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## THE ENERGY SPECTRUM OF THE COMBINED SYSTEMS

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**ABSTRACT:** In the article are mentioned combined discrete dynamic systems and analysis of their energy spectrum especially aimed to the event activated part of the system. The fusion result of time-driven and event activated systems are combined dynamic systems, called also hybrid systems. System consists of a combination of cyclically recurring processes in each period of processing and the stochastically emerging events. This theorem assumes stochastic events generation corresponding to the Poisson distribution. The energy picture of stochastic events generated by the Poisson distribution in the combined systems is created using the analogy from electrical engineering associated with telegraph signal transmission.

**KEYWORDS:** combined systems, events, Poisson distribution, probability, density distribution

### ❖ INTRODUCTION – PURPOSE OF THE ARTICLE

In the article are mentioned combined discrete dynamic systems and analysis of their energy spectrum especially aimed to the event activated part of the system. This theorem assumes stochastic events generation corresponding to the Poisson distribution.

The energy picture of stochastic events generated by the Poisson distribution in the combined systems is created using the analogy from electrical engineering associated with telegraph signal transmission.

### ❖ COMBINED DISCRETE SYSTEMS

Combined systems include two parts of discrete systems: discrete time-dynamic-activated systems, as well as discrete events systems. The fusion result of time-driven and event activated systems are combined dynamic systems, called also hybrid systems. In case that notion “Hybrid system” is used in connection perhaps between distributed control system and programmable logical controllers, neural networks, genetic algorithms and fuzzy logics, or combination of electric and mechanical power units (Ždánky, 2009), for better prediction and better identification was introduced the concept of “Combined dynamic system”.

This system consists of a combination of cyclically recurring processes in each period of processing and the stochastically emerging events. These events affect the management and substantially influence the selection of a suitable sampling period. Sampling period must be designed so all necessary control instructions and utilities, which are responding to emerging events, are processed. Given the sampling period, combined dynamical systems are classified into 2 groups (Strémy, 2010):

- a) with the constant sampling period - when setting its value, it is necessary to determine the ratio of ingredient event  $T_p$  and the component of a time-driven system  $T_R$
- b) with changing sampling period in which it is necessary to determine the impact, respectively the relationship between the constant component (time-controlled system) and random event component (various depending on emerging events).

Event component of the sampling period significantly affects the behavior of the entire system. It affects controllability, stability and overall dynamic properties of the combined systems that are designed by ratio of events and time-controlled discrete systems.

Time-controlled part of the combined discrete systems is realized by a constant cyclic monitoring, processing and evaluation of inputs and states of the system. Event part is in automated control systems implemented by service interruptions, called upon the occurrence of any of the events. A particular problem in such systems appears to be how to determine the impact of stochastically changing event component of their dynamic properties and quality control (Strémy, 2010).

❖ ENERGY SPECTRUM OF THE COMBINED SYSTEMS

The energy picture of stochastic events generated by the Poisson distribution in the combined systems is created using the analogy from electrical engineering associated with telegraph signal transmission (ЛЕВИН, 1960).

Signal (Fig.1) represents events generated with amplitude  $S(t)$  over time  $t$ . In the case of interrupts detection, which represents events generated in the system, the signal amplitude changes to the  $h$  value and stays unchanged during the operation period of the utilities associated with stochastic events. Every new event in the system is stored in an events buffer, in which all events waiting for utilities operations are sorted. The sequence of utilities operations in the events buffer is affected by the time of event generation and by the priority associated to individual interrupts. If there was not any new interrupt detected in the system, respectively there is no new event in the events buffer, the signal amplitude stabilizes itself on the 0 value. The utilities operation period in the given moment is represented by the time period  $t_u$ . If there was not any new event detected in the system, the periodical operations of the time activated part of combined systems takes place.

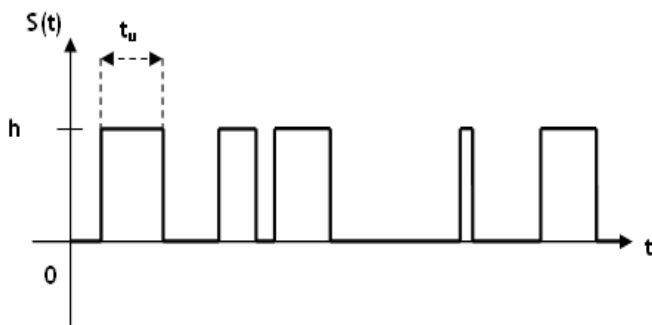


Figure 1. The events signal in the combined systems

This theorem assumes stochastic events generation corresponding to the Poisson distribution (Fig.2). If the mean value of the events generation is  $\nu$ , then the product of  $\nu t$  represents the mean value of the events generation over time  $t$ . The following formula is used for calculation of the Poisson mean parameter, which is the mean value of the events generation in an arbitrary time interval  $\langle t_1, t_2 \rangle$ :

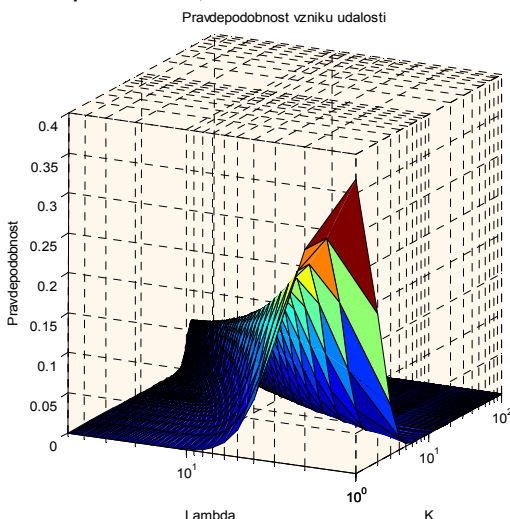


Figure 2. The Poisson distribution of probability

$$\lambda = \int_{t_1}^{t_2} \nu(t) dt$$

The autocorrelation function of the presented signal between the two discrete moments  $t_1, t_2$  corresponds to the formula:  $K(t_1, t_2) = h^2 e^{-2 \int_{t_1}^{t_2} \nu(t) dt}$

If the signal is stationary, which means  $\nu(t) = \nu_0 = const$ , then the correlation function depends only from the difference of these two discrete moments  $\tau = t_2 - t_1$ , and the formula is simplified:  $K(\tau) = h^2 e^{-2\nu_0|\tau|}$

Based up on the autocorrelation function could be the energetic spectrum of the stationary stochastic signal calculated using the Wiener-Chinchin equation:

$$F(\omega) = 4h^2 \int_0^{\infty} e^{-2\nu_0\tau} \cos \omega\tau d\tau = \frac{8h^2\nu_0}{\omega^2 + 4\nu_0^2}$$

The representation of the autocorrelation function and its spectral density of the energy distribution are shown in Fig. 3.

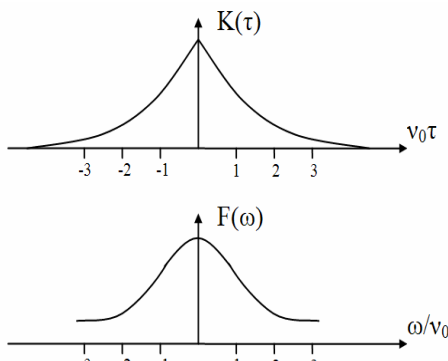


Figure 3. Autocorrelation function  $K(\tau)$  and corresponding spectral density  $F(\omega)$

The event generation probability in the system is assumed instead of signal amplitude. In the case of event detection in the system and consequent utilities are processed, the variable  $h$  is equal to 1. After substitution  $h=1$  into the autocorrelation function, we get the following formula:

$$K(\tau) = e^{-2\nu_0|\tau|}$$

and its representation for different values of  $\nu_0$  changes in the dependence shown in Fig. 4.

After substituting the considered probability  $h$  into the original formula of energy density distribution, the formula for energy density distribution of the stochastic signal could be written as follows:

$$F(\omega) = \frac{8\nu_0}{\omega^2 + 4\nu_0^2}$$

Graphical representation of the probability distribution density with the mean value  $\lambda$  and the events count  $k$  is shown in Fig. 5.

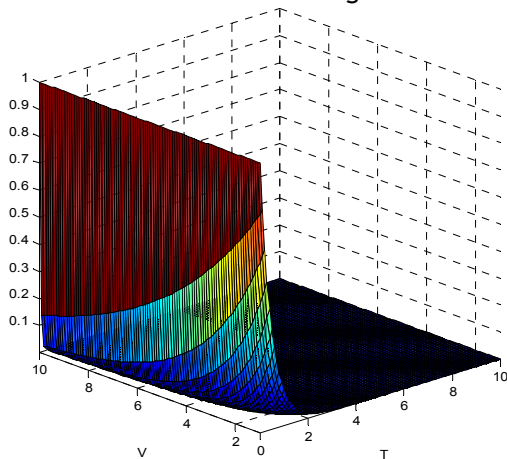


Figure 4. Autocorrelation function  $K(\tau)$  for different values of  $\nu_0$

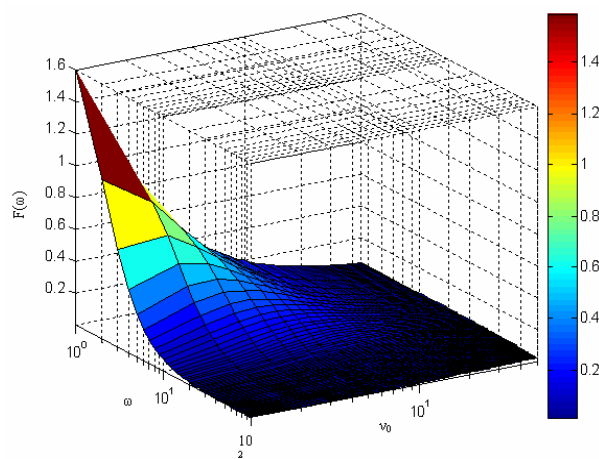


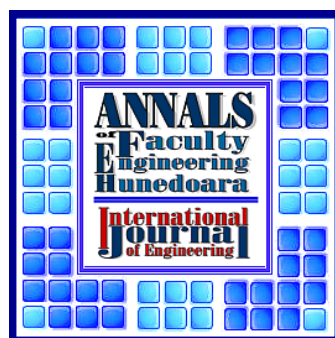
Figure 5. The probability distribution density  $F(\omega)$

#### ❖ CONCLUSIONS

From the viewpoint of the analysis and synthesis is important to know the dynamic properties of the combined systems. With application of the normal probability distribution to the events part of the combined systems can be described static behavior of the system, while using the Poisson distribution laws and energy spectrum can be captured dynamics of the probability of the events generation in the system.

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