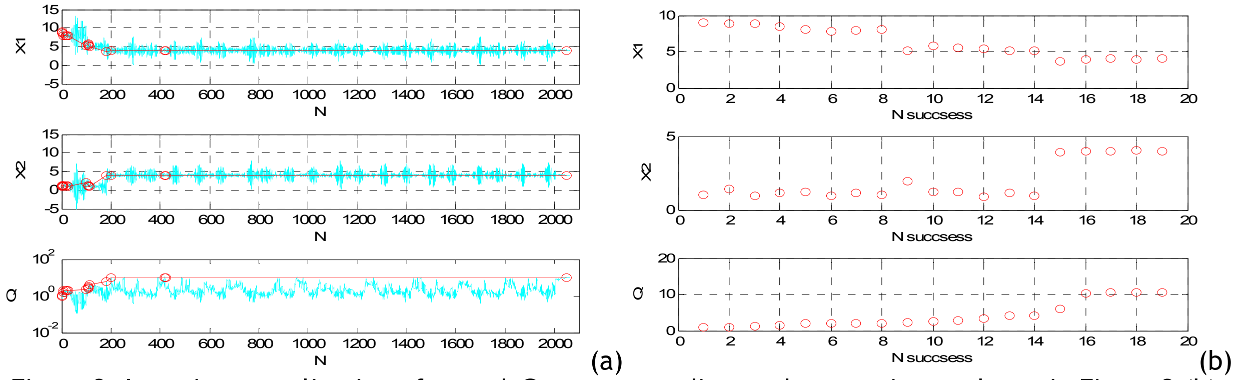



Figure 3. Iterative actualization of $x$ and $Q$, corresponding to the experiment shown in Figure 2 (b): (a) all iterations including successful and unsuccessful steps and (b) the iterations, presenting the successful steps only. The sign "o" denotes the successful iterations.

Experiments were conducted additionally for localizing the global maximum of the 4-dimension case of the Shekel function. The following values of the parameters were chosen: $k_1 = 1$ and $k_2 = 0.001$, $\delta_Q = 0.1$ and $\delta_x = 0.1h^0 = 0.04$. Three versions of the random search algorithm with back steps were used. Table 1 shows the representation of these algorithms when starting from 10 different starting points, located outside the area of attraction of the global maximum. By '*' the successful localization of the global maximum is marked. As expected, the classical random search algorithm with back steps cannot find the global optimum. The chaotic forcing of the random search with back steps increases the probability for finding the global optimum, and the proposed in the paper n-sphere-based modified version of it (13) has a greater success compared to (11) (for the particular examples: 100% vs. 40%, respectively).

Table 1. Three versions of the random search algorithm, used for optimization of the Shekel function - 4D case

| n = 4 | Random search with back step | | |
|---|---|---|---|
| Initial point | Classical case (6) | With chaotic intervention (11) | With modified chaotic intervention (13) |
| (6,7,8,9) | - | * | * |
| (9,9,6,7) | - | * | * |
| (6,7,6,7) | - | * | * |
| (7,7,7,7) | - | * | * |
| (4,9,4,9) | - | - | * |
| (2,8,2,8) | - | - | * |
| (9,1,9,1) | - | - | * |
| (5,9,5,9) | - | - | * |
| (9,4,9,4) | - | - | * |
| (1,9,1,9) | - | - | * |

## ❖ CONCLUSIONS

The proposed n-sphere-based modification of a chaotic intervention into the random search method with back step leads to a greater success in localizing the global extremum compared to the considered two random search options: (1) classical, without chaotic intervention, and (2) with chaotic intervention, as proposed in [7]. The representation of the methods with chaotic intervention depends crucially on the choice of a certain number of parameters. The future investigations in the field of the problem under consideration will include: comparative analysis based on success, time consumption and calculation difficulties of the proposed method and other popular methods of global search; expansion of the search method with the availability of both areal and functional restrictions; etc.

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