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MHD FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF CHEMICAL REACTION

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ABSTRACT: *Theoretical solution of hydromagnetic effects on flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of chemical reaction of first order has been studied. The plate temperatures as well as concentration near the plate are raised linearly with respect to time t. The plate is exponentially accelerated with a prescribed velocity against gravitational field. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration profiles are studied for different physical parameters like magnetic field parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, time and a. It is observed that the velocity decreases with increasing magnetic field parameter or chemical reaction parameter.*
KEYWORDS: *Accelerated, isothermal, vertical plate, exponential, heat & mass transfer, chemical reaction, magnetic field*

INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil,and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conduction fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al (1979). MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al (1981). MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh (1983). The dimensionless governing equations were solved using Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Gupta (1960) studied flow of an electrically conducting fluid near an uniformly accelerated vertical plate in the presence of uniform magnetic field. Raptis et al (1981) discussed MHD flow past an accelerated vertical plate with variable suction and heat flux. Singh (1984) analyzed MHD flow past an exponentially accelerated vertical plate with uniform temperature.

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young (1958) analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al. (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. (1999). The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Hence it is proposed to study the effects on unsteady MHD flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL FORMULATION

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform mass diffusion in the presence of magnetic field has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis taken normal to the plate. At time $t' \leq 0$ the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane against gravitational field. The temperature from the plate as well as concentration near the plate is made to increase linearly with respect to time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - k_1 C' \tag{3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

The prescribed initial and boundary conditions are

$$\begin{array}{llll} u=0, & T=T_\infty, & C'=C'_\infty & \text{for all } y, t'\leq 0 \\ t'>0: u=u_0\exp(a't'), & T=T_\infty+(T_w-T_\infty)At', & C'=C'_\infty+(C'_w-C'_\infty)At' & \text{at } y=0 \\ u\to 0, & T\to T_\infty, & C'\to C'_\infty & \text{as } y\to\infty \end{array} \tag{4}$$

where, $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$\begin{gathered} U=\frac{u}{u_0},\ t=\frac{t'u_0^2}{\nu},\ Y=\frac{yu_0}{\nu},\ \vartheta=\frac{T-T_\infty}{T_w-T_\infty}, \\ Gr=\frac{g\beta\nu(T_w-T_\infty)}{u_0^3},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3}, \\ Pr=\frac{\mu C_p}{k},\ a=\frac{a'\nu}{u_0^2},\ Sc=\frac{\nu}{D},\ M=\frac{\sigma B_0^2\nu}{\rho u_0^2},\ K=\frac{Vk_1}{u_0^2} \end{gathered} \tag{5}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\vartheta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \tag{6}$$

$$\frac{\partial \vartheta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \vartheta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The negative sign of K in the last term of the equation (8) indicates that the chemical reaction takes place from higher level of concentration to lower level of concentration.

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llll} U=0, & \vartheta=0, & C=0 & \text{for all } Y,\ t\leq 0 \\ t>0:\ U=\exp(at), & \vartheta=t, & C=t & \text{at } Y=0 \\ U\to 0, & \vartheta\to 0, & C\to 0 & \text{as } Y\to\infty \end{array} \tag{9}$$

SOLUTION PROCEDURE

The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\vartheta = t\left[(1+2\eta^2 Pr)erfc(\eta\sqrt{Pr}) - \frac{2}{\sqrt{\pi}}\eta\sqrt{Pr}\,exp(-\eta^2 Pr)\right] \tag{10}$$

$$\begin{aligned} C = {} & \frac{t}{2}\left[exp(2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt}) + exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\ & -\frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt}) - exp(2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt})\right] \end{aligned} \tag{11}$$

$$\begin{aligned} U = {} & \frac{exp(at)}{2}\left[exp(2\eta\sqrt{(M+a)t})\,erfc(\eta+\sqrt{(M+a)t}) + exp(-2\eta\sqrt{(M+a)t})\,erfc(\eta-\sqrt{(M+a)t})\right] \\ & +[(e+d)+(ec+bd)t]\left[exp(2\eta\sqrt{Mt})\,erfc(\eta+\sqrt{Mt}) + exp(-2\eta\sqrt{Mt})\,erfc(\eta-\sqrt{Mt})\right] \\ & -\frac{(ec+bd)\eta\sqrt{t}}{\sqrt{M}}\left[exp(-2\eta\sqrt{Mt})\,erfc(\eta-\sqrt{Mt}) - exp(2\eta\sqrt{Mt})\,erfc(\eta+\sqrt{Mt})\right] \\ & -d\,exp(bt)\left[exp(2\eta\sqrt{(M+b)t})\,erfc(\eta+\sqrt{(M+b)t}) + exp(-2\eta\sqrt{(M+b)t})\,erfc(\eta-\sqrt{(M+b)t})\right] - 2d\,erfc(\eta\sqrt{Pr}) \\ & -e.exp(ct)\left[exp(2\eta\sqrt{(M+c)t})\,erfc(\eta+\sqrt{(M+c)t}) + exp(-2\eta\sqrt{(M+c)t})\,erfc(\eta-\sqrt{(M+c)t})\right] \\ & -2bdt\left[(1+2\eta^2 Pr)erfc(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}exp(-\eta^2 Pr)\right] \\ & +d\,exp(bt)\left[exp(2\eta\sqrt{Pr\,bt}).erfc(\eta\sqrt{Pr}+\sqrt{bt}) + exp(-2\eta\sqrt{Pr\,bt}).erfc(\eta\sqrt{Pr}-\sqrt{bt})\right] \\ & -(1+ct)e\left[exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}+\sqrt{Kt}) + exp(-2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}-\sqrt{Kt})\right] \\ & +\frac{ce\eta\sqrt{Sct}}{\sqrt{K}}\left[exp(-2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}-\sqrt{Kt}) - exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc}+\sqrt{Kt})\right] \\ & +e\,exp(ct)\left[exp(2\eta\sqrt{Sc(K+c)t})\,erfc(\eta\sqrt{Sc}+\sqrt{(K+c)t}) + exp(-2\eta\sqrt{Sc(K+c)t})\,erfc(\eta\sqrt{Sc}-\sqrt{(K+c)t})\right] \end{aligned} \tag{12}$$

where $b = \frac{M}{Pr-1}$, $c = \frac{M-KSc}{Sc-1}$, $d = \frac{Gr}{2b^2(1-Pr)}$, $e = \frac{Gc}{2c^2(1-Sc)}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

Discussion Of Results

For physical understanding of the problem numerical computations are carried out for different physical parameters M, a, Gr, Gc, K, Sc and t upon the nature of the flow and transport. The value of Prandtl number Pr is chosen such that they represent water (Pr = 7.0). The numerical values of the velocity are computed for different physical parameters like magnetic filed parameter, chemical reaction parameter, a, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profile for different values of time (t = 0.2, 0.4, 0.6, 0.8), Sc = 0.6, Gr = Gc = 5, M = 2, K = 5 and a = 1 are shown in Figure 1. It is observed that velocity increases with increasing values of the time.

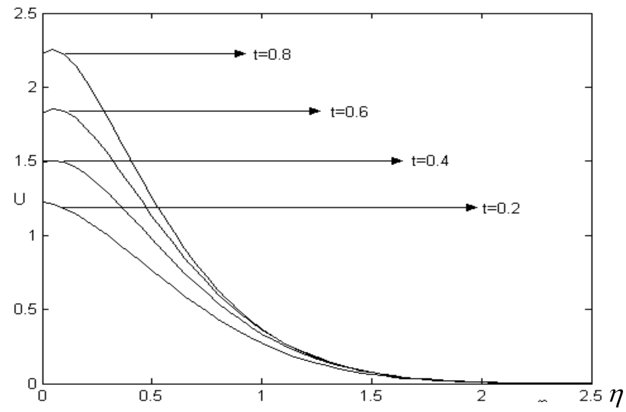

Figure 1. The velocity profile for different values of time

Figure 2 demonstrates effects of the magnetic field parameter on the velocity when M = (0.2, 2, 5), Sc = 0.6, Gr = Gc = 5, t = 0.2, K = 5 and a = 0.1. It is observed that velocity increases with decreasing magnetic field parameter.

The velocity profile for different values of a = (0.2, 0.5, 1), Sc = 0.6, Gr = Gc = 5, M = 2, K = 5 and t = 0.2 are studied and presented in Figure 3. It is observed that the velocity increases with decreasing values of a. Figure 4 represents the effect of concentration profiles for different values of t = (0.2, 0.4, 0.6), K = 3 and Sc = 0.6. It is observed that the concentration increases with increasing values of time 't'.

Figure 5 demonstrates the velocity profiles for different values of thermal Grashof number Gr = (2, 5, 8), mass Grashof number Gc = (5, 10, 15), t = 0.2, Sc = 0.6, M = 2 and a = 1. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

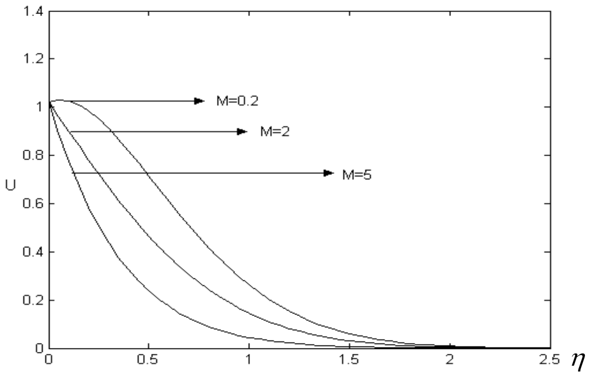

Figure 2. The velocity profile for different values of M

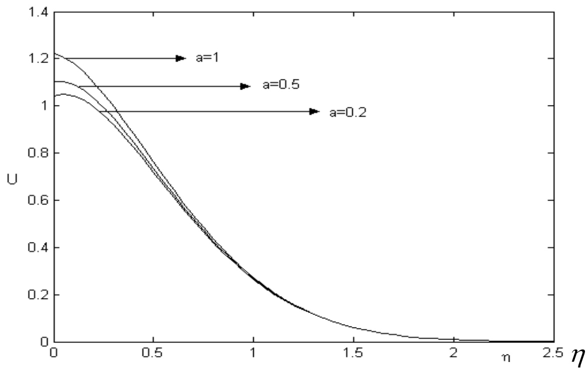

Figure 2. The velocity profile for different values of a

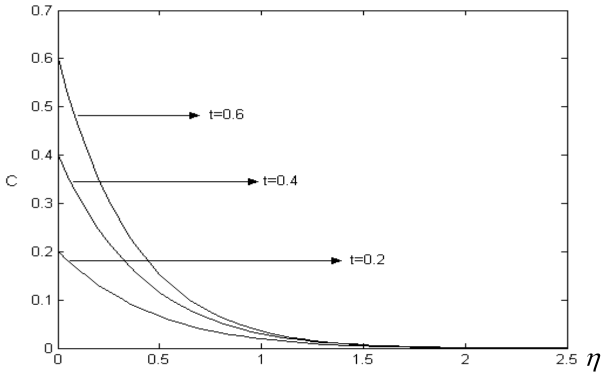

Figure 4. Concentration profiles for different values of time

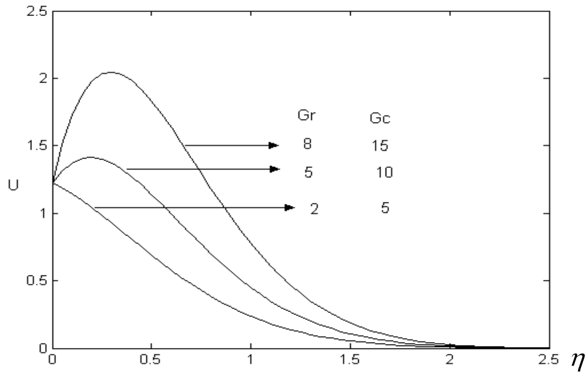

Figure 5. Velocity profiles for different values of Gr, Gc

CONCLUSIONS

An exact analysis of hydromagnetic flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number and t are studied graphically. The conclusions of the study are as follows:

(i) The velocity decreases with increasing magnetic field parameter M (or) chemical reaction parameter.

(ii) The velocity increases with increasing values of thermal Grashof number (or) mass Grashof number and time t in the presence of magnetic field parameter.

(iii) The wall concentration increases with increasing time t.

(iv) The temperature increases with increasing values of time 't'.

(v) The effect of heat transfer more in the presence of air than in water.

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