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[1.] Stephan VELCHEV, [2.] Ivan KOLEV, [3.] Krasimir IVANOV

INVESTIGATING THE INFLUENCE OF THE RAKE ANGLE ON THE SPECIFIC CUTTING FORCE BY TURN LATHING

[1-3.] DEPARTMENT OF MACHINE TOOLS & MANUFACTURING, "ANGEL KANCHEV" UNIVERSITY OF RUSE,
8 STUDENTSKA STR., 7017 RUSE, BULGARIA

ABSTRACT: Research has been carried out to determine the influence of the rake angle on the specific cutting force of different processed materials. New mathematical models are applied that give an adequate description of the physical nature of that relationship. The precision of the empiric mathematical models is estimated by value of the relative error.
KEYWORDS: cutting, specific cutting force, mathematical models, rake angle

❖ INTRODUCTION

The influence of the rake angle on the specific cutting force is usually estimated by the correction coefficient, which according to the reference [10] is determined according to the formula:

$$k_{\gamma o} = 1 - \frac{\gamma_o - \gamma_k}{66{,}7}, \tag{1}$$

where γ_o is the rake anglee, for which the specific cutting force has to be determined. $\gamma_k = 6^o$ for steels, $\gamma_k = 2^o$ for cast irons. The formula is valid for $\gamma_o \in \{-20^o \div 30^o\}$.

According to [7,8,12] it is accepted that by increasing the rake angle with 1^0, the main force, hence the specific cutting force decreases with about 1%.

For approximate estimations of the cutting forces for steels with an average hardness it is recommended [5] to estimate the influence of the rake angle in the following way: by quick cutting instruments the rake angle decreasing with 1^0 causes increasing of the main cutting force with 1,5%, which corresponds to formula (1), while by hard alloyed instruments the increasing is with 0,75%.

It is thus assumed that the dependence of the specific force on the rake angle is approximated by means of a mathematical model linear function of the type:

$$k_c = c_o^{'} - c_1^{'}\gamma_o, \tag{2}$$

where the coefficient $c_1^{'}$, according to different bibliographic sources has values $c_1^{'}$ = 0,0075 ÷0,015.

The influence of the rake angle on the specific cutting force is more convenient to be described by using the angle of cutting $\delta_o = 90^o - \gamma_o$. In that case the mathematical model (2) obtained is expressed as follows:

$$k_c = c_o + c_1\,\delta_o. \tag{3}$$

In order to approximate the relationship of the main cutting force and the angle of cutting, a mathematical model power function is also used, which after transformation for a specific cutting force has the following appearance:

$$k_c = c_o\,\delta_o^{c_1}. \tag{4}$$

The exponent c_1 has values c_1 = 0,83 ÷0,92 [4] or c_1 = 0,95÷1,05 [1]. It is distinguished that at higher cutting forces these values can be smaller, i.e. the influence of δ_o, respectively of γ_o on the cutting force is weaker.

A theoretical determination of the specific cutting force of plastics in dependence of the different parameters, influencing the cutting process, privately influenced by the rake angle, is given in [9]. The theoretical and simulation investigation by means of the suggested lathe turning model is based on the viscose plastic theory. The application of the latter requires different physical and mechanical parameters of the processed material and their temperature dependencies, which therefore have to be determined by means of rather complex experiments.

The aim of the present investigation is to achieve more accurate cutting force estimation by using of new mathematical models of the specific cutting force dependency on the main rake angle of the cutting tool. The following tasks should be accomplished in order to achieve this objective: physical foundation of hypothetical mathematical models of the correlation $k_c = f(\delta_o)$ by changing the cutting angle (the rake angle) in a wide range of values; experimental investigation of the specific cutting force dependency on the cutting angle by means of a turn lathing of different processed materials; obtaining and investigating the proposed empirical mathematical models of this correlation.

❖ PHYSICAL GROUNDS OF HYPOTHETICAL MATHEMATICAL MODELS

The specific cutting force can be analyzed as obtained from three components:

$$k_c = k_{c\gamma\, n} + k_{c\gamma\, f} + k_{c\alpha\, f}\,, \qquad (5)$$

where $k_{c\gamma\, n}$, $k_{c\gamma\, f}$ and $k_{c\alpha\, f}$ are constituents of the specific cutting force, obtained from the projections on the cutting speed direction, correspondingly of the normal force on the front surface, the friction force on the front surface and the friction force on the back surface.

When the rake angle values increase in their positive range $(\delta_o < \pi/2)$ at given cut layer thickness and cutting speed, the constituent $k_{c\gamma\, n}$ decreases. It can be assumed that the constituent $k_{c\alpha\, f}$ does not depend on the rake angle and has a determined constant value. By large rake angle values $\gamma_o \to \pi/2$ ($\delta_o \to 0$), the value of $k_{c\gamma\, n}$ tends to zero. Therefore, a condition can be defined, to which the empiric mathematical models of approximating the specific cutting force dependency on the cutting angle should correspond:

$$\text{at } \delta_o \to 0\,;\; k_{c,\delta_o \to 0} = k_{c\gamma\, f} + k_{c\alpha\, f} = const\,. \qquad (6)$$

The physical sense of the specific cutting force value $k_{c,\delta_o \to 0}$ is that it is obtained from the friction forces on the front and back surfaces.

When the rake angle values decrease in their negative range $(\delta_o \geq \pi/2)$ the constituent $k_{c\gamma\, n}$ increases, while the constituent $k_{c\gamma\, f}$ is headed in the opposite direction, furthermore the friction coefficient decreases. It can be assumed that the increase of $k_{c\gamma\, n}$ is very strongly expressed and on the total specific cutting force increases. This, for example, is confirmed of the experiments by hardened steel lathe turning. By a considerable interpretation of the obtained results, given in [13], it can be determined that decreasing of the rake angles values in the interval of $\gamma_o \in \{-30^\circ \div (-75^\circ)\}$, obtained from the cutting wedge radius of curvature at small thicknesses, a considerable (repeated) increase of the specific cutting force is obtained.

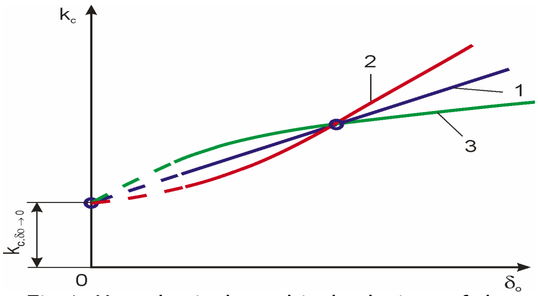

Fig.1. Hypothetical graphical relations of the specific cutting force from the cutting angle

It is very difficult to figure out the intensity of the specific cutting force increasing physical considerations. It can be assumed that depending on the type and mechanical properties of the processed material, the cut layer thickness and the cutting speed, it is possible to approximate the correlation $k_c = f(\delta_o)$ by means of a concave, convex or linear function. Fig. 1 shows hypothetical graphical relations of the specific cutting force on the cutting angle, answering the relation (6) and the upper considerations. They are monotonously growing under the condition that the cutting is executed without stratum-formation. The mathematical models, through which these hypothetical graphical relations can possibly be approximated, are selected from [6], some of them are modified in terms of structure under the conditions that $c_o, c_1, c_2, > 0$ (table.1).

Table .1 Hypothetical Mathematical Models

№	Mathematical model	$k_{c,\delta_o \to 0}$
1.	$k_c = c_o\, \delta_o^{c_1}$	0
2.	$k_c = c_o + c_1\, \delta_o$	c_o
3.	$k_c = c_o + c_1\, \delta_o^{c2}$	$c_o + c_1$
4.	$k_c = c_o\, e^{c_1 \delta_o}$	c_o
5.	$k_c = c_o\, e^{c_1 \delta_o^{c2}}$	c_o

The mathematical model 1 is involved for comparison, although it does not satisfy the physical condition (6), as it is often used to approximate the relation $k_c = f(\delta)$.

❖ RESEARCH METHODOLOGY

The research methodology is given in more details in [3] and the abstract is given in [2].

❖ EXPERIMENTAL RESEARCH RESULTS

The experimental investigations of the rake angle influence on the specific cutting force are conducted on processed materials, given in table 2.

Table 2. Processed Materials

Processed material	Steel AISI W1-1.0C	Steel ASTM5140	Grey iron GG15	Grey iron GG25	Bronze CuSn7P0.7	Aluminium alloy AlCu4.5Mn0.5Mg1.6
Hardness, HB	170 -180	226-233	205	233	68-78	115-122
$v_c, m/\min$	105	96	50	63	98	100
$f, mm/r$	0,452	0,452	0,452	0,452	0,34	0,34

Straight blade knives are used with soldered hard alloyed disks of the P30 type and geometrical parameters $\alpha_0 = 10^o$, $\kappa_r = 70^o$, $\kappa_r^{'} = 20^o$ and $r_\varepsilon = 1{,}25 - 1{,}35\,mm$. The experiments are carried out when the rake angle variation is from $\gamma_0 = -20^o$ to $\gamma_0 = 30^o$ at every 10^o and at a nominal cutting depth of $a = 2{,}0\,mm$, while the cutting speeds and the feedings for the different materials are given in table 2. The main cutting force is measured by means of a dynamometric device at a threefold repetition of each experiment. Using the specifically developed for the purpose computer program by calculations the real cutting depth, the real cut layer cross section area and the specific cutting force were determined.

Fig.2 shows the correlations, graphically expressed according to the average values for each experiment, between the specific cutting force and the cutting angle (the rake angle) for the different processed materials. The correlations show that the specific cutting force increase, while increasing the cutting angle (decreasing of the rake angle), happens at various intensity for a given processed material.

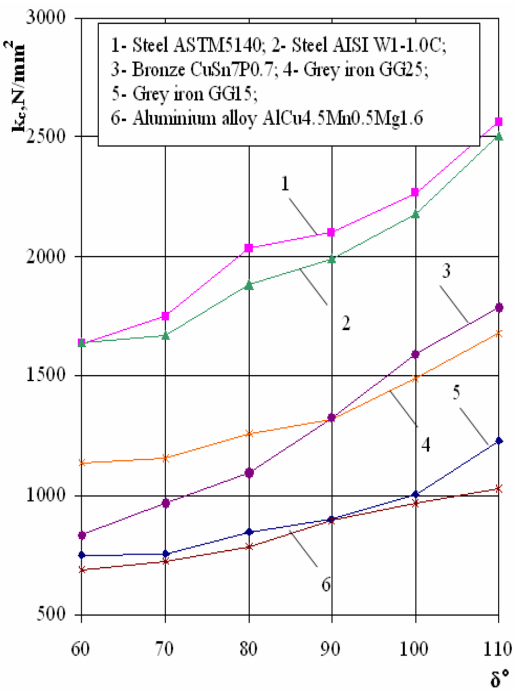

Fig. 2. Correlation of the specific cutting force and the cutting angle for different processed materials

To approximate the dependency of the specific cutting force from the cutting angle, hypothetical mathematical models are obtained and analyzed, given in table 1. The coefficients of these models, obtained after mathematical processing of the experimental data by using a computer program, are given in table 3. In order to select and evaluate a model, which best approximates the experimental results, the Fisher criterion F, having a critical value F_k at a significance level $\alpha = 0{,}05$, the correlation coefficient R, the maximum relative error $|\Delta k_{c\max}|\%$ and the physical condition (6) are applied (table 3). In the columns (PhC) physical condition and (AD) adequacy with a (+) or a (-) is marked the fulfillment of the conditions.

Table 3. Coefficients of the mathematical models and statistical criteria

Processed material	Model	C0	C1	C2	F	Fk	R	$\lvert\Delta k_{c\max}\rvert$ □	ФУ	АД
Steel AISI W1-1.0C	1	94,70	0,686	-	5,80	3,26	0.965	4,92	-	-
	2	555,02	16,89	-	4,92	3,26	0.970	4,28	+	-
	3	1476,30	$4{,}27.10^{-4}$	3,14	2,54	3,49	0.985	5,23	+	+
	4	963,10	$8{,}41.10^{-3}$	-	3,57	3,26	0.978	3,75	+	-
	5	1426,80	$4{,}27.10^{-6}$	2,38	2,45	3,49	0.985	5,14	+	+
40X	1	83,00	0,722	-	5,33	3,26	0.985	2,91	-	-
	2	532,69	17,93	-	4,70	3,26	0.987	3,20	+	-
	3	1137,92	0,445	1,71	4,06	3,49	0.988	4,27	+	+
	4	965,25	$8{,}8.10^{-3}$	-	3,97	3,26	0.989	4,23	+	-
	5	965,96	9.10^{-3}	1,01	3,97	3,49	0.989	4,29	+	+
Grey iron GG15	1	26,28	0,80	-	63,04	3,26	0.945	7,94	-	-
	2	121,81	9,36	-	54,5	3,26	0.953	8,54	+	-
	3	717,79	$3{,}4.10^{-8}$	4,98	3,20	3,49	0.997	2,88	+	+
	4	388,42	$9{,}9.10^{-3}$	-	34,77	3,26	0.970	5,70	+	-
	5	706,41	$3{,}49.10^{-9}$	4,02	3,06	3,49	0.997	2,62	+	+
Grey iron GG25	1	80,39	0,63	-	29,41	3,26	0,950	5,79	-	-
	2	425,03	10,74	-	23,41	3,26	0,960	6,06	+	-
	3	1097,19	$2{,}65.10^{-7}$	4,58	1,77	3,49	0,997	1,89	+	+
	4	679,67	$7{,}7.10^{-3}$	-	15,80	3,26	0,973	4,71	+	-
	5	1082,7	$8{,}9.10^{-9}$	3,77	1,90	3,49	0,997	1,69	+	+
Bronze CuSn7P0.7	1	4,02	1,293	-	6,34	3,26	0,993	5,78	-	-
	2	-402,6	19,64	-	8,40	3,26	0,991	6,73	-	-
	3	506,9	$2{,}5.10^{-2}$	2,307	2,47	3,49	0,997	2,96	+	+
	4	321,0	$1{,}57.10^{-2}$	-	2,77	3,26	0,997	2,94	+	+
	5	294,2	$2{,}30.10^{-2}$	0,931	2,77	3,49	0,997	3,13	+	+
Al. alloy AlCu4.5 Mn0.5Mg1.6	1	36,65	0,708	-	5,10	3,26	0.989	4,11	-	-
	2	229,8	7,27	-	4,15	3,26	0.991	3,63	+	-
	3	472,7	$2{,}14.10^{-1}$	1,676	3,41	3,49	0.992	2,66	+	+
	4	403,0	$8{,}6.10^{-3}$	-	3,56	3,26	0.992	2,57	+	≈
	5	372,6	$1{,}5.10^{-2}$	0,892	3,56	3,49	0.992	2,73	+	≈

The mathematical model 1 for all investigated processed materials, apart from not meeting the physical condition (6), it is also not adequate according to the Fisher criterion. The latter is valid for model 2 as well. The mathematical model 4 for all processed materials, with the exception of Bronze CuSn7P0.70, is not adequate either. The mathematical models 3 and 5 are almost equal valued to the evaluation criteria. It is more appropriate to choose the mathematical model 3 for all processed materials, as it has a simpler structure (table. 4). The intensity of growing varies by the different processed materials.

Table 4. Recommended mathematical models

№	Processed material	Mathematical model	$k_{c.\delta_o=\frac{\pi}{2}}$ N/mm^2	$k_{c.\delta_o\to\infty}$ N/mm^2
1.	Steel AISI W1-1.0C	$\hat{k}_c = 1474{,}3 + 4{,}27.10^{-4}\,\delta_o^{3{,}12}$	2008	1474
2.	Steel ASTM5140	$\hat{k}_c = 1137{,}9 + 0{,}445.\delta_o^{1{,}71}$	2115	1138
3.	Grey iron GG15	$\hat{k}_c = 717{,}8 + 3{,}40.10^{-8}\,\delta_o^{4{,}98}$	901	718
4.	Grey iron GG25	$\hat{k}_c = 1097{,}2 + 2{,}65.10^{-7}\,\delta_o^{4{,}58}$	1334	1097
5.	Bronze CuSn7P0.7	$\hat{k}_c = 506{,}9 + 2{,}50.10^{-2}\,\delta_o^{2{,}31}$	1324	507
6.	Aluminum alloy AlCu4.5Mn0.5Mg1.6	$\hat{k}_c = 472{,}7 + 2{,}14.10^{-1}\,\delta_o^{1{,}68}$	883	473

❖ INVESTIGATING THE ACCURACY OF THE EMPIRICAL MATHEMATICAL MODELS

The research objective is to evaluate the precision of some used empirical models, related to the specific cutting force and the rake angle, as mentioned in the reference data, given in [10], [12], [7]. The models are marked respectively as SF, SK and CTM. The proposed new model of this dependency is used as a basis for this assessment (table 4.), applying the methodology, given in [3].

The deviations of the empirical models, according to reference data, from the proposed new model, are estimated by the following formula:

$$\Delta k_{c\gamma} = \frac{k_{c\gamma} - k_{co\gamma}}{k_{co\gamma}} \cdot 100\% \qquad (7)$$

where $k_{c\gamma}$ and $k_{co\gamma}$ are correspondingly correction coefficients, accounting the influence of the rake angle, respectively according to reference data and the new model.

The deviations dependancies $\Delta k_{c\gamma}$, calculated by the formula (7), from the rake angle according to model SF- formula (1), for the steel types Steel AISI W1-1.0C and Steel ASTM5140 and for the cast iron types Grey iron GG15 and Grey iron GG25, are given on fig. 3. The deviations of this model are considerable:

$\Delta k_{c\gamma} = -25{,}0 \div 6{,}7$ % for steel type Steel AISI W1-1.0C;
$\Delta k_{c\gamma} = -20{,}9 \div 10{,}7$ % for steel type Steel ASTM5140;
$\Delta k_{c\gamma} = -30{,}9 \div 1{,}2$ % for Grey iron GG15;
$\Delta k_{c\gamma} = -33{,}2 \div 4{,}6$ % for Grey iron GG25.

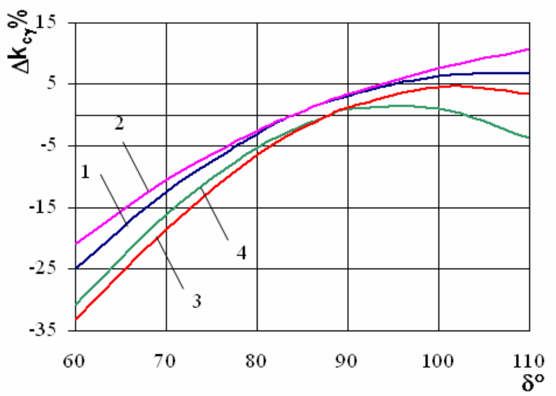

Fig. 3. Deviation dependency $\Delta k_{c\gamma}$ from the cutting angle δ, according to the SF-model for the different processed materials, as follows: 1 - Steel AISI W1-1.0C, 2- Steel ASTM5140, 3 - Grey iron GG15, 4 - Grey iron GG25

The deviations of the SK and CTM models are considerably smaller – for steel and cast iron types they reach up to 12%, for Bronze CuSn7P0.7 $\Delta k_{c\gamma} = -16{,}6 \div 9{,}7$ %, and for aluminum alloy AlCu4.5Mn0.5Mg1.6 they are inconsiderable ($\Delta k_{c\gamma} = -6{,}5 \div 1{,}2$ %).

❖ CONCLUSIONS

The following conclusions can be made from the investigations carried out:

a. A condition is defined from physical considerations, which is to be met by the mathematical models of approximating the dependency of the specific cutting force from the rake angle. It consists of introducing a new parameter - the initial specific force, regarding the rake angle. The physical meaning of this parameter is that it is obtained only from the friction forces on the front and rear surface.

b. Hypothetical graphical dependencies $k_c = f(\delta_o)$ and hypothetical mathematical models for their approximation are offered, meeting the defined condition.

c. New mathematical models for approximating the dependency of the specific cutting force from the cutting angle when turn lathing of different processed materials are obtained through experimental investigations and mathematical processing of the experimental data. A mathematical model is recommended as the best one, related to structure, adequacy and accuracy.

d. The investigations carried out on the empirical mathematical models, according to reference date and in comparison to the offered new model for the specific force, it was determined that in some cases the deviations are inconsiderable, but in other cases they can reach up to 20-30%. This is why when estimating the cutting forces, applying such models, considerable errors can be made.

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